Coherent Coulomb excitation of relativistic nuclei in aligned crystal targets

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We study coherent Coulomb excitation of ultrarelativistic nuclei passing through the aligned crystal target. We develop multiple scattering theory description of this process which consistently incorporates both the specific resonant properties of particle-crystal interactions and the shadowing effect typical of the diffractive scattering. We emphasise that the effect of quantum mechanical diffraction makes the physics of ultrarelativistic nuclear excitations entirely different from the physics of non-relativistic atomic excitations experimentally studied so far. It is found that at small transverse momenta $q_\perp$ the shadowing effect drastically changes the dependence of coherent amplitudes on the crystal thickness L, from the widely discussed growth $\propto L$ typical of the Born approximation to the inverse thickness attenuation law. At relatively large $q_\perp$ no attenuation effect is found but the coherency condition is shown to put stringent constrain on the growth of the transition rate with growing L.

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1. There were several proposals of experiments on the coherent Coulomb excitation of relativistic nuclei in crystal target [1 – 4]

$$A\gamma \to A^*. \tag{1}$$

The central idea of [1 – 4] is that in a nucleus (the ground state $|0\rangle$ and the excited state $|1\rangle$) under periodical perturbation $V \sin \nu t$ with the frequency ν equal to the level splitting $\Delta E = E_1 - E_0$ there develop quantum beats with the oscillation frequency $\omega = \langle 1|V|0\rangle$. If the perturbation V is weak, then for $\omega t \ll 1$ the $|0\rangle \to |1\rangle$ transition probability $P(t)$ increases rapidly with the time $P(t) \propto \omega^2 t^2$. If one could subject nuclei to a high frequency field, $\nu \simeq \Delta E$, the rates of transition can be enhanced substantially. The high monochromaticity is an evident condition to sustain the fast growth of $P(t)$ over large time scale. It has been suggested in [1 – 4] that all of the above requirements are met best in Coulomb interaction of a high-energy nucleus propagating in a crystal along the crystallographic axis. Here the rôle of "time" is played by the crystal thickness L. For ultrarelativistic particles ν is enhanced due to the Doppler shift, $\nu' = \gamma\nu$, where γ is the Lorentz factor. It is the Lorentz factor which can boost ν to the hundreds keV's range.

In [1 – 4] the Coulomb field of a crystal was evaluated in the Weizsäker-Williams approximation, and then applied to the calculation of the transition amplitude in the plane wave Born approximation. It was claimed [1, 2, 4] that the law $P(N) \propto N^2$, holds up to the crystal thicknesses $N = L/a \sim 10^4 - 10^5$ (hereafter a stands for the lattice spacing). However, in a consistent treatment of coherent $A \to A^*$ transitions one needs to include distortions due to the initial state Coulomb interactions of the nucleus A and final state Coulomb interactions of the excited nucleus A^*.

In this paper we show that at small transverse momenta the effect of these interactions (multiple diffractive scattering off atomic row) is reminiscent the well known Glauber-Gribov shadowing effect [5, 6] and entails an attenuation of the coherent excitation amplitudes with growing crystal thickness. The early discussion of shadowing in the total cross section of elastic high-energy particle-crystal scattering is found in [7].

Here we emphasise that it is the diffraction phenomenon which makes the physics of ultrarelativistic nuclear excitations with $\gamma \sim 10$ entirely different from the physics of coherent excitation of atoms [8, 9], where typically $\gamma \ll 1$.

It should be noticed, however, that in contrast to the high-energy hadronic scattering which is described by predominantly imaginary amplitudes, the shadowing in Coulomb scattering does not imply a simple "absorption", but a redistribution of scattered nuclear waves in the phase space. Therefore the depletion of the domain of small-$q_\perp$ means the enhancement of the large-$q_\perp$ region. At relatively large transverse momenta no shadowing is found but the nuclear waves scattered by different atoms get out of tune easily. Here the structure factor of crystal puts stringent constrain on the coherent

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excitation rate which at the projectile energies available would not exhibit any sizable enhancement effect.

2. Consider the small-angle Coulomb scattering of ultrarelativistic nucleus (the mass number A, the charge Z_1 and the four-momentum p) moving along a crystal axis. The projectile-nucleus undergoes a correlated series of soft collisions which give rise to diagonal ($A \to A$, $A^* \to A^*$) and off-diagonal ($A \to A^*$, $A^* \to A$) transitions. The interatomic distances, $a \sim 3 - 5$ Å, are large, compared to the Thomas-Fermi screening radius $r \simeq 0.468 Z_2^{-1/3}$ Å, where Z_2 is the atomic number of the target atom and $\alpha = 1/137$. The amplitude of thermal vibrations of the lattice $\langle u^2\rangle^{1/2}$ estimated from the Debye approximation equals to $\langle u^2\rangle^{1/2} \sim (0.05 - 0.1)$ Å for most commonly studied crystals at room temperature [10]. The relevant impact parameters, b, satisfy as we shall see $b \ll a$. This implies that the amplitudes of scattering by different atomic strings parallel to a given crystallographic axis are incoherent.

In [3, 1, 2] it has been proposed to study the electric dipole transition in ^{19}F, the excitation of the state $|J^\pi = 1/2^-\rangle$ from the ground state $|1/2^+\rangle$. Let us accept this proposal and consider the phenomenological matrix element of the transition $1/2^+ \to 1/2^-$

$$\mathcal{M} = d\bar{u}(p')\gamma_5\sigma_{\mu\nu}u(p)q^\nu\varepsilon^\mu, \tag{2}$$

where both $u(p')$ and $u(p)$ are bispinors of initial and final states of the projectile-nucleus. By definition, $\sigma_{\mu\nu} = \frac{1}{2}[\gamma_\mu, \gamma_\nu]$, d is the transition dipole moment, $p' = p + q$, $\mathbf{q} = (\mathbf{q}_\perp, \kappa)$, $\mathbf{q}_\perp = (q_\perp\cos\phi, q_\perp\sin\phi)$ and ε is the photon polarisation vector. Then after a series of quite standard high-energy approximations one readily finds the helicity-flip Born amplitude $t_B(\mathbf{q}_\perp)$ of the transition $1/2^+ \to 1/2^-$ in the nucleus-atom collision

$$t_B(\mathbf{q}_\perp) = \sqrt{\alpha} d Z_2 \frac{q_\perp e^{i\phi}}{q_\perp^2 + \lambda^2}, \tag{3}$$

where $\lambda^2 = \mu^2 + \kappa^2$ and $\mu = r^{-1}$. Denoted by κ is the longitudinal momentum transfer

$$\kappa = M\Delta E + q_\perp^2/2p, \tag{4}$$

which determines the coherency length $l_c \sim \kappa^{-1}$. In eq.(4) M is the mass of projectile-nucleus. The excitation energy is $\Delta E \simeq 110$ keV. Normalisation of amplitudes is such that

$$d\sigma/dq_\perp^2 = |t(\mathbf{q}_\perp)|^2. \tag{5}$$

The integral $\sigma_B = \int dq_\perp^2 |t_B(\mathbf{q}_\perp)|^2$ diverges at large $q_\perp$ and need be regularised. The natural regulator is the inverse amplitude of thermal vibrations of the lattice.

Making use of the impact parameter representation simplifies the summation of diagrams of multiple Coulomb scattering. Then, in the eikonal approximation the full nucleus-atom amplitude to all orders in $\alpha Z_1 Z_2$ reads

$$t(\mathbf{q}_\perp) = \sqrt{\alpha} d Z_2 \lambda e^{i\phi} \int b db J_1(q_\perp b) K_1(\lambda b) \exp[i\chi(b)], \tag{6}$$

where $J_1(x)$ and $K_{0,1}(x)$ are the Bessel functions and the screened Coulomb phase shift function is

$$\chi(b) = -\beta K_0(\mu b), \tag{7}$$

where $\beta = 2\alpha Z_1 Z_2$.

The only phenomenological parameter of eq.(2), dipole moment of the $1/2^+ \to 1/2^-$ transition, denoted by d, can be determined from the width Γ of the 110 keV level $^{19}F(1/2^-)$ which is $\Gamma = d^2\Delta E^3/\pi + \mathcal{O}(\Delta E/M)$. Then the measured life-time $\tau = \Gamma^{-1} = (0.853 \pm 0.010) \cdot 10^{-9}$ s [11] yields $d \simeq 4.3 \cdot 10^{-8}$ keV^{-1}. Two useful conclusions can be drawn immediately. First, because of large value of τ the decay of excited state inside the target can be safely neglected. Second, due to the smallness of d, the excitation amplitude is much smaller than the elastic Coulomb amplitude for all $q_\perp$ up to $q_\perp \sim \sqrt{\alpha} Z_1/d$ and can be considered as a perturbation. Thus the multi-channel problem reduces to the one-channel one.

Then, in the static lattice approximation the evaluation of the full transition amplitude on a string of N identical atoms reads [12]

$$T(\mathbf{q}_\perp) = \sqrt{\alpha} d Z_2 \lambda S(q_\perp) e^{i\phi} I(q_\perp), \tag{8}$$

where

$$I(q_\perp) = \int b db\, J_1(q_\perp b) K_1(\lambda b) \exp[-i\beta N K_0(\mu b)]. \tag{9}$$

In eq.(8) $S(q_\perp)$ is the structure factor of the lattice

$$S(q_\perp) = \frac{\sin(\kappa N a/2)}{\sin(\kappa a/2)}, \tag{10}$$

and κ is given by eq.(4).

Split integration over b in eq.(9) into the two domains: $\mu^{-1} \lesssim b \lesssim a$ and $0 < b < \mu^{-1}$. In the former domain it suffices to use the asymptotics $K_{0,1}(x) \sim \exp(-x)$ which upon the slight readjustment of screening parameters, $\mu \to \mu' = \mu(1 + \frac{1}{2}\log(2\xi/\pi))$ and $\lambda \to \lambda' = \mu(1 + \frac{1}{2}\log(2\xi/\pi))$ at the relevant $b \sim \mu^{-1}\xi$ proves numerically very accurate. Hereafter, $\xi = \log(\beta N/\Delta)$ and $\Delta = \lambda/\mu = \sqrt{1 + \kappa^2/\mu^2}$. Then, the steepest descent from the saddle-point at

$$b_0 = \mu^{-1}(\xi + i\pi/2),$$

yields for $\xi^2 \gg \mu/\lambda$ and $q_\perp \lesssim \mu\xi^{-1}$

$$I(q_\perp) \simeq \frac{q_\perp}{\mu^3}\sqrt{\frac{2\pi}{\Delta}}e^{-\Delta}\log^2\left(\frac{i\beta N}{\Delta}\right)\left(\frac{\Delta}{i\beta N}\right)^{\Delta} \propto$$

$$\propto \log^2 N\left(\frac{1}{N}\right)^{\Delta}. \qquad (11)$$

Consequently, for small transverse momenta $q_\perp \lesssim \mu\xi^{-1}$ as soon as $\xi \gg 1$ which holds for all practical purposes one has the attenuation of the coherent excitation amplitude.

For higher transverse momenta $q_\perp \gg \mu\xi^{-1}$ making use of the stationary phase approximation with the above reservation about the substitution $\mu \to \mu'$ and $\lambda \to \lambda'$ yields

$$I(q_\perp) \simeq \frac{\sqrt{\eta}}{\mu q_\perp}\exp(-\Delta\eta)\exp\left[-i\frac{q_\perp}{\mu}(\eta+1)\right] \propto$$

$$\propto \sqrt{\log N}\left(\frac{1}{N}\right)^{\Delta}, \qquad (12)$$

where $\eta = \log(\mu\beta N/q_\perp) \gg 1$. As before we find no enhancement but the attenuation of the coherent transition amplitude. Indeed, let the projectile momentum satisfies the resonance condition [1 – 4]

$$M\Delta E/p = 2\pi n/a\,, \;\; n = 0,\, 1,\, 2... \qquad (13)$$

In [1, 2, 4] it has been suggested to look for the coherent transitions in the W-crystal at $n = 3$. This regime corresponds to $\gamma \simeq 10$ and $\Delta = \lambda/\mu \simeq \sqrt{1 + 4\pi^2 n^2/(a\mu)^2} \simeq 1.2$. Still higher n discussed in [1 – 4] would correspond to higher momentum transfers and to much stronger suppression of the coherent excitation amplitude.

Now consider the contribution to $I(q_\perp)$ from the second domain, $0 < b < \mu^{-1} \equiv r$. The region of small impact parameters is affected by the lattice thermal vibrations which are known to suppress the coherent amplitude. Let us make use of the fact that for some commonly used crystals at a room temperature the root-mean-square one-dimensional displacement u is such that $u \ll r$. For example, for the diamond lattice suggested as a target in [3] $u/r \simeq 0.16$ [10]. The integral in $r.h.s.$ of eq.(9) reads

$$I(q_\perp) = \int_0^r bdb\, J_1(q_\perp b)K_1(\lambda b)\exp[-i\beta N K_0(\mu b)] \simeq$$

$$\simeq \frac{1}{\lambda q_\perp}\left(\frac{\mu}{q_\perp}\right)^{i\beta N}\int\limits_0^{rq_\perp} dz J_1(z) z^{i\beta N}. \qquad (14)$$

The contribution to $I(q_\perp)$ non-vanishing with growing N comes from $z \sim z_0 = \beta N$, where z_0 is the point of stationary phase. Hence, the requirement $q_\perp \gg \mu\beta N$. The width, $\delta z \simeq \sqrt{\beta N}$, of the region, the major contribution to (14) comes from, is much smaller than the interval of integration in (14) which is $\Delta z \gg \beta N$. Then, for $\beta N \gg 1$ one has

$$I(q_\perp) \simeq \frac{1}{\lambda q_\perp}\exp\left[-i\beta N\log\frac{q_\perp}{2\mu} + 2i\varphi\right], \qquad (15)$$

where

$$\exp[2i\varphi] = \frac{\Gamma(1 + i\beta N/2)}{\Gamma(1 - i\beta N/2)}. \qquad (16)$$

Thus, for $q_\perp \gg \mu\beta N$ one finds no attenuation effect. However, at large $q_\perp$ the structure factor of crystal (10) enters the game. The resonance condition (13) implies fine tuning of phases of scattered waves. If $q_\perp$ becomes large the phases get out of tune easily. At large projectile momentum p the structure factor (10) allows variation of $q_\perp^2$ within a rather wide band which is however $\propto N^{-1}$. In the neighbourhood of the resonance

$$S(q_\perp) \simeq N[1 - B^2 q_\perp^4], \qquad (17)$$

where $B = aN/4\sqrt{6}p$. Hence, the excitation cross section $\sigma = \int dq_\perp^2 |T(\mathbf{q}_\perp)|^2$ which varies as

$$\sigma \propto N^2 \log(N_c/N), \qquad (18)$$

where

$$N_c \ll N_n = \left[\frac{4\sqrt{6}p_n}{a\mu^2\beta^2}\right]^{1/3} \qquad (19)$$

and $p_n = aM\Delta E/2\pi n$. For example, for the W crystal $N_3 \simeq 30$ and for the diamond target crystal one has $N_3 \simeq 300$. With certain reservations about the effect of lattice thermal vibrations, one can conclude that not only the $q_\perp$-dependence of the coherent transition amplitude differs dramatically from the early predictions [2, 3], but the effect of the coherent enhancement is much weaker than that predicted in [1, 2, 4].

Notice, that for $N \ll N_n$ one has

$$p/q_\perp^2 \gg L \qquad (20)$$

and the straight paths approximation which we rely upon in our analysis holds true.

We conclude that in quantitative analysis of high-energy particle-crystal interactions due allowance must be made to the multiple scattering effects which dramatically change the pattern of the coherent Coulomb excitation compared to widely used approaches based on the Born approximation.

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1. V. V. Okorokov and S. V. Proshin, *Investigation of the coherent excitation of the relativistic nuclei in a crystal*, Moscow, ITEP-13-1980.
2. Yu. L. Pivovarov, A. A. Shirokov, and S. A. Vorobev, Nucl. Phys. **A509**, 800 (1990); Yu. L. Pivovarov and A. A. Shirokov, Sov. J. Nucl. Phys. **37**, 653 (1983).
3. R. Fusina and J. C. Kimball, Nucl. Instrum. Meth. **B33**, 77 (1988).
4. V. V. Okorokov, Yu. L. Pivovarov, A. A. Shirokov, and S. A. Vorobev, *Proposal of experiment on coherent excitation of relativistic nuclei in crystals*, Moscow, ITEP-90-49, Fermilab Library.
5. R. J. Glauber, in *Lectures in Theoretical Physics*, Eds. W. E. Brittin et al., Interscience Publishers, Inc., New York, vol.1,1959, p. 315.
6. V. N. Gribov, Sov. Phys. JETP **29**, 483 (1969); **30**, 709 (1970).
7. N. P. Kalashnikov, E. A. Koptelov, and M. I. Ryazanov, Sov. Phys. JETP **63**, 1107 (1972); N. P. Kalashnikov and V. D. Mur, Sov. J. Nucl. Phys. **16**, 613 (1973).
8. V. V. Okorokov, JETP Lett. **2**, 111 (1965); Sov. J. Nucl. Phys. **2**, 719 (1966).
9. V. V. Okorokov, D. L. Tolchenkov, Yu. P. Cheblukov et al., JETP Lett. **16**, 415 (1972); Phys. Lett. **A43**, 485 (1973); M. J. Gaillard, J. C. Poizat, J. Remillieux, and M. L. Gaillard, Phys. Lett. **A45**, 306 (1973); H. G. Berry, D. S. Gemmel, R. E. Holland et al., Phys. Lett. **A49**, 123 (1975); M. Mannami, H. Kudo, M. Matsushita, and K. Ishii, Phys. Lett. **A64**, 136 (1977); S. Datz, C. M. Moak, O. H. Crawford et al., Phys. Rev. Lett. **40**, 843 (1987); C. M. Moak, S. Datz, O. H. Crawford et al., Phys. Rev. **A19**, 977 (1979); F. Fujimoto, Nucl. Instr. Methods **B40/41**, 165 (1989); Y. Iwata, K. Komaki, Y. Yamazaki et al., Nucl. Instr. Methods **48**, 163 (1990).
10. D. S. Gemmel, Rev. Mod. Phys. **46**, 129 (1974).
11. F. Ajzenberg-Selove, Nucl. Phys. **A190**, 1 (1972).
12. V. R. Zoller, JETP Lett. **64**, 788 (1996).