Pis'ma v ZhETF, vol. 76, iss. 8, pp. 576 – 578

Theory of exchange coupling in disordered magnetic multilayers

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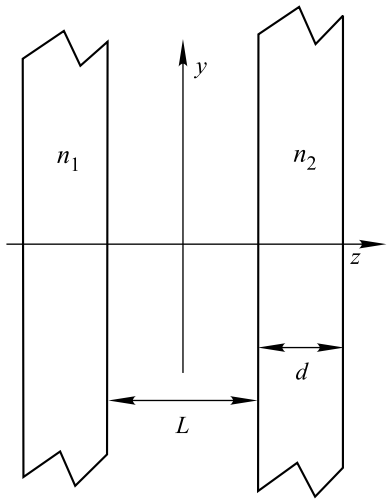
Submitted 11 September 2002

We consider mechamism of exchange coupling based on interaction between electrons in nonmagnetic layer. Depending on ratio of inverse time of diffusion of electrons between ferromagnetic layers and ferromagnetic splitting of conducting electrons this mechanism describes transition from ferromagnetic to noncollinear ordering of magnetizations of ferromagnetic layers.

PACS: 75.70.Cn

1. Introduction and main results. In the metallic ferromagnet-nonferromagnet-ferromagnet multilayers (see figure) magnetic structure oscillates between ferromagnetic and antiferromagnetic orientations of the ferromagnets' magnetizations as a function of thickness of nonmagnetic metal L with a period of order of the Fermi wave length [1 – 6]. The explanation of this phenomenon is based on the fact that the interlayer coupling is due to Ruderman-Kittel interaction between electron spins in different ferromagnets.



Further investigations discovered the structures with perpendicular orientations of the ferromagnets' magnetizations (see for rev. [7]). Often the phenomenological coupling between the magnetizations of ferromagnetic layers in the such structures can be represented as sum of bilinear and biquadratic contributions

$$\Omega(\varphi) = J_1 \cos\varphi + J_2 \cos^2\varphi. \qquad (1)$$

Here φ is an angle between the directions of magnetizations of ferromagnetic films. The bilinear constant J_1 oscillates as function of the interlayer distance L. In the case of the large positive biquadratic constant J_2 the minimum of $\Omega(\varphi)$ corresponds to $\varphi = \pi/2$. As explained by Slonzevskii the large positive biquadratic coupling might be the result of spatial fluctuations of the bilinear coupling J_1 due to the ferromagnet-nonferromagnet surface roughness [7].

In disordered system, where L is larger than the electron mean free path l, RKKY interaction $\langle J_1 \rangle$, averaged over the realizations of scattering potential exponentially decreases [8]. At the same time fluctuations of local exchange become much larger than $\langle J_1 \rangle$ [9] giving rise to biquadratic contribution $J_2 \gg |\langle J_1 \rangle|$ [10].

Here we propose mechanism of coupling in the disordered multilayers based on interaction between electrons in the nonmagnetic layer. Spin fluctuations in the system of interacting electrons give rise to the contribution to thermodynamic potential [11], which depends on magnetic field or, in our case, on the relative orientation of the magnetizations in ferromagnetic layers. Here we show that in magnetic multilayer this mechanism describes transition between the ferromagnetic and noncolinear ordering with increasing the distance between ferromagnetic layers or the value of ferromagnetic splitting of conducting electrons.

We assume that the magnetic multilayer can be described by the Hamiltonian

$$H = H_0 + \epsilon_{ex} \int\limits_F d\mathbf{r} \Psi_\alpha^+(\mathbf{r})\, \mathbf{n}(z)\, \sigma_{\alpha\beta} \Psi_\beta(\mathbf{r}) + H_{int}. \qquad (2)$$

Here H_0 is the Hamiltonian of free electrons in random field. The second term describes the exchange field in ferromagnetic layers. ϵ_{ex} is ferromagnetic splitting of the conducting electrons; $\mathbf{n}(z)$ is unit vector of the direction of magnetization of ferromagnetic layers; $\mathbf{n}(z) = \mathbf{n}_1$ at $z < -L/2$ and $\mathbf{n}(z) = \mathbf{n}_2$ at $z > L/2$ as it is shown on figure; $\Psi_\alpha^+(\mathbf{r})$ and $\Psi_\beta(\mathbf{r})$ are creation and annihilation

operators, $\sigma_{\alpha\beta}$ are Pauli matrixes. Integration in the second term is over the ferromagnetic layers. The last term H_{int} describes Coulomb interaction between electrons in nonmagnetic layer. We assume that the interaction in the ferromagnetic layers is taken into account self consistently in ϵ_{ex}.

The details of calculation are given in the last part of the paper. Here we present the main results. Characteristic energies in the problem are ferromagnetic splitting of conducting electrons ϵ_{ex} and Thouless energy $E_c \equiv D/L^2$. D is diffusion constant of conduction electrons. We assume that it is the same in nonmagnetic and ferromagnetic layers.

In the case of small thickness when $E_c > \epsilon_{ex}$ the coupling between ferromagnetic layers has bilinear form and the coupling energy per unit area is

$$\Omega(\varphi) = -\frac{F}{8(4\pi L)^2}\frac{\epsilon_{ex}^2}{E_c}\cos\varphi. \qquad (3)$$

Here F is characteristic constant of interaction in the diffusion channel [11]. It is positive for the Coulomb repulsion between electrons. Let us note that in this regime coupling (3) does not depend on L. The minimum of (3) corresponds to ferromagnetic orientation of the magnetizations in multilayer $\varphi = 0$. Note that the result is obtained in limit when $L > l$, or ϵ_{ex} smaller that the inverse mean free time D/l^2.

At the larger distance L when $E_c < \epsilon_{ex}$ the coupling has biquadratic form and the coupling energy per unit area is

$$\frac{D}{L^2} \ll \epsilon_{ex} \quad L \gg L \quad \Omega(\varphi) \simeq \frac{F}{(4\pi L)^2}E_c\cos^2\varphi. \qquad (4)$$

This quantity decreases as L^{-4} with increasing distance. The minimum of coupling energy corresponds to the noncollinear state $\varphi = \pi/2$.

Both expressions are given for the case of infinite thickness of the ferromagnetic layers. Calculation shows that in the case $d > \sqrt{D/\epsilon_{ex}}$ the coupling weakly depends on d.

Results (3) and (4) are valid at low temperature T, when $L < \sqrt{D/T}$. At larger temperature coupling energy decreases exponentially as $\exp\left(-\sqrt{(8\pi T/D)}L\right)$.

Let us compare results (3) and (4) with biquadratic contribution due to the mesoscopic fluctuations of RKKY interaction [10], which is $J_2 \sim \frac{1}{L^2}\frac{E_c^2}{Ad}$ at $E_c < \epsilon_{ex}$ and

$$J_2 \sim \frac{1}{L^2}\frac{\epsilon_{ex}^3}{AdE_c} \text{ at } E_c > \epsilon_{ex}.$$

Here A is an intralayer ferromagnetic stiffness and thickness $d > \sqrt{D/\epsilon_{ex}}$.

The quantity J_2 decreases with L much faster than (4). Also for $\epsilon_{ex}/Ad \ll 1$, $F \approx 1$ the coupling energy given by expressions (3) and (4) is larger than the biquadratic contribution due to the mesoscopic fluctuations of RKKY in the whole range of the distances. In this case with increasing distance $L \gg l$ the system is undergo the transition between the ferromagnetic and noncollinear $\varphi = \pi/2$ ordering. Such transition was observed in [12].

2. Derivation of results. The correction to thermodynamic potential which depends on $\epsilon_{ex}\mathbf{n}(z)$ is given by an expression [11]

$$\Omega(\varphi) = \frac{F}{4}T\sum_{|\omega_n\tau|<1;\alpha,\beta}|\omega_n|\int\frac{d^2\mathbf{q}}{(2\pi)^2}\times$$
$$\times\int_{|z|<L/2}dz D_{\beta\beta}^{\alpha\alpha}(z,z,\mathbf{q},\omega_n). \qquad (5)$$

Here constant F describes the screened Coulomb interaction in diffusion channel. $\omega_n = 2\pi nT$ is Matsubara frequency. τ is the electron mean free time.

The diffusion ladder satisfies equation

$$\left(-D\frac{d^2}{dz^2} + Dq^2 + |\omega_n|\right)D_{\mu\eta}^{\alpha\beta} +$$
$$+i\epsilon_{ex}\mathbf{n}(z)\left(\sigma_{\alpha\gamma}D_{\mu\eta}^{\gamma\beta} - D_{\mu\gamma}^{\alpha\beta}\sigma_{\gamma\eta}\right)sign\omega_n = \qquad (6)$$
$$= \delta(z-z')\delta_{\alpha\beta}\delta_{\mu\eta}.$$

It is convenient to present the solution of equation (6) at $|z| < L/2$ in the form

$$D_{\mu\eta}^{\alpha\beta} = A_{\mu\eta}^{\alpha\beta}\exp(-Qz) + U_{\alpha\gamma}^{+}C_{\mu\gamma}^{\gamma\beta}U_{\gamma\eta}\exp(Qz) +$$
$$+\frac{\exp(-Q|z-z'|)}{2DQ}\delta_{\alpha\beta}\delta_{\mu\eta}. \qquad (7)$$

Here we introduce $Q = \sqrt{q^2 + |\omega_n|/D}$, U is matrix of the relative rotation of the magnetizations of ferromagnetic layers. In the case when direction of the magnetization in ferromagnetic layer $z < -L/2$ is directed along z axes $\mathbf{n}(z) = (0,0,1)$ and at $z > L/2$ direction is $\mathbf{n}(z) = (\sin\varphi, 0, \cos\varphi)$, it is matrix of the rotation along y axes $U = \exp(i\varphi\sigma_y/2)$.

For simplicity we consider the limit of the semiinfinite ferromagnetic layers. More detail consideration shows that at $d > \sqrt{D/\epsilon_{ex}}$ the results weakly depend on the thickness of the ferromagnetic layers. It is convenient to introduce the boundary conditions for the diffusion ladder at ferromagnet-nonferromagnet interfaces taking into account that according to equation (6) in the coordinate system where spins are directed along the

magnetization, components of the ladder with antiparallel spins decreases as $\exp(-Q_1|z|)$ and $\exp(-Q_1^*|z|)$ at $|z| > L/2$, where $Q_1 = \sqrt{q^2 + (\omega + i\epsilon_{ex})/D}$. The components of the ladder with parallel spins decreases as $\exp(-Q|z|)$ at $|z| > L/2$. At $z = -L/2$ where $\mathbf{n}(z) = (0,0,1)$ the boundary conditions are

$$\left(\frac{d}{dz} - Q_1\right) P^+_{\alpha\gamma} D^{\gamma\beta}_{\mu\gamma} P^-_{\gamma\eta} = \left(\frac{d}{dz} - Q_1^*\right) P^-_{\alpha\gamma} D^{\gamma\beta}_{\mu\gamma} P^+_{\gamma\eta} = 0, \qquad \left(\frac{d}{dz} - Q\right) P^{\pm}_{\alpha\gamma} D^{\gamma\beta}_{\mu\gamma} P^{\pm}_{\gamma\eta} = 0. \tag{8}$$

Here we introduce projectors of the spins on z-axes $P_{\pm} = (1 \pm \sigma_z)/2$.

The same kind of boundary conditions can be introduced for the rotated diffusion ladder $U_{\alpha\gamma} D^{\gamma\beta}_{\mu\gamma} U^+_{\gamma\eta}$ at $z = L/2$. Solving the system of equations (6), (8) we obtain

$$\Omega(\varphi) = -\frac{F}{2} T \sum_{|\omega_n \tau| < 1} |\omega_n| \int \frac{d^2\mathbf{q}}{(2\pi)^2} \frac{L}{DQ} \times$$
$$\times \frac{\left[\left(|\Lambda|^2 - (\operatorname{Re}\Lambda)^2\right)\cos\varphi + \left(|\Lambda|^4 - \frac{1}{2}(\operatorname{Re}\Lambda)^2\right)\cos^2\varphi - \frac{1}{2}(\operatorname{Re}\Lambda)^2 + \operatorname{Re}\Lambda\left(1 - \frac{1}{2}|\Lambda|^2 - \frac{1}{2}|\Lambda|^2\cos^2\varphi\right)\frac{\sinh QL}{QL}\right]}{\left[1 - (\operatorname{Re}\Lambda)^2 + 2\left(|\Lambda|^2 - (\operatorname{Re}\Lambda)^2\right)\cos\varphi + \left(|\Lambda|^4 - (\operatorname{Re}\Lambda)^2\right)\cos^2\varphi\right]}. \tag{9}$$

Here $\Lambda = [(Q_1 - Q)/(Q_1 + Q)]\exp(-QL)$. Expression (9) contains the divergent terms, which do not depend on φ and must be subtracted.

In the limit of the large exchange splitting when $|Q_1| > Q$ the parameter $\Lambda = \exp(-QL)$ is real. In this case the energy is function of $\cos^2\varphi$. Subtracting in expression (9) terms which do no depend on the angle we obtain

$$\Omega(\varphi) = \frac{F}{4} T \sum_{|\omega_n \tau| < 1; \alpha, \beta} |\omega_n| \times$$
$$\times \int \frac{d^2\mathbf{q}}{(2\pi)^2} \frac{L\left(1 - \Lambda\frac{\sinh QL}{QL}\right)}{DQ} \frac{\Lambda^2\cos^2\varphi}{(1 - \Lambda^2\cos^2\varphi)}. \tag{10}$$

The main contribution in expression (10) is from the region where $\Lambda < 1$. The denominator therefore gives only small correction. Neglecting it we obtain the expression (4).

In the opposite limit of the small exchange splitting $\operatorname{Re}\Lambda \sim \epsilon_{ex}^2$, $|\Lambda| \sim \epsilon_{ex}$ and to the order ϵ_{ex}^2 the coupling energy is proportional to $\cos\varphi$

$$\Omega(\varphi) = -\frac{F}{2} T \sum_{|\omega_n \tau| < 1; \alpha, \beta} |\omega_n| \int \frac{d^2\mathbf{q}}{(2\pi)^2} \frac{L|\Lambda|^2}{DQ} \cos\varphi. \tag{11}$$

Calculating (11) at zero temperature we obtain (3). The transition between the limits (10) and (11) occurs at $\epsilon_{ex} \sim D/L^2$.

This work is supported by Russian Fund for Fundamental Research grant # 01-02-17794.

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