

Transition from non Fermi liquid behavior to Landau Fermi liquid behavior induced by magnetic fields

Yu. G. Pogorelov, V. R. Shaginyan^{*1)}

Departamento de Física, Universidade do Porto, 4150 Porto, Portugal

** Petersburg Nuclear Physics Institute RAS, 188300 Gatchina, Russia*

Submitted 23 September 2002

We show that a strongly correlated Fermi system with the fermion condensate, which exhibits strong deviations from Landau Fermi liquid behavior, is driven into the Landau Fermi liquid by applying a small magnetic field B at temperature $T = 0$. This field-induced Landau Fermi liquid behavior provides the constancy of the Kadowaki-Woods ratio. A re-entrance into the strongly correlated regime is observed if the magnetic field B decreases to zero, then the effective mass M^* diverges as $M^* \propto 1/\sqrt{B}$. At finite temperatures, the strongly correlated regime is restored at some temperature $T^* \propto \sqrt{B}$. This behavior is of general form and takes place in both three dimensional and two dimensional strongly correlated systems. We demonstrate that the observed $1/\sqrt{B}$ divergence of the effective mass and other specific features of heavy-fermion metals are accounted for by our consideration.

PACS: 71.10.Hf, 71.27.+a, 74.72.-h

Recently, a peculiar critical point was observed in heavy-fermion metal YbRh_2Si_2 at low temperatures T [1]. This critical point is driven by the magnetic field B which suppresses the antiferromagnetic order when reaches the critical value, $B = B_{c0}$, while the effective mass M^* diverges as $M^* \propto 1/\sqrt{B - B_{c0}}$ [1]. The study of the magnetic field dependence of the coefficients A , γ_0 , and χ_0 in the resistivity, $\rho(T) = \rho_0 + \Delta\rho$, with $\Delta\rho = A(B)T^2$, specific heat, $C/T = \gamma_0(B)$, and the magnetic ac susceptibility, has revealed that YbRh_2Si_2 behaves as a true Landau Fermi liquid (LFL) for $B > B_{c0}$ and the well-known Kadowaki-Woods ratio A/γ_0 [2] is preserved [1]. In contrast, for $B = 0$, YbRh_2Si_2 demonstrates a non Fermi liquid (NFL) behavior, and the resistivity follows a quasilinear temperature dependence $\Delta\rho \sim T$ down to 80 mK at which antiferromagnetic (AF) order takes place. At lower temperatures, the resistivity in AF-ordered state is described by $\Delta\rho \sim T^2$. A similar picture is observed in heavy-fermion compounds CeMIn_5 ($M = \text{Ir, Co, and Rh}$), where the electronic specific heat C shows more pronounced metallic behavior at sufficiently high magnetic fields [3]. These observations are consistent with the de Haas van Alphen (dHvA) studies of CeIrIn_5 , which find that the effective mass decreases with increasing B [3, 4].

It is pertinent to note that heavy fermion metals are more likely three dimensional (3D) than two dimensional (2D). The origin of NFL behavior observed in heavy-

fermion metals is still a subject of controversy [5]. Moreover, the observed constancy of the Kadowaki-Woods ratio when $B \rightarrow B_{c0}$ [1] leads to the failure of the standard model of heavy-fermion metals, when the mass renormalization is supposed to come from the exchange by soft magnetic fluctuations in a 2D spin fluid [6]. As a result, we are left with even more complicated and challenging problems in the physics of strongly correlated electrons.

In this Letter, we study the nature of the critical behavior, assuming that the fermion condensation phase transition (FCQPT) [7] plays the role of the critical point. Analyzing the appearance of the fermion condensate (FC) which occurs beyond the point of FCQPT in an electron Fermi liquid and induces the NFL behavior, we show that the liquid is driven, by applying a weak magnetic field, back into a specific LFL state with effective mass $M^* \propto 1/\sqrt{B}$. The LFL behavior induced by rather low magnetic fields, provides constancy of the Kadowaki-Woods ratio. But the strongly correlated regime is restored when the magnetic field B decreases to zero, while the effective mass M^* diverges as $M^* \propto 1/\sqrt{B}$. Also the strongly correlated regime is restored at some finite temperature $T^*(B) \propto \sqrt{B}$. Such a behavior is of general form and takes place in both three dimensional and two dimensional strongly correlated systems. We demonstrate that the observed crossover from NFL to LFL in certain heavy-fermion metals is accounted for by our consideration.

For the reader's convenience we first outline the NFL behavior of Fermi systems with FC and the main proper-

¹⁾e-mail: vrshag@thd.pnpi.spb.ru

ties of FCQPT using, as an example, a two-dimensional electron liquid in the superconducting state induced by FCQPT [7, 8]. At $T = 0$, the ground state energy $E_{gs}[\kappa(\mathbf{p}), n(\mathbf{p})]$ is a functional of the superconducting order parameter $\kappa(\mathbf{p})$ and of the quasiparticle occupation function $n(\mathbf{p})$ and is determined by the known equation of the weak-coupling theory of superconductivity (see e.g. [9])

$$E_{gs} = E[n(\mathbf{p})] + \int \lambda_0 V(\mathbf{p}_1, \mathbf{p}_2) \kappa(\mathbf{p}_1) \kappa^*(\mathbf{p}_2) \frac{d\mathbf{p}_1 d\mathbf{p}_2}{(2\pi)^4}. \quad (1)$$

Here $E[n(\mathbf{p})]$ is the ground-state energy of normal Fermi liquid, $n(\mathbf{p}) = v^2(\mathbf{p})$ and $\kappa(\mathbf{p}) = v(\mathbf{p})\sqrt{1 - v^2(\mathbf{p})}$. It is assumed that the pairing interaction $\lambda_0 V(\mathbf{p}_1, \mathbf{p}_2)$ is weak. Minimizing E_{gs} with respect to $\kappa(\mathbf{p})$ we obtain the equation connecting the single-particle energy $\varepsilon(\mathbf{p})$ to the superconducting gap $\Delta(\mathbf{p})$

$$\varepsilon(\mathbf{p}) - \mu = \Delta(\mathbf{p}) \frac{1 - 2v^2(\mathbf{p})}{2\kappa(\mathbf{p})}, \quad (2)$$

including the chemical potential μ . Here the single-particle energy $\varepsilon(\mathbf{p})$ is determined by the Landau equation [10]

$$\varepsilon(\mathbf{p}) = \frac{\delta E[n(\mathbf{p})]}{\delta n(\mathbf{p})}, \quad (3)$$

while the equation for the superconducting gap $\Delta(\mathbf{p})$ takes the form

$$\Delta(\mathbf{p}) = -\lambda_0 \int V(\mathbf{p}, \mathbf{p}_1) \kappa(\mathbf{p}_1) \frac{d\mathbf{p}_1}{4\pi^2}. \quad (4)$$

If $\lambda_0 \rightarrow 0$, then, the maximum value of the superconducting gap $\Delta_1 \rightarrow 0$, and Eq. (2) reduces to that proposed in Ref. [11]

$$\varepsilon(\mathbf{p}) - \mu = 0, \text{ if } \kappa(p) \neq 0 \text{ (} 0 < n(p) < 1 \text{)} \quad (5)$$

$$\text{for } p \in L_{FC} : p_i \leq p \leq p_f.$$

At $T = 0$, Eq. (5) defines a new state of Fermi liquid with FC, such that the modulus of the order parameter $|\kappa(\mathbf{p})|$ has finite values in the FC range of momenta L_{FC} : $p_i \leq p \leq p_f$, while the superconducting gap can be infinitely small, $\Delta_1 \rightarrow 0$, in this range [7, 11, 12]. Such a state can be considered as superconducting, with infinitely small value of Δ_1 so that the entropy of this state is zero. This state, created by the quantum phase transition, disappears at $T > 0$. The FCQPT can be considered as a “pure” quantum phase transition because it cannot take place at finite temperatures. Generally, this

quantum critical point should not represent the termination at $T > 0$ of a line of continuous transitions. However it corresponds to a certain critical value of density $x = x_{FC}$ (the critical point of FCQPT) which is determined also by Eq. (5). Notice that at finite temperatures the FCQPT continues to have a strong impact on the system properties, up to a certain temperature T_f above which FC effects become insignificant [7, 13]. FCQPT does not violate rotational or translational symmetry of the order parameter $\kappa(\mathbf{p})$. It follows from Eq. (5) that the quasiparticle system “splits” into two quasiparticle subsystems: one in the L_{FC} range, occupied by the quasiparticles with enhanced effective mass $M_{FC}^* \propto 1/\Delta_1$, and another with LFL effective mass M_L^* at $p < p_i$. If $\lambda_0 \neq 0$, then Δ_1 becomes finite, and the finite value of the effective mass M_{FC}^* in L_{FC} can be obtained from Eq. (2) as [7, 8]

$$M_{FC}^* \simeq p_F \frac{p_f - p_i}{2\Delta_1}, \quad (6)$$

while the effective mass M_L^* is only weakly disturbed. Here p_F is the Fermi momentum. It follows from Eq. (6) that the quasiparticle dispersion can be presented by two straight lines characterized by the effective masses M_{FC}^* and M_L^* respectively. These lines intersect near the electron binding energy E_0 which defines an intrinsic energy scale of the system:

$$E_0 = \varepsilon(p_f) - \varepsilon(p_i) \simeq \frac{(p_f - p_i)p_F}{M_{FC}^*} \simeq 2\Delta_1. \quad (7)$$

Let us assume that FC has just taken place, that is $p_i \rightarrow p_F \leftarrow p_f$, the deviation $\delta n(p)$ from LFL occupation function is small (though finite), and $\lambda_0 \rightarrow 0$. Expanding the functional $E[n(p)]$ in Eq. (3) in Taylor’s series with respect to $\delta n(p)$ and retaining the leading terms, we have

$$\Delta E = \sum_{\sigma_1} \int \varepsilon_0(\mathbf{p}_1, \sigma_1) \delta n(\mathbf{p}_1, \sigma_1) \frac{d\mathbf{p}_1}{(2\pi)^2} + \quad (8)$$

$$+ \sum_{\sigma_1 \sigma_2} \int F_L(\mathbf{p}_1, \mathbf{p}_2, \sigma_1, \sigma_2) \delta n(\mathbf{p}_1, \sigma_1) \delta n(\mathbf{p}_2, \sigma_2) \frac{d\mathbf{p}_1 d\mathbf{p}_2}{(2\pi)^4},$$

where $F_L(\mathbf{p}_1, \mathbf{p}_2, \sigma_1, \sigma_2) = \delta^2 E / \delta n(\mathbf{p}_1, \sigma_1) \delta n(\mathbf{p}_2, \sigma_2)$ is the Landau interaction, and σ denotes the spin states. Varying both sides of Eq. (8) with respect to the functions $\delta n(p)$ and taking into account Eq. (5), one obtains the FC equation

$$\mu = \varepsilon(\mathbf{p}, \sigma) = \varepsilon_0(\mathbf{p}, \sigma) + \quad (9)$$

$$+ \sum_{\sigma_1} \int F_L(\mathbf{p}, \mathbf{p}_1, \sigma, \sigma_1) \delta n(\mathbf{p}_1, \sigma_1) \frac{d\mathbf{p}_1}{(2\pi)^2},$$

$$p_i \leq p \leq p_f \in L_{FC}.$$

Equation (9) acquires non-trivial solutions at the density $x = x_{FC}$ if the Landau amplitude F_L (depending on density) is positive and sufficiently large, so that the potential energy integral on the right hand side of Eq. (9) prevails over the kinetic energy $\varepsilon_0(\mathbf{p})$ [11]. Note, that in case of heavy fermion metals, this condition can be easily satisfied because of the huge effective mass. It is also seen from Eq. (9) that the FC quasiparticles form a collective state, since their energies are defined by the macroscopical number of quasiparticles within L_{FC} , and vice versa. The shape of their spectrum is not affected by the Landau interaction, which, generally speaking, depends on the system properties, including the collective states, impurities, etc. The only thing defined by the interaction is the width $p_i - p_f$ of L_{FC} (provided it exists). Thus, we can conclude that the spectra related to FC are of universal form.

At temperatures $T \geq T_c$ when Δ_1 disappears, Eq. (6) for the effective mass M_{FC}^* is changed for [7, 8]

$$M_{FC}^* \simeq p_F \frac{p_f - p_i}{4T}. \quad (10)$$

The energy scale separating the slow dispersing low energy part, defined by the effective mass M_{FC}^* , from the faster dispersing relatively high energy part, defined by the effective mass M_L^* , can be estimated as $E_0 \simeq p_F(p_f - p_i)/M_{FC}^*$ [7, 8], so for the case of Eq. (10) it is

$$E_0 \simeq 4T. \quad (11)$$

It follows from Eq. (10) that M^* depends on the temperature, and the width γ of the single-particle excitations results $\gamma \sim T$, leading to a linear temperature dependence $\Delta\rho \sim T$ [13]. This contrasts with the well-known LFL relations: $\gamma \sim T^2$, and $\Delta\rho \sim T^2$.

Now we consider the behavior of an electronic system with FC in magnetic fields, supposing the coupling constant $\lambda_0 \neq 0$ to be infinitely small. As we have seen above, at $T = 0$, the superconducting order parameter $\kappa(\mathbf{p})$ is finite in the FC range, while the maximum value of the superconducting gap $\Delta_1 \propto \lambda_0$ is infinitely small. Therefore, any small magnetic field $B \neq 0$ will destroy the coherence of $\kappa(\mathbf{p})$ and thus FC itself. Also, the existence of FC can not be compatible with the evident Zeemann splitting of quasiparticle energy bands $\varepsilon(\mathbf{p}, \sigma) = \varepsilon(\mathbf{p}) - \sigma\mu_{eff}B$ (see below for μ_{eff}). To define the type of FC restructuring, simple energy reasons are invoked. On the one hand, the energy gain ΔE_B due to the magnetic field B is $\Delta E_B \propto B^2$ and tends to zero with $B \rightarrow 0$. On the other hand, occupying the finite range L_{FC} in the momentum space, FC delivers a finite gain in the ground state energy [11]. Thus, a new state

replacing FC should be very close in its ground state energy to the former state. Such a state is given by the multiconnected Fermi sphere, where the smooth quasiparticle distribution function $n(\mathbf{p})$ in the L_{FC} range is replaced by a multiconnected distribution $\nu(\mathbf{p})$ [14]

$$\nu(\mathbf{p}) = \sum_{k=1}^n \theta(p - p_{2k-1})\theta(p_{2k} - p), \quad (12)$$

where the parameters $p_i \leq p_1 < p_2 < \dots < p_{2n} \leq p_f$ are adjusted to obey the normalization conditions:

$$\int_{p_{2k}}^{p_{2k+3}} \nu(\mathbf{p}) \frac{d\mathbf{p}}{(2\pi)^3} = \int_{p_{2k}}^{p_{2k+3}} n(\mathbf{p}) \frac{d\mathbf{p}}{(2\pi)^3}. \quad (13)$$

For the definiteness sake, we consider the most interesting case of 3D system, while the consideration of a 2D system goes along the same line. We note that the idea of multiconnected Fermi sphere, with production of new, interior segments of the Fermi surface, has been considered already [15, 16]. Let us assume that the thickness of each interior block is approximately the same $p_{2k+1} - p_{2k} \simeq \delta p$ and δp is defined by B . Then, the single-particle energy in the region L_{FC} is given can be fitted by

$$\varepsilon(\mathbf{p}) - \mu \sim \mu \frac{\delta p}{p_F} \left[\sin\left(\frac{p}{\delta p}\right) + b(p) \right]. \quad (14)$$

The blocks are formed since all the single particle states around the minimum values of the fast sine function are occupied and those around its maximum values are empty, the average occupation being controlled by a slow function $b(\mathbf{p}) \approx \cos[\pi n(\mathbf{p})]$. It is seen from Eq. (14) that the effective mass m^* at each internal Fermi surface is of the order of the bare mass m_0 , $m^* \sim m_0$. Upon substituting $n(\mathbf{p})$ in Eq. (8) by $\nu(\mathbf{p})$, defined by Eqs. (12) and (13), and taking into account the Simpson's rule, we obtain that the minimum loss in the ground state energy due to the formation of blocks is about $(\delta p)^4$. This result can be understood if one bears in mind that the continuous FC function $n(\mathbf{p})$ delivers the minimum value to the energy functional, Eq. (8), while the approximation $\nu(\mathbf{p})$ by steps of size δp produces the minimum error of the order of $(\delta p)^4$. On the other hand, this loss must be compensated by the energy gain due to the magnetic field. Thus, we arrive at

$$\delta p \propto \sqrt{B}. \quad (15)$$

With an account taken of the Zeemann splitting in the dispersion law, Eq. (14), each of the blocks is polarized, since its outer areas are occupied only by spin-up qua-

siparticles. The width of this areas in the momentum space δp_0 is given by

$$\frac{p_F \delta p_0}{m^*} \sim B \mu_{eff}, \quad (16)$$

where $\mu_{eff} \sim \mu_B$ is the effective moment. We can consider such a polarization, without perturbing the previous estimates, since it is seen from Eq. (15) that $\delta p_0 / \delta p \ll 1$. The total polarization ΔP is obtained multiplying δp_0 by the number N of the blocks which is proportional to $1/\delta p$, $N \sim (p_f - p_i)/\delta p$. Taking into account Eq. (15), we obtain

$$\Delta P \sim m^* \frac{p_f - p_i}{\delta p} B \mu_{eff} \propto \sqrt{B}, \quad (17)$$

and thus it prevails over $\sim B$ contribution from the LFL part. On the other hand, this quantity can be expressed as

$$\Delta P \propto M^* B, \quad (18)$$

where M^* is the ‘‘average’’ effective mass related to the finite density of states at the Fermi level,

$$M^* \sim N m^* \propto \frac{1}{\delta p}. \quad (19)$$

We can also conclude that M^* defines the specific heat.

Otherwise Eq. (15) can be examined, starting from a different point surmising that multiconnected Fermi sphere can be approximated by a single block. Let us put $\lambda_0 = 0$. Then, the energy gain due to the magnetic field $\Delta E_B \sim B^2 M^*$. The energy loss ΔE_{FC} because of the restructuring of the FC state can be estimated by using the Landau formula which directly follows from Eqs. (8) and (9)

$$\Delta E_{FC} = \int (\varepsilon(\mathbf{p}) - \mu) \delta n(\mathbf{p}) \frac{d\mathbf{p}^3}{(2\pi)^3}. \quad (20)$$

As we have seen above, the region occupied by variation $\delta n(\mathbf{p})$ has the length δp , while $(\varepsilon(\mathbf{p}) - \mu) \sim (p - p_F) p_F / M^*$. As a result, we have, $\Delta E_{FC} = \delta p^2 / M^*$. Upon equating ΔE_B and ΔE_{FC} and taking into account Eq. (19), we arrive at the following equation

$$\frac{\delta p^2}{M^*} \propto \delta p^3 \propto \frac{B}{\delta p}, \quad (21)$$

which coincides with Eq. (15).

It follows from Eqs. (17) and (18) that the effective mass M^* diverges as

$$M^* \propto \frac{1}{\sqrt{B}}. \quad (22)$$

Equation (22) shows that, by applying a magnetic field B , the system can be driven back into LFL with the effective mass $M^*(B)$ dependent on the magnetic field. It was demonstrated that the constancy of the Kadowaki-Woods ratio is obeyed by systems in the strongly correlated regime when the effective mass is sufficiently large [17]. Therefore, we are led to the conclusion that, by applying magnetic fields, the system is driven back into LFL where the constancy of the Kadowaki-Woods ratio is obeyed. Since the resistivity $\Delta \rho \propto (M^*)^2$ [17], we obtain from Eq. (22)

$$\Delta \rho \propto \frac{1}{B}. \quad (23)$$

At finite temperatures, the system persists to be LFL, but there is a temperature $T^*(B)$ at which the polarized state is destroyed. To calculate the function $T^*(B)$, we observe that the effective mass M^* characterizing the single particle spectrum cannot be changed at $T^*(B)$. In other words, at the crossover point, we have to compare the effective mass defined by T^* , Eq. (10), and that defined by the magnetic field B

$$\frac{1}{M^*} \propto T^* \propto \sqrt{B}. \quad (24)$$

As a result, we obtain

$$T^*(B) \propto \sqrt{B}. \quad (25)$$

At temperatures $T \geq T^*$, the system comes back into the state with M^* defined by Eq. (10), and we observe the NFL behavior. It follows from Eq. (25), that a heavy fermion system at some temperature T can be driven back into LFL by applying strong enough magnetic field $B \geq B_{cr} \propto (T^*)^2$. We can also conclude, that at finite temperature T , the effective mass of a heavy fermion system is relatively field independent at magnetic fields $B \leq B_{cr}$ and show a more pronounced metallic behavior at $B \geq B_{cr}$ since the effective mass is decreased, see Eq. (22). The same behavior of the effective mass can be observed in the dHvA measurements. We note that our consideration is valid up to temperatures $T \ll T_f$.

Now we are in position to consider the nature of the field-induced quantum critical point in YbRh₂Si₂. The properties of this AF heavy fermion metal with the ordering Neel temperature $T_N = 70$ mK were recently investigated in Refs. [1, 18]. In AF state, this metal shows LFL behavior. As soon as the weak AF order is suppressed either by a tiny volume expansion or by temperature, pronounced deviations from LFL behavior are observed. The experimental facts show that the spin density wave picture is failed when dealing with

the obtained data [1, 6, 18]. We assume that the electron density in YbRh_2Si_2 is close to the critical value $(x_{\text{FC}} - x)/x_{\text{FC}} \ll 1$ [19], so that this system can be easily driven across FCQPT. Then, in the AF state, the effective mass is given by Eq. (22) and the electron system of YbRh_2Si_2 possesses LFL behavior. When the AF state is suppressed at $T > T_N$ the system comes back into NFL. By tuning $T_N \rightarrow 0$ at a critical field $B = B_{c0}$, the itinerant AF order is suppressed and replaced by spin fluctuations [18]. Thus, we can expect absence of any long-ranged magnetic order in this state, and the situation corresponds to a paramagnetic system with strong correlation in the field $B = 0$. As a result, the FC state is restored and we can observe NFL behavior at any temperatures in accordance with experimental facts [1]. As soon as an excess magnetic field $B > B_{c0}$ is applied, the system is driven back into LFL. To describe the behavior of the effective mass, we can use Eq. (22) substituting B by $B - B_{c0}$

$$M^* \propto \frac{1}{\sqrt{B - B_{c0}}}. \quad (26)$$

Equation (26) demonstrates the $1/\sqrt{B - B_{c0}}$ divergence of the effective mass, and therefore the coefficients $\gamma_0(B)$ and $\chi_0(B)$ should have the same behavior. Meanwhile the coefficient $A(B)$ diverges as $1/(B - B_{c0})$, being proportional to $(M^*)^2$ [17], and thus preserving the Kadowaki-Woods ratio, in agreement with experimental findings [1]. To construct $T - B$ phase diagram for YbRh_2Si_2 we use the same replacement $B \rightarrow B - B_{c0}$ in Eq. (25)

$$T^*(B) \propto \sqrt{B - B_{c0}}. \quad (27)$$

The phase diagram given by Eq. (27) is in good qualitative agreement with the experimental data [1]. We recall that our consideration is valid at temperatures $T \ll T_f$. The experimental phase diagram shows that the behavior $T^* \propto \sqrt{B - B_{c0}}$ is observed up to 150 mK [1] and

allows us to estimate the magnitude of T_f which can reach at least 1 K in this system.

To conclude, we have demonstrated that a new type of the quantum critical point observed in heavy-fermion metal YbRh_2Si_2 can be identified as FCQPT.

This work was supported in part by the Russian Foundation for Basic Research, # 01-02-17189.

1. P. Gegenwart et al., Phys. Rev. Lett. **89**, 056402 (2002); P. Gegenwart et al., cond-mat/0207570.
2. K. Kadowaki and S. B. Woods, Solid State Commun. **58**, 507 (1986).
3. J. S. Kim et al., Phys. Rev. **B64**, 134524 (2001).
4. Y. Haga et al., Phys. Rev. **B63**, 060503 (2001).
5. P. Coleman et al., J. Phys. Condens. Matter **13**, R723 (2001).
6. P. Coleman, cond-mat/0206003.
7. M. Ya. Amusia and V. R. Shaginyan, JETP Lett. **73**, 232 (2001); S. A. Artamonov and V. R. Shaginyan, JETP **92**, 287 (2001); M. Ya. Amusia and V. R. Shaginyan, Phys. Rev. **B63**, 224507 (2001).
8. M. Ya. Amusia, S. A. Artamonov, and V. R. Shaginyan, JETP Lett. **74**, 435 (2001).
9. D. R. Tilley and J. Tilley, *Superfluidity and Superconductivity*, Bristol, Hilger, 1975.
10. L. D. Landau, Sov. Phys. JETP **3**, 920 (1956).
11. V. A. Khodel and V. R. Shaginyan, JETP Lett. **51**, 553 (1990); V. A. Khodel, V. R. Shaginyan, and V. V. Khodel, Phys. Rep. **249**, 1 (1994).
12. G. E. Volovik, JETP Lett. **53**, 222 (1991).
13. J. A. Dukelsky et al., Z. Phys. **B102**, 245 (1997).
14. S. A. Artamonov, V. R. Shaginyan, and Yu. G. Pogorelov, JETP Lett. **68**, 942 (1998).
15. M. de Llano and J. P. Vary, Phys. Rev. **C19**, 1083 (1979); M. de Llano, A. Plastino, and J. G. Zabolitsky, Phys. Rev. **C20**, 2418 (1979).
16. M. V. Zverev and M. Baldo, cond-mat/9807324.
17. V. A. Khodel and P. Schuck, Z. Phys. **B104**, 505 (1997).
18. K. Ishida et al., Phys. Rev. Lett. **89**, 107202 (2002).
19. V. R. Shaginyan, cond-mat/0208568.