Non-BCS pairing in anisotropic strongly correlated electron systems in solids

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The problem of pairing in anisotropic electron systems possessing patches of fermion condensate in the vicinity of the van Hove points is analyzed. Attention is directed to opportunities for the occurrence of non-BCS pairing correlations between the states belonging to the fermion condensate. It is shown that the physical emergence of such pairing correlations would drastically alter the behavior of the single-particle Green function, the canonical pole of Fermi-liquid theory being replaced by a branch point.

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The ground state of conventional superconductors at zero temperature is known to be a condensate of Cooper pairs with total momentum $\mathbf{P}=0$. In Fermi-liquid theory, the familiar BCS structure of the ground state is associated with the logarithmic divergence of the particle-particle propagator at $\mathbf{P}=0$ and is independent of the details of the pairing interaction. However, a markedly different situation can exist in strongly correlated systems in which the necessary stability condition for the Landau state is violated and the Landau quasiparticle momentum distribution suffers a rearrangement. Under certain conditions, this rearrangement leads to a fermion condensate (FC): a continuum of dispersionless single-particle (sp) states whose energy $\epsilon(\mathbf{p})$ coincides with the chemical potential μ over a finite (and in general disconnected) domain $\mathbf{p}\in\mathcal{F}$ in momentum space [1 – 6]. In such a case, the preference for pairing with $\mathbf{P}=0$ comes into question because of the degeneracy of the FC sp spectrum, and the nature of pairing depends on the configuration assumed by the FC.

Here we study a two-dimensional square-lattice system having lattice constant l, in which the FC is situated in domains adjacent to four van Hove points with coordinates $(\pm\pi/l,0)$ and $(0,\pm\pi/l)$, while the sp states with ordinary dispersion are concentrated around diagonals of the Brillouin zone [2, 5]. To proceed efficiently, we shall focus on the nature of particle-particle correlations in the FC subsystem and ignore contributions from the sp states with nonzero dispersion. It is assumed that all the particle-hole contributions have already been taken into account in terms of an effective single-particle Hamiltonian with sp spectrum $\xi(\mathbf{p})=\epsilon(\mathbf{p})-\mu$. Accordingly, only pairing contributions should be incorporated in the equation for the Green function $G_{\alpha\beta}(x,x')=-i\langle T\psi_\alpha(x)\psi^\dagger_\beta(x')\rangle$. For simplicity, spin indices α, β, etc. will henceforth be omitted. The Green function is then expressed as $G(\mathbf{p},\varepsilon)=\left[G_o^{-1}(\mathbf{p},\varepsilon)-\Sigma(\mathbf{p},\varepsilon)\right]^{-1}$ in terms of the free Green function $G_o^{-1}(\mathbf{p},\varepsilon)=\varepsilon-\xi(\mathbf{p})$ and a self-energy or mass operator $\Sigma(\mathbf{p},\varepsilon)$. In superfluid electron systems with a FC, the familiar Cooper pair ("C-pair") of BCS theory, which by definition has momentum $\mathbf{P}=0$, can form only from sp states of diagonally opposite patches of the FC. The electron mass operator Σ is given by the usual formula $\Sigma=-\Delta G_o^-\Delta$, where $G_o^-(\mathbf{p},\varepsilon)=-\left[\varepsilon+\xi(\mathbf{p})\right]^{-1}$ and Δ is the amplitude associated with generation of the BCS pair. In this case the electron Green function G has the conventional pole, and the sp spectrum, given by $E(\mathbf{p})=\left[\xi^2(\mathbf{p})+\Delta^2\right]^{1/2}$, possesses a gap specified by Δ.

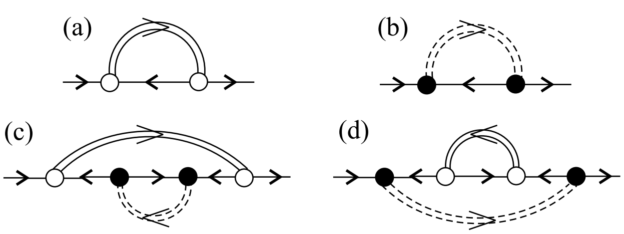
In anisotropic electron systems inhabiting crystalline materials that exhibit fermion condensation, all FC patches should in principle be treated on an equal footing. It follows that pairing correlations affecting sp states located in neighboring FC patches may become important. Since the fraction of the Brillouin zone occupied by the FC is small, these correlations are specified by the antiferromagnetic vector $\mathbf{Q}=(\pi/l,\pi/l)$. In the conventional situation, the BCS coupling constant prevails, and the formation of "Q-pairs" having momentum $\mathbf{Q}$ is irrelevant. However, in the antiferromagnetic scenario for fermion condensation [5], wherein the scattering amplitude Γ is approximated by the well-known spin-fluctuation-exchange term [7]

$$N_0\Gamma(\mathbf{q}\to\mathbf{Q},\omega)=-\frac{\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2}{\kappa^2p_F^{-2}(\mathbf{q}-\mathbf{Q})^2+\beta^2+i\omega/\omega_0},\quad (1)$$

the constant λ_C associated with C-pair formation coincides with the Q-pair coupling constant λ_Q. Upon supplementing Eq. (1) by regular terms, the ratio λ_Q/λ_C may vary in either direction. This prompts us to investigate the condition for stability of the BCS state against perturbations Δ' of the BCS gap function characterized by vectors close to $\mathbf{Q}$ (see below).

If this condition is violated, then Q-pairs must enter the picture. The most likely outcome is the Larkin-Ovchinnikov-Fulde-Ferrell (LOFF) [8, 9] scenario, in which the Q-pair condensate simply replaces the C-pair condensate of the BCS description. In the LOFF scenario, the new ground state usually ceases to be homogeneous. However, in the present case involving the single commensurate vector $\mathbf{Q}$, the system remains homogeneous. Another possibility is that the new ground state becomes a "cocktail" composed of C- and Q-condensates. In this nonabelian exemplar of the pairing problem, the whole band of many-particle–many-hole states, envisioned as a conglomerate of C- and Q-pairs, comes into play. A similar situation occurs in the microscopic theory of rotation treated as a collective excitation [10].

To gain insight into the problem, let us represent the relevant mass operator Σ in terms of Feynman diagrams, as illustrated in Figure. The propagator of a C-pair is depicted by a double solid line; the propagator



Diagrammatic representation of contributions to the mass operator Σ

of a Q-pair, by a double dashed line; and that of an electron, by a single solid line. The open circle stands for the usual block of the Feynman diagrams representing the amplitude Δ for generation of a C-pair, while the solid circle stands for the block D of similar diagrams describing generation of a Q-pair. Within a BCS-like approximation, only diagrams (a) and (b) are relevant, as in the conventional case of interband transitions.

In drawing more complicated diagrams contributing to the mass operator, these restrictions must be obeyed:

(i) Emission of a pair of either type (C or Q) must be compensated by its absorption, ensuring particle conservation. In fact, pair emission and pair absorption must alternate from left to right in any diagram.

(ii) The direction of the single electron line reverses upon passing through a circle, whether open or solid.

(iii) The first (leftmost) and the last (rightmost) circles must be of the same type.

(iv) The first two circles cannot be of the same type; otherwise the diagram is reducible. Likewise, the last two circles cannot be of the same type.

With these restrictions, the equation for the mass operator Σ can be expressed in closed form, since only two different degrees of freedom are involved. We obtain a system of three equations,

$$\begin{aligned} \Sigma &= -\Delta^+ G_C^- \Delta - D^+ G_Q^- D, \\ G_C &= G_o - G_o D^+ G_Q^- D G_C, \\ G_Q &= G_o - G_o \Delta^+ G_C^- \Delta G_Q. \end{aligned} \tag{2}$$

Upon introducing the dimensionless quantities $X_C = G_C/G_o$, $X_Q = G_Q/G_o$, and

$$\begin{aligned} K_C &= G_o \Delta^+ G_o^- \Delta, \\ K_Q &= G_o D^+ G_o^- D, \end{aligned} \tag{3}$$

this system can be conveniently rewritten

$$\begin{aligned} \Sigma G_o &= -K_C X_C - K_Q X_Q, \\ X_C &= 1 - K_Q X_C X_Q^-, \\ X_Q^- &= 1 - K_C X_C X_Q^-. \end{aligned} \tag{4}$$

The second of Eqs. (4) gives $X_C X_Q^- = (1 - X_C)/K_Q$; insertion of this relation into the third equation of the set leads to $K_C X_C^2 + (K_Q - K_C + 1) X_C - 1 = 0$. Analogous operations yield $K_Q X_Q^2 + (K_C - K_Q + 1) X_Q - 1 = 0$. These quadratic equations have respective solutions

$$\begin{aligned} X_C &= \frac{K_C - 1 - K_Q + \left[(K_C - 1 - K_Q)^2 + 4K_C\right]^{1/2}}{2K_C}, \\ X_Q &= \frac{K_Q - 1 - K_C + \left[(K_Q - 1 - K_C)^2 + 4K_Q\right]^{1/2}}{2K_Q}. \end{aligned} \tag{5}$$

The Green function G is now easily evaluated from the first of Eqs. (4), with the result

$$G = \frac{G_o}{\left[(K_C - K_Q + 1)^2 + 4K_Q\right]^{1/2}}. \tag{6}$$

The same result may be derived from the equations of motion [11]. We observe that the conventional Fermi-liquid-theory pole has been replaced by a branch point. The new Green function (6) possesses a nonzero imaginary part over a finite interval in ε delimited by the two zeros $E_\pm$ of the denominator of G. This is our primary result.

For the sake of clarity, let us neglect the inclination of the FC plateau in the spectrum of the sp excitations due to pairing and set $\xi(\mathbf{p} \in F) \equiv 0$, just as in the system with a FC present but without pairing. We then find

$$\left[(K_C - K_Q + 1)^2 + 4K_Q\right]^{1/2} = \\ = \varepsilon^{-2}\left[(\varepsilon^2 - E_+^2)(\varepsilon^2 - E_-^2)\right]^{1/2}, \quad (7)$$

where $E_\pm(\Delta \pm D)^2$, and Eq. (6) takes the form

$$G(\varepsilon) = \frac{\varepsilon}{\left[(\varepsilon^2 - E_+^2)(\varepsilon^2 - E_-^2)\right]^{1/2}}, \qquad \mathbf{p} \in \mathcal{F}. \quad (8)$$

Another interesting result concerns the topological charge N introduced by Volovik [2] to analyze the structure of the sp Green function of Fermi systems. Suppressing a trace over spin and band indices, this quantity is given by

$$N = \oint_C \frac{dl}{2\pi i} G(\omega, \mathbf{p}) \partial_l G^{-1}(\Omega, \mathbf{p}), \quad (9)$$

where the behavior of G is considered on an imaginary semi-axis of frequencies $\varepsilon = i\Omega$ and the integral is evaluated along an arbitrary contour C in the space $(\Omega, \mathbf{p})$ enclosing the singularity (a linear singularity – the Fermi line – occurring at $\Omega = 0$). The topological charge has the value $N = 1$ for a normal Fermi liquid and remains unchanged for marginal and Luttinger Fermi liquids. However, when a fermion condensate is present, its value shrinks to $N = 1/2$ [2]. If we now generalize the definition of N to apply to superfluid systems, asserting simply that the integration contour embraces the singularity of G, the topological charge evaluated for the Green function (8) is again $N = 1/2$, since the full variation of the argument of G over the contour amounts only to π. We conclude that topological charge is conserved in the superfluid phase transition induced by non-BCS pairing.

Beyond these formal results, there is the pivotal question of whether non-BCS pairing of the kind described here can, in reality, win the contest with ordinary BCS pairing when the effective interaction Γ takes the form (1) and therefore entails comparable values of the two coupling constants λ_C and λ_Q. Resolution of this issue requires a knowledge of the actual gap functions Δ and D. Generalized gap equations determining the two gap functions may be derived by summation of the appropriate diagrams of the scattering amplitude in the particle-particle channel, as is done in the diagrammatic foundation of BCS theory. Explicitly, these equations read

$$\Delta(\mathbf{p}) = -\int \mathcal{V}(\mathbf{p}, \mathbf{p}_1, \mathbf{P} = 0) G_C^-(\mathbf{p}_1, \varepsilon) \times \\ \times \Delta(\mathbf{p}_1) G(\mathbf{p}_1, \varepsilon) dv_{\mathbf{p}_1} \frac{d\varepsilon}{2\pi i},$$

$$D(\mathbf{p}) = -\int \mathcal{V}(\mathbf{p}, \mathbf{p}_1, \mathbf{P} = \mathbf{Q}) G_Q^-(\mathbf{p}_1 + \mathbf{Q}, \varepsilon) \times \\ \times D(\mathbf{p}_1) G(\mathbf{p}_1, \varepsilon) dv_{\mathbf{p}_1} \frac{d\varepsilon}{2\pi i}, \quad (10)$$

where $\mathcal{V}(\mathbf{p}, \mathbf{p}_1, \mathbf{P} = 0)$ and $\mathcal{V}(\mathbf{p}, \mathbf{p}_1, \mathbf{P} = \mathbf{Q})$ are the respective blocks of scattering-amplitude diagrams irreducible in the particle-particle channel and specified by pair momenta 0 and $\mathbf{Q}$, while $dv_\mathbf{p}$ denotes the FC momentum-space volume element. At nonzero temperature T, the usual factor $\tanh(\varepsilon/2T)$ is to be inserted in the integrands of Eqs. (10).

In the familiar case with $D = 0$, the second equation in the set (10) disappears, and we are left with the single gap equation of BCS theory. Conversely, if $\Delta = 0$, only the second equation in (10) survives, and we are led to the LOFF type of pairing. A third possibility is the emergence of a "cocktail" with both $\Delta \neq 0$ and $D \neq 0$. To decide which of the competing scenarios prevails in a given case, we should compare the respective superfluid corrections δE_N and $\delta E_{\rm BCS}$ to the ground-state energy (i.e., we should compare the condensation energies for the different pairing alternatives, where N labels one or another non-BCS scenario).

It is instructive to treat a simple model in which the blocks $\mathcal{V}(\mathbf{p}, \mathbf{p}_1, \mathbf{P} = 0)$ and $\mathcal{V}(\mathbf{p}, \mathbf{p}_1, \mathbf{P} = \mathbf{Q})$ are approximated by respective constants λ_C and λ_Q in the FC region, while vanishing outside. The solutions Δ and D of Eqs. (10) are then also constants in this domain and zero outside. The non-BCS condensation energy is given by the formula

$$\delta E_N(\lambda) = -\int_0^\lambda \frac{\Delta^2 + D^2}{\lambda_1^2} d\lambda_1, \quad (11)$$

which may be derived in the same manner as the analogous formula for $\delta E_{\rm BCS}$ appearing in the Landau-Lifshitz textbook [12]. In obtaining this result, the ratio λ_Q/λ_C has been fixed.

A comprehensive study of the problem requires a knowledge of the sp spectrum $\xi(\mathbf{p}) = \epsilon(\mathbf{p}) - \mu$, since when pairing occurs, $\xi(\mathbf{p})$ necessarily differs from zero even in the momentum region $\mathcal{F}$ occupied by the FC [1]. In evaluating $\xi(\mathbf{p})$, one can employ the standard relation

$$\delta\epsilon(\mathbf{p}) = \int f(\mathbf{p}, \mathbf{p}_1) \delta n(\mathbf{p}_1) dv_{\mathbf{p}_1}, \qquad \mathbf{p} \in \mathcal{F}, \quad (12)$$

where $\delta n = n - n_0$ is the difference between the momentum distributions for the superfluid and nonsuperfluid states of the system *with* the fermion condensate present, $\delta\epsilon$ is the corresponding difference in the sp spectra, and f is the effective interaction in the particle-hole channel.

In analyzing the problem, we exploit the fact that the strength f of the effective interaction $f(\mathbf{p},\mathbf{p}_1)$ in the particle-hole channel exceeds its strength in the particle-particle channel. This allows one to expand $\xi(\mathbf{p})$ as given by Eq. (12) in a Taylor series with respect to the order parameter Δ and evaluate the coefficients of the expansion by equating terms of the same power in this parameter. To illustrate the procedure, consider the situation in which the ordinary C-pairing of BCS theory prevails. In this case,

$$\Delta = -\lambda_C \int\limits_{\mathcal{F}} n(\mathbf{p})\,(1-n(\mathbf{p}))\,dv_{\mathbf{p}}, \tag{13}$$

where

$$n(\mathbf{p}) = \frac{E(\mathbf{p})-\xi(\mathbf{p})}{2E(\mathbf{p})} \tag{14}$$

and $E(\mathbf{p}) = \left[\xi^2(\mathbf{p})+\Delta^2\right]^{1/2}$. Let us now insert the expansion

$$\xi(\mathbf{p}) = \xi_1(\mathbf{p})\Delta + \xi_2(\mathbf{p})\Delta^2 + \ldots\,, \quad \mathbf{p}\in\mathcal{F} \tag{15}$$

first into Eq.(14) and then into Eq. (12), thereby obtaining

$$\xi_1(\mathbf{p})\Delta + \xi_2(\mathbf{p})\Delta^2 + \ldots = \int\limits_{\mathcal{F}} f(\mathbf{p}_1,\mathbf{p}_1)\,\times \tag{16}$$

$$\times\left[\frac{1}{2}-\frac{\xi_1(\mathbf{p}_1)+\xi_2(\mathbf{p}_1)\Delta+\ldots}{2\left[((\xi_1(\mathbf{p}_1)+\xi_2(\mathbf{p}_1)\Delta+\ldots)^2+1\right]}-n_0(\mathbf{p}_1)\right]dv_{\mathbf{p}_1}.$$

Here we have neglected an insignificant variation of the chemical potential of order Δ^2. Since the gap value Δ is small, one can expand the r.h.s. of Eq. (16) into a Taylor series in Δ. Every term of the latter expansion must necessarily coincide with the respective term of the Taylor expansion on the l.h.s. of the equation. Focusing on the terms of zeroth power in Δ, which are absent from the l.h.s. of Eq. (16), this fact is seen to require that the term $1-\xi_1(p)/\left[\xi_1^2(p)+1\right]^{1/2}-2n_0(p)$ on the r.h.s. is identically zero, which in turn yields

$$\begin{aligned} \xi_1(\mathbf{p}) &= \frac{(1-2n_0(\mathbf{p}))]}{2\left[n_0(\mathbf{p})\,(1-n_0(\mathbf{p}))\right]^{1/2}}, \\ E_1(\mathbf{p}) &= \frac{\Delta}{2\left[n_0(\mathbf{p})\,(1-n_0(\mathbf{p}))\right]^{1/2}}, \qquad \mathbf{p}\in\mathcal{F}. \end{aligned} \tag{17}$$

These conditions are virtually equivalent to the coincidence of $n_0(\mathbf{p})$ in the FC region, in the limit $\Delta\to 0$. Equating the terms linear in Δ on the left and right of Eq. (16), one can find the quantity ξ_2, and so on to higher orders as needed. The analysis shows that the dimensionless ratio $\xi_2(\mathbf{p})\Delta/\xi_1(\mathbf{p})$, which is proportional to the ratio $(\mathcal{V}/f)\sim(T_c/T_f)$ of the critical temperature T_c of the pairing transition to the characteristic temperature T_f of fermion condensation, always remains rather small because the interaction f must be very strong for a fermion condensate to form. It is thus a reasonable approximation to retain only the term ξ_1 in the expansion (15).

One can proceed analogously in the general case with $D\neq 0$. The same argumentation can in fact be applied to evaluate the variation of the FC spectrum with T in the normal state at temperatures near T_c. The result becomes especially transparent if one may ignore damping effects, in which case the standard formula $n(\mathbf{p},T) = \left[1+\exp\left(\xi(\mathbf{p},T)/T\right)\right]^{-1}$ may be employed in Eq. (12) to obtain [3, 1]

$$\xi(\mathbf{p},T) = T\ln\frac{1-n_0(\mathbf{p})}{n_0(\mathbf{p})}, \quad \mathbf{p}\in\mathcal{F}. \tag{18}$$

The key equations (10) and (12) are cumbersome to analyze and solve. However, their treatment is facilitated if we work in the temperature region close to the critical temperature T_c, since one of the gap functions Δ or D vanishes, while the other satisfies a linear equation yielding the corresponding critical temperature, T_{c1} (for $\Delta\equiv 0$) or T_{c2} (for $D\equiv 0$). If BCS pairing is victorious, this equation takes the customary form [13]

$$1 = -\lambda_C\int\frac{1-2n(\mathbf{p},T_{c1})}{2\xi(\mathbf{p},T_{c1})}dv_{\mathbf{p}}. \tag{19}$$

Suppressing an insignificant variation of the momentum distribution $n(\mathbf{p},T)$ with T and inserting $\xi(\mathbf{p},T)$ from Eq. (18), we arrive at the relation

$$T_{c1} = -\frac{\lambda_C}{2}\int\frac{1-2n_0(\mathbf{p})}{\ln\left[1-n_0(\mathbf{p})\right]-\ln n_0(\mathbf{p})}\,dv_{\mathbf{p}}\,, \tag{20}$$

which determines T_{c1}. In the opposite case, for which the Q-condensate disappears at $T_{c2}>T_{c1}$, the analog of Eq. (20) is found to be

$$T_{c2} = -\lambda_Q\,\times \tag{21}$$

$$\times\int\limits_{\mathcal{F}}\frac{1-n_0(\mathbf{p})-n_0(\mathbf{p}+\mathbf{Q})}{\ln(1-n_0(\mathbf{p}))-\ln n_0(\mathbf{p})+\ln(1-n_0(\mathbf{p}+\mathbf{Q}))-\ln n_0(\mathbf{p}+\mathbf{Q})}dv_{\mathbf{p}}.$$

We see that the outcome of the contest between C- and Q-condensates at sufficiently high temperatures depends crucially on the arrangement of the FC.

What conditions ensure the occurrence of the cocktail solution of the gap equations (10)? We answer this question for the case $T = 0$ by considering the stability condition

$$\Delta'(\mathbf{k},\omega) \equiv -(\mathcal{V}F') = - \int \mathcal{V}[G^-(\mathbf{p},\varepsilon)G(\mathbf{p}+\mathbf{k},\varepsilon+\omega) + F(\mathbf{p},\varepsilon)F(\mathbf{p}+\mathbf{k},\varepsilon+\omega)]\Delta'(\mathbf{k},\omega)dv_{\mathbf{p}}\frac{d\varepsilon}{2\pi i} \quad (22)$$

for the BCS state in the particle-particle channel, derived from the first of Eqs. (10) with D set identically to zero. Violation of stability is signaled by the emergence of imaginary frequencies $\omega(\mathbf{k})$ in solutions of this equation. In the most dangerous case, the wave vector $\mathbf{k}$ associated with the perturbation Δ' coincides with $\mathbf{Q}$ and involves the block $\mathcal{V}(\mathbf{p},\mathbf{p}_1,\mathbf{Q})$, which we treat as a parameter λ_Q. The stability condition is violated if the coupling constant λ_Q exceeds a critical value λ_Q^{cr}. The equation fixing λ_Q^{cr} is

$$1 = -\lambda_Q^{\mathrm{cr}} \int_{\mathcal{F}} [G(\mathbf{p},\varepsilon)G(\mathbf{Q}-\mathbf{p},-\varepsilon) + F(\mathbf{p},\varepsilon)F(\mathbf{Q}-\mathbf{p},-\varepsilon)]dv_{\mathbf{p}}\frac{d\varepsilon}{2\pi i}, \quad (23)$$

where G and F are the pair of Green functions entering the system of Gor'kov equations. Upon substituting the explicit forms for these functions, Eq. (23) may be converted into

$$1 = -\lambda_Q^{\mathrm{cr}} \int_{\mathcal{F}} \frac{E(\mathbf{p})E(\mathbf{p}+\mathbf{Q}) + \xi(\mathbf{p})\xi(\mathbf{p}+\mathbf{Q}) + \Delta^2}{2E(\mathbf{p})E(\mathbf{p}+\mathbf{Q})[E(\mathbf{p})+E(\mathbf{p}+\mathbf{Q})]}dv_{\mathbf{p}}, \quad (24)$$

the gap $\Delta(\lambda_C)$ and the spectra $E(\mathbf{p},\lambda_C)$ and $\xi(\mathbf{p},\lambda_C)$ being given by Eqs. (13) and (17). Eq. (24) serves to determine the critical constant λ_Q^{cr} for given λ_C, and the pure BCS vacuum is destroyed if $\lambda_Q > \lambda_Q^{\mathrm{cr}}$.

Violation of the stability condition for the state with a pure Q-condensate may be analyzed along the same lines. In this case, one employs the second of Eqs. (10) with Δ set identically zero, and determines the critical constant λ_C^{cr} responsible for destroying the pure Q-pairing state as a function of λ_Q. Now, suppose that the two curves $\lambda_C^{\mathrm{cr}}(\lambda_Q)$ and $\lambda_Q^{\mathrm{cr}}(\lambda_C)$ are plotted on the plane (λ_C,λ_Q). If a region is found in which *both* the C- and Q-condensates loose their stability, then the cocktail solution of Eqs. (10) must prevail throughout that region.

Finally, we turn briefly to possible experimental consequences of non-BCS pairing. In conventional superconductors, the linewidth is known to be very narrow, but this would not be the case if non-BCS pairing were to occur (see Eq. (8)). A significant broadening of the sp line is predicted to accompany the cocktail solution. Such a spectral broadening would affect many prominent experimental signatures of pairing, notably the falloff of the specific heat $C(T)$ as $T \to 0$ (slower than in the BCS case), the dependence of the gap value on T, and the behavior of the penetration depth. Another specific feature of the non-BCS solutions is related to possible violation of the property of time-reversal invariance. This property is of course intrinsic to BCS theory, since the ground-state is time-reversal invariant by construction. However, if the total momentum $\mathbf{P}$ of the pairs involved differs from zero, special restrictions are needed to maintain the invariance.

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