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Coherent final state interaction in jet production in nucleus-nucleus collisions

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We study the coherent final state interaction of an energetic parton produced in AA collisions caused by the change in the cutoff scale and running coupling constant from the vacuum to quark-gluon plasma. We demonstrate that the contribution of this new mechanism to the energy loss may be of the order of magnitude of the induced gluon radiation. However, an accurate evaluation of this medium effect is a difficult task because there is a strong cancellation between the cutoff and running coupling constant effects. The uncertainties in the contribution of the coherent final state interaction restrict strongly the accuracy of jet tomographic analyses of the matter density produced in AA reactions.

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1. Introduction. In recent years there has been much work done on the energy loss of fast partons in hot QCD medium due to gluon radiation induced by multiple scattering (for a review, see [1]). This is of great importance for understanding final state interaction in hard reactions in high energy nucleus-nucleus collisions which are under active investigation at RHIC, and will be studied in future experiments at LHC.

The theoretical calculations show that the energy loss in quark-gluon plasma (QGP) exceeds considerably the one in hadronic medium [2 – 4]. Since gluon radiation softens the parton fragmentation functions of energetic partons produced in hard reactions in the initial stage of AA collisions it should lead to significant suppression of the high p_T hadronic spectra in AA collisions with respect to pp collisions (so-called jet quenching) if a hot QGP is formed [5, 6]. Such a suppression was indeed recently discovered by the PHENIX experiment [7] at RHIC for π^0 spectra at $p_T \lesssim 4\,\text{GeV}$ in central $\text{Au}+\text{Au}$ collisions at $\sqrt{s} = 130\,\text{GeV}$. Because the energy loss is sensitive to the density of hot medium it looks quite natural to use experimental data on high p_T spectra for jet tomographic analysis of the matter density produced in AA reactions [8 – 10].

To understand the range of uncertainty in the jet tomographic analyses it is important to study other possible final state interaction effects in jet production. One mechanism of potential interest is the in-medium modification of the parton cascade without gluon exchanges between the fast partons and thermal partons (this mechanism we call the coherent final state interaction (CFSI)). The reason is evident: jet splitting in vacuum is the major mechanism of the energy loss of energetic partons, and if the medium affects the parton cascading one can expect a significant modification of the fragmentation functions. For example, such a modification should inevitably arise as a mass effect due to different infrared cutoff scales in the vacuum and QGP. Another obvious source of the CFSI is the in-medium modification of the running coupling constant[1] $\alpha_s(k)$. Although at large virtualities the running coupling constants in the vacuum and QGP are close to each other this should not be the case at low k where different background environments in which the gluon bremsstrahlung occurs can lead to a difference in the running coupling constants in these two cases. Note that both these medium effects should lead to transition radiation very similar to that of photon radiation in QED. The purpose of the present work is to address the CFSI for RHIC conditions within a simple model for gluon radiation which will be discussed in detail below.

2. Cutoff scales and running coupling constants. Let us first discuss the magnitudes of the cutoff scales for parton splitting in the vacuum and QGP. In the QCD vacuum the natural cutoff is the inverse gluon correlation radius $R_c^{-1} \sim 0.8-1\,\text{GeV}$ [11, 12]. Namely at such a virtuality scale the perturbative cascade stops in the Monte Carlo programs like JETSET, and the string fragmentation takes over. Note also that introduction of the effective gluon mass $m_{g,v} \sim R_c^{-1}$ (hereafter we use the index v for vacuum quantities, and for the plasma quantities below we use the index p) allows one to describe the HERA data on the low-x proton structure

[1] Note that these CFSI effects differ from the coherent double gluon exchanges which are usually included in the induced gluon radiation to insure the unitarity [3].

function [13]. The analysis of inclusive radiative decays of the J/Ψ and Υ [14] also gives $m_{g,v} \sim 0.7 - 1.2\,\mathrm{GeV}$.

In the QGP phase the nonperturbative fluctuations are suppressed, and the natural cutoff for radiation of transverse gluons, which propagate through QGP as quasiparticles, is the thermal gluon mass. At high temperature it reads $m^2_{g,p} = (g^2T^2/2)\,(N_c/3 + N_f/6)$. An analysis of the results of the lattice calculations show that in the temperature range $T \sim (1-3)T_c$ ($T_c \approx 170\,\mathrm{MeV}$ is the temperature of the confinement phase transition), which is of relevance to AA collisions at RHIC, the nonberturbative effects are still important [15]. Using a quasiparticle picture with massive gluons and quarks in the above temperature window the authors of Ref. [15] obtained $m_{g,p} \approx 0.4\,\mathrm{GeV}$, and $m_{q,p} \approx 0.3\,\mathrm{GeV}$. Thus we see that there is a considerable difference in the cutoffs in the vacuum and QGP.

Let us recall now the situation with the running coupling constant at low k. There are some indications that in the vacuum nonperturbative effects stop the growth of the running coupling constant at $k \lesssim k_c \sim 1\,\mathrm{GeV}$ [16–19]. Phenomenologically the magnitude of $\alpha_{s,v}$ at $k \lesssim k_c$ can be estimated from, say, the analysis of the heavy quark energy losses which gives [17]

$$\int\limits_0^{2\mathrm{GeV}} dk \frac{\alpha_{s,v}(k)}{\pi} \approx 0.36\,\mathrm{GeV}. \qquad (1)$$

For the simplest prescription with frozen $\alpha_{s,v}$ at $k < k_c$ (the so-called F-model [17]), using the one-loop expression at $k > k_c$ one can obtain from (2) $\alpha_{s,v}(k < k_c) = \alpha^{fr}_{s,v} \approx 0.7$, and $k_c \approx 0.82\,\mathrm{GeV}$. These values are for $\Lambda_{QCD} = 0.3\,\mathrm{GeV}$.

Unfortunately, at present there is no accurate information on $\alpha_s(k)$ for gluon emission from a fast parton in QGP. Available pQCD calculations are performed in the static limit (see for example [20–22] and references therein). The running coupling constant obtained in [21, 22] has a pole at $k/\Lambda_{QCD} \sim 3$ at $T \sim 250\,\mathrm{MeV}$. Thus, in pQCD, even for the static case, the situation with k-dependence of the in-medium running coupling constant at low k is unclear. On the other hand, analysis of the lattice results within quasiparticle model gives the thermal α_s with a smooth T-dependence, and at $T \sim 250\,\mathrm{MeV}$ $\alpha_s \approx 0.5$ [15]. In the present paper in the absence of accurate information on the in-medium running coupling constant for fast partons we perform calculations using the above F-model with different values of $\alpha^{fr}_{s,p}$.

3. Evaluation of coherent medium correction to gluon spectrum. Let us now discuss technical aspects of our analysis of the CFSI. We consider the gluon radiation from a fast quark (the generalization to the radiation from a gluon is trivial). We neglect multiple emission and consider only the leading order splitting $q \to gq$. It is reasonable since the effect is dominated by the gluons with small transverse momenta $k \lesssim 1-2\,\mathrm{GeV}$. Note that we choose the z-axis along the momentum of the initial fast parton, so for the central rapidity region in AA collisions our L is the ordinary transverse distance between the jet production point and the boundary of QGP.

We consider a fast quark with energy E_i produced at $z = 0$ which eventually splits at some $z > 0$ into the gluon and final quark with the energies $E_g = xE_i$ and $E_f = (1-x)E_i$ respectively. The corresponding matrix element can be written in the form (below for simplicity we drop color factors)

$$T = i\int\limits_0^{\infty} dz \int d\boldsymbol{\rho}\, g\bar{\psi}_f(\boldsymbol{\rho}, z)\gamma^\mu A_\mu(\boldsymbol{\rho}, z)\psi_i(\boldsymbol{\rho}, z), \qquad (2)$$

where $\psi_{i,f}(z, \boldsymbol{\rho})$ are the wave functions of the initial and final quarks, and A_μ is the wave function of the emitted gluon, $\boldsymbol{\rho}$ is the transverse coordinate. In (2) we do not explicitly indicate the z and k dependence of the running coupling constant g. We evaluate the matrix element (2) for small emission angles. Then, at high energies $E_j \gg m_q$ the quark wave functions using the ordinary light-cone spinor basis can be written as

$$\psi_j(\boldsymbol{\rho}, z) = \exp(iE_j z)\hat{U}_j \phi_j(z, \boldsymbol{\rho}), \qquad (3)$$

where the operator $\hat{U}_j$ reads

$$\hat{U}_j = \left(\sqrt{2E_j} + \frac{\boldsymbol{\alpha}\mathbf{p} + \beta m_q}{\sqrt{2E_j}}\right)\chi_j. \qquad (4)$$

Here χ_j is the quark spinor (normalized to unity), $\boldsymbol{\alpha} = \gamma^0\boldsymbol{\gamma}$, $\beta = \gamma^0$, and $\mathbf{p} = -i\nabla_\perp$. The gluon wave function can be written in a form similar to (3) (up to an obvious change of the spin operator). The transverse quark wave function $\phi_j(\boldsymbol{\rho}, z)$ entering (3) is governed by the two-dimensional Schrödinger equation in which z plays the role of time

$$i\frac{\partial\phi_j(z, \boldsymbol{\rho})}{\partial z} = \frac{(\mathbf{p}^2 + m_q^2)}{2E_j}\,\phi_j(z, \boldsymbol{\rho}). \qquad (5)$$

A similar equation holds for the gluon wave function.

Without a loss of generality we can take for the initial quark the plane wave state in the $\boldsymbol{\rho}$-plane and set $\mathbf{p}_i = 0$. Then all the transverse wave functions can be written as

$$\phi_j(z, \boldsymbol{\rho}) = \exp\left\{i\left[\mathbf{p}_j\boldsymbol{\rho} - \int_0^z d\xi \frac{(\mathbf{p}_j^2 + m_j^2(\xi))}{2E_j}\right]\right\}. \quad (6)$$

Eventually, the $\boldsymbol{\rho}$-integration in (2) will give $\mathbf{p}_g + \mathbf{p}_f = \mathbf{p}_i = 0$. Note that since the quark mass is of only marginal significance (for light quarks) in the gluon radiation we will neglect the z-dependence of m_q and use the same quark mass in the vacuum and in QGP. However, for the gluon transverse wave function we use the z-dependent gluon mass: $m_g(z < L) = m_{g,p}$ and $m_g(z > L) = m_{g,v}$.

Using the above formulas for the transition amplitude and wave functions with the help of the standard Fermi golden rule one can obtain after some simple calculations for the gluon distribution (below $\mathbf{k} = \mathbf{p}_g$)

$$\frac{dN}{dx d\mathbf{k}^2} = \frac{dN^{(0)}}{dx d\mathbf{k}^2} + \frac{dN^{(1)}}{dx d\mathbf{k}^2}, \tag{7}$$

$$\frac{dN^{(0)}}{dx d\mathbf{k}^2} = \frac{C_F \alpha_s^v(k)}{\pi x}\left(1 - x + \frac{x^2}{2}\right)\frac{\mathbf{k}^2}{(\mathbf{k}^2 + \mu_v^2)^2}, \tag{8}$$

$$\frac{dN^{(1)}}{dx d\mathbf{k}^2} = \frac{2C_F \alpha_s^v(k)}{\pi x}\left(1 - x + \frac{x^2}{2}\right) \times \left[1 - \cos\left(\frac{(\mathbf{k}^2 + \mu_p^2)L}{2E_i x(1-x)}\right)\right] \times \frac{\mathbf{k}^2 r(k)[(r(k)-1)\mathbf{k}^2 + r(k)\mu_v^2 - \mu_p^2]}{(\mathbf{k}^2 + \mu_p^2)^2(\mathbf{k}^2 + \mu_v^2)}, \tag{9}$$

where $\mu_i^2 = m_q^2 x^2 + m_{g,i}^2(1-x)$, $r(k) = \sqrt{\alpha_{s,p}(k)/\alpha_{s,v}(k)}$. The first term on the r.h.s of (7) is the gluon spectrum in the vacuum, and the second one gives the medium correction of interest. In deriving (8), (9) we neglected the small spin-flip contribution ($\propto m_q^2$).

Recall that the gluon formation length is $L_f \sim 2E_i x(1-x)/(\mathbf{k}^2 + \mu^2)$ (we do not specify here the medium index since μ_p and μ_v are of the same order). Thus we see that the argument of cosine in the r.h.s. of (9) is $\sim L/L_f$. Obviously, at $L_f \ll L$ the rapidly oscillating cosine as function of L will vanish upon averaging over the production point of the fast quark, and one gets L-independent correction to the vacuum term. It can be written in the following physically transparent form

$$\left.\frac{dN^{(1)}}{dx d\mathbf{k}^2}\right|_{L_f \ll L} \approx \left.\frac{dN^{(0)}}{dx d\mathbf{k}^2}\right|_p - \left.\frac{dN^{(0)}}{dx d\mathbf{k}^2}\right|_v + \frac{dN^{\mathrm{tran}}}{dx d\mathbf{k}^2}, \tag{10}$$

where the first two terms describe simply modification of the spectrum due to the change in m_g and α_s (in an infinite QGP), and the last term is the contribution of the transition radiation which reads

$$\frac{dN^{\mathrm{tran}}}{dx d\mathbf{k}^2} = \frac{C_F \alpha_s^v(k)}{\pi x}\left(1 - x + \frac{x^2}{2}\right)\left(\frac{\mathbf{k} r(k)}{\mathbf{k}^2 + \mu_p^2} - \frac{\mathbf{k}}{\mathbf{k}^2 + \mu_v^2}\right)^2. \tag{11}$$

It can be derived from (2) taking for the lower limit of the z-integral $-\infty$. Note that the change in m_g and α_s both cause the transition radiation.

On the other hand, for the gluons with $L_f \gg L$ expanding cosine in (9) one gets the correction $\propto L^2$

$$\left.\frac{dN^{(1)}}{dx d\mathbf{k}^2}\right|_{L_f \gg L} \approx \frac{C_F \alpha_s(k) L^2}{\pi x}\left(1 - x + \frac{x^2}{2}\right) \times \frac{\mathbf{k}^2 r(k)[(r(k)-1)\mathbf{k}^2 + r(k)\mu_v^2 - \mu_p^2]}{4E_i^2 x^2 (1-x)^2 (\mathbf{k}^2 + \mu_v^2)}. \tag{12}$$

In this regime one cannot separate the transition radiation. Note that the above formulas demonstrate that the decrease of the cutoff and coupling constant in QGP work in opposite directions. One can also see that the relative contribution of the medium correction in (7) is larger for gluons with $L_f \lesssim L$. These facts are consistent with intuitive expectation.

4. Numerical results. In numerical calculations we take: $m_{g,p} = 0.4$ and $m_{g,v} = 0.8\,\mathrm{GeV}$, for the quark mass we take $m_{q,p} = m_{q,v} = 0.3\,\mathrm{GeV}$. As was mentioned above in the absence of accurate information on the in-medium α_s at low k we perform numerical calculations parametrazing it in the same F-model as for $\alpha_{s,v}$ for several values of $\alpha_{s,p}^{fr}$. To understand the sensitivity of CFSI to $\alpha_{s,p}^{fr}$ we use for it four values: 0.7, 0.5, 0.4, and 0.3. The first version corresponding to $\alpha_{s,p} = \alpha_{s,v}$ is unlikely to be realistic since it neglects the in-medium modification of $\alpha_s(k)$, but it allows to see the magnitude of the pure mass effect. The last value is also unlikely to be realistic. Indeed, the results of the analysis of the lattice data within the quasiparticle model of QGP [15] say that $\alpha_s \approx 0.3$ occurs for $T \approx 2.2T_c$. Since the QGP can have a temperature above this value only during a short time in the initial stage of its evolution the value of $\alpha_{s,p}^{fr} = 0.3$ is probably too small for evaluation of the CFSI. The value $\alpha_{s,p}^{fr} = 0.5$ seems to be most reasonable. For instance, it is close the value $\alpha_s \approx 0.47$ obtained in [15] at $T \approx 1.5T_c \approx 250\,\mathrm{MeV}$. Of course, it is not obvious that the thermal α_s can be extrapolated safely to higher energies. However, one can expect that in the splitting of fast partons the effect of thermostat on α_s can only be weaker than that for the thermal partons. If this is the case, the CFSI correction may be larger than our estimate.

In Fig.1 we show the x – dependence of the ratio $R(x) = \frac{dN}{dx} / \frac{dN^{(0)}}{dx}$ for several k-windows at $E_i = 40\,\mathrm{GeV}$ averaged over L in the interval $[0, 6]\,\mathrm{fm}$. As can be seen from Fig.1 for $\alpha_{s,p} = \alpha_{s,v}$ the mass effect alone enhances considerably the probability of gluon emission at low x and $k \lesssim 1\,\mathrm{GeV}$. The curves for smaller values of $\alpha_{s,p}^{fr}$ show that the enhancement of the radiation

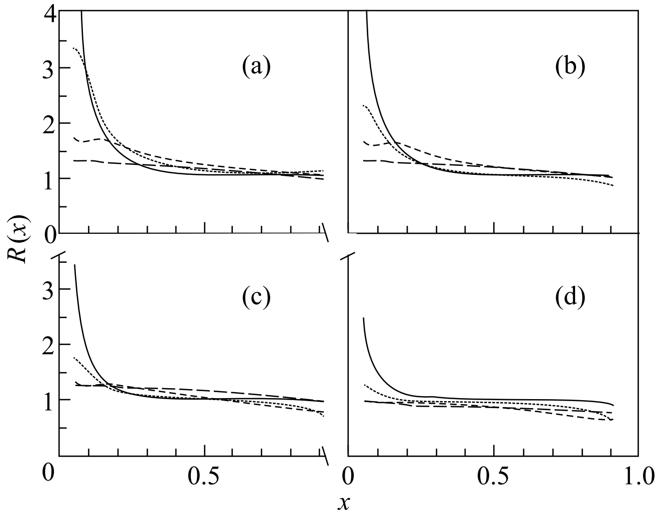

Fig.1. The ratio $R(x) = \frac{dN}{dx} \,/\, \frac{dN^{(0)}}{dx}$ at $E_i = 40\,\text{GeV}$ for $\alpha_{s,p}^{fr}$ =0.7 (a), 0.5 (b), 0.4 (c), and 0.3 (d) evaluated using Eqs. (7)–(9) for the k-windows: [0,0.5] (solid line), [0.5,1] (dotted line), [1,1.5] (dashed line), and [1.5,2] (long dashed line) GeV. The x-distributions obtained averaging over L in the interval [0,6] fm.

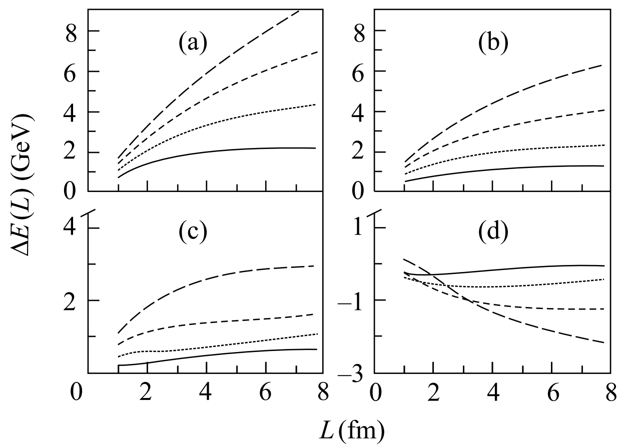

Fig.2. The quark energy loss ΔE due to the CFSI as a function of L for $\alpha_{s,p}^{fr}$ =0.7 (a), 0.5 (b), 0.4 (c), and 0.3 (d) at E_i =10 (solid line), 20 (dotted line), 40 (dashed line), and 80 (long dashed line) GeV

due to smaller cutoff in QGP is strongly compensated by the effect of smaller coupling constant in QGP, and for, probably unrealistic, $\alpha_{s,p}^{fr} = 0.3$ there is a kinematic region where CFSI suppresses the gluon radiation.

Using (9) we also calculated the energy loss, defined as

$$\Delta E = E_i \int\limits_{x_{\min}}^{x_{\max}} dx \int\limits_{0}^{k^2_{\max}} d\mathbf{k}^2 x \frac{dN^{(1)}}{dx d\mathbf{k}^2}.$$

For the limits of the x- and $\mathbf{k}^2$-integration we take $x_{\min} = m_g/E_i$, $x_{\max} = m_q/E_i$, and $k^2_{\max} = \min[E_i^2 x^2, E_i^2(1-x)^2]$. In Fig.2 we show the results for ΔE as a function of L at E_i =10, 20, 40, and 80 GeV. As one can see for $\alpha_{s,p}^{fr}$ =0.7, 0.5, and 0.4 the energy loss is positive and rising with L and E_i. For $\alpha_{s,p}^{fr} = 0.3$ ΔE becomes negative which shows that suppression of the gluon radiation due to a small coupling constant becomes stronger than the enhancement caused by the mass effect. This strong cancellation between the two competing effects makes it difficult to give definitive predictions for the effect of CFSI in the region $\alpha_{s,p}^{fr} \sim 0.3 - 0.4$. Note that our ΔE for $\alpha_{s,p}^{fr} = 0.5$ appears to be of the same order of magnitude as the GLV prediction [8] for the energy loss due to the induced radiation.

5. Summary. We have shown that the final state interaction due to the change in the cutoff scale and running coupling constant from the vacuum to QGP modifies the gluon radiation from fast partons produced in AA collisions. The contribution to the energy loss of this mechanism may be of the same order of magnitude as the induced gluon radiation. However, accurate evaluation of the CFSI is a difficult task since there are strong cancellations between the mass and running coupling constant effects, and the results depend strongly on the assumptions on the k-dependence of the in-medium $\alpha_s(k)$.

The results of the present paper raise a practical question, whether the jet tomographic analyses based on the theory of the induced gluon radiation can be used for extracting the density of hot QCD medium produced in AA collisions. At present, one cannot exclude the possibility that the CFSI may appear to be even more important than the induced radiation. To clarify the situation it is highly desirable to study the influence of QGP on the running coupling constant for fast partons.

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1. R. Baier, D. Schiff, and B. G. Zakharov, Ann. Rev. Nucl. Part. **50**, 37 (2000).
2. R. Baier et al., Nucl. Phys. **B483**, 291 (1997); **B484**, 265 (1997); **B531**, 403 (1998); JHEP **0109**, 033 (2001).
3. B. G. Zakharov, JETP Lett. **63**, 952 (1996); **65**, 615 (1997); **70**, 176 (1999); Phys. Atom. Nucl. **61**, 838 (1998); hep-ph/9807396.
4. M. Gyulassy, P. Levai, and I. Vitev, Nucl. Phys. **B594**, 371 (2001).
5. X.-N. Wang, Phys. Rev. **C58**, 2321 (1998).

6. I. P. Lokhtin and A. M. Snigirev, Phys. Lett. **B440**, 163 (1998).
7. K. Adcox et. al. [PHENIX Collaboration], Phys. Rev. Lett. **88**, 022301 (2002).
8. M. Gyulassy, P. Levai, and I. Vitev, Phys. Lett. **B538**, 282 (2002).
9. C. S. Salgado and U. A. Wiedemann, hep-ph/0204221, (2002).
10. E. Wang and X.-N. Wang, hep-ph/0202105 (2002).
11. E. V. Shuryak, Rev. Mod. Phys. **65**, 1 (1993).
12. E. V. Shuryak, hep-ph/0205031 (2002).
13. N. N. Nikolaev and B. G. Zakharov, Phys. Lett. **B327**, 149 (1994).
14. J. H. Field, hep-ph/0101158 (2002).
15. P. Levai and U. Heinz, Phys. Rev. **C57**, 1879 (1998).
16. A. C. Mattingly and P. M. Stevenson, Phys. Rev. **D49**, 437 (1994).
17. Yu. L. Dokshitzer, V. A. Khoze, and S. I. Troyan, Phys. Rev. **D53**, 89 (1996).
18. Yu. L. Dokshitzer, G. Marchesini, and B. R. Webber, Nucl. Phys. **B469**, 93 (1996).
19. D. V. Shirkov and I. L. Solovtsov, Phys. Rev. Lett. **79**, 1209 (1997).
20. R. Baier, B. Pire, and D. Schiff, Phys. Lett. **B238**, 367 (1990).
21. M. A. van Eijick, C. R. Stephens, and Ch. G. van Weert, Mod. Phys. Lett. **A9**, 309 (1994).
22. M. Chaichian and M. Hayashi, Acta Phys. Polon. **27**, 1703 (1996).