Anatomy of Isolated Monopole in Abelian Projection of $SU(2)$ Lattice Gauge Theory

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We study the structure of the isolated static monopoles in the maximal Abelian projection of $SU(2)$ lattice gluodynamics. Our estimation of the monopole radius is: $R^{\rm mon} \approx 0.06\,{\rm fm}$.

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1. The monopole confinement mechanism in $SU(2)$ lattice gauge theory is confirmed by many numerical calculations (see e.g. reviews [1]). In the maximal Abelian projection monopole currents form one big cluster and several small clusters. The big cluster, infrared (IR) cluster, percolates and has a nontrivial fractal dimension, $D_f > 1$ [2]. The properties of small, ultraviolet (UV), clusters differs much from those of the IR cluster, it can be shown that the IR monopole cluster is responsible for the confinement of quarks [3]. As it was shown in the recent publication [4] the structure of IR and UV monopoles is completely different, and monopoles in IR clusters are condensed due to their special anatomy. In this publication we study the structure of Abelian monopoles in $SU(2)$ lattice gauge theory in a different way than it was done in ref. [4]. We study the structure of the isolated monopoles, the results show that nontrivial monopole anatomy plays crucial role in the confinement phenomenon.

2. The plaquette action of the compact electrodynamics (cQED),

$$S^P_{cQED} = \beta_{U(1)} \cos\theta_P, \qquad (1)$$

is close to the action of $SU(2)$ lattice gauge theory in the maximal Abelian projection at small values of the bare charge g (in the continuum limit of gluodynamics). The proof is as follows. By definition the maximal Abelian projection corresponds to the maximization of the functional R with respect to all gauge transformations Ω:

$$\max_\Omega R[U_l^\Omega], \quad U_l^\Omega = \Omega^+ U_l \Omega,$$
$$R[U_l] = \sum_l Tr[\sigma_3 U_l^+ \sigma_3 U_l] = \sum_l \cos 2\varphi_l, \qquad (2)$$

here we use the standard parametrization of the link matrix, $U_{l,11} = U^*_{l,22} = \cos\varphi_l e^{i\theta_l}$, $U_{l,12} = U^*_{l,21} = \sin\varphi_l e^{-i\chi_l}$. Thus, the maximization of R, eq.(2), corresponds to the maximization of the modules of the diagonal elements $U_{l,11}$, $U_{l,22}$. The $SU(2)$ plaquette action is $S^P_{SU(2)} = \beta\cdot\frac{1}{2}\,{\rm Tr}\,U_P = \beta\cos\theta_l\cos\varphi_l$, and at large values of β in the maximal Abelian projection $\cos\varphi_l$ is close to unity (due to (2)), φ_l is small and $SU(2)$ plaquette action has the form:

$$S^P_{SU(2)} = \\ = \beta\left[\cos\theta_P \cos\varphi_1 \cos\varphi_2 \cos\varphi_3 \cos\varphi_4 + O(\sin\varphi_l)\right]. \qquad (3)$$

3. The larger value of β, the smaller $\sin\varphi_l$, and $S^P_{SU(2)}$ (3) coincides with $S^P_{\rm cQED}$ (1) in the limit $\beta\to\infty$. On the other hand at small values of the bare charge (at large values of $\beta_{U(1)}$) the compact electrodynamics is in the deconfinement phase, and gluodynamics is in the confinement phase; on the other hand the actions of both theories are close to each other. The explanation of this paradox was given in Refs. [5, 4], it was shown that the action of the non-diagonal gluons, $S^{\rm off}$, on the plaquettes near the monopole from IR clusters is negative, and the full non-Abelian action, $S^{SU(2)} = S^{\rm off} + S^{\rm Abel}$, is smaller than the Abelian part of the action. The standard qualitative proof of the existence of the deconfinement phase transition in cQED is the representation of the partition function as the sum over the monopole trajectories of length L:

$$\mathcal{Z} = \sum_L \exp\{-\beta L c\}(7)^L, \qquad (4)$$

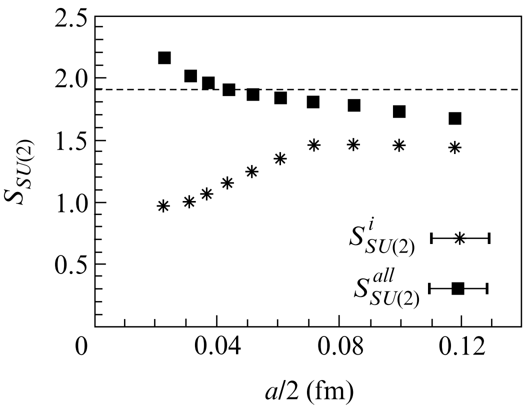
here c is the action of the unit length of the monopole trajectory, 7^L is the entropy of the line of the length L drawn on $4D$ hypercubic lattice. It is clear that at $\beta = \beta_c \equiv \ln 7/c$ there exists the phase transition in the sum (4). This phase transition is absent in lattice gluodynamics since in this case the monopoles have nontrivial structure and the action of the unit of monopole trajectory in lattice units depends on β: $c = c(\beta)$ the sum (4) is always divergent, the monopoles are condensed and form the percolating cluster. The monopole condensation was proven in gluodynamics by several independent calculations [6].

4. In ref. [4] the average nonabelian action on the plaquettes near the monopole trajectory in IR clusters has been measured. Since the lattice spacing a depends on β the calculations at various β correspond to the measurement of the field strength at various distances, $a(\beta)/2$, from the monopole center. Below we present the results of another measurement, we calculate the average field strength on the plaquettes closest to the monopole center for monopoles which satisfy the following two conditions:

(i) the link with the monopole current has the same direction as the previous and subsequent monopole current links;

(ii) there are no other monopoles at the distance less than $2a$ from the considered monopole, except of monopoles discussed at point (i).

Thus we study "static" and "standing along monopoles", we call such monopoles as *isolated* monopoles. The results of the calculations are shown on Figure where we plot the dependence of $S^i_{SU(2)} = 6\beta \cdot \frac{1}{2}(< \mathrm{Tr}U_P^{\mathrm{imon}} > - < \mathrm{Tr}U_P >)$, on $a/2$; U_P^{imon} are the plaquette matrices corresponding to plaquettes closest to the isolated monopole,



The dependence of $S^i_{SU(2)}$ (stars) and $S^{all}_{SU(2)}$ (squares) on $a/2$. The dashed line corresponds to $S = \ln 7$

the normalization of $S^i_{SU(2)}$ is such that it exactly corresponds to the action of the unit length of the monopole trajectory. If $S^i_{SU(2)} < \ln 7$ the isolated monopoles are condensed (see the discussion of the partition function (4)). On Figure we also show the quantity $S^{\mathrm{all}}_{SU(2)} = 6\beta \cdot \frac{1}{2}(< \mathrm{Tr}U_P^{\mathrm{allmon}} > - < \mathrm{Tr}U_P >)$, here U_P^{allmon} is the matrix corresponding to plaquettes closest to all monopoles (isolated and not isolated).

The main conclusion from Figure is that the action of isolated monopoles decreases when we approach the monopole center, and that these monopoles are condensed. Our numerical results also show that the *abelian* part of the action of the isolated monopoles increases when we approach the monopole center. Thus the contribution of nondiagonal gluons to nonabelian action of monopole is negative, and just due to that these monopoles differs from monopoles in cQED and are condensed at any value of β.

5. Following ref. [4] we can estimate the radius of the isolated monopole, R_m, as the point of the maximal derivative of the function $S^i_{SU(2)}(a/2)$, we thus get: $R_m \approx 0.065$ fm. Note that other dimensional numbers which characterize the gluodynamic vacuum are an order of magnitude larger. For example, the average intermonopole distance [4], which can be estimated from the results of ref. [3] is: $R_m \approx 0.5$ fm; the width of the abelian confining flux tube is: $R_t \approx 0.3$ fm [7]; the average instanton radius is: $R_I \approx 0.3$ fm (see [8] and references therein). Thus we see that in the QCD vacuum there exists a rather small scale, 0.065 fm, such small scale was already discussed in various studies of QCD vacuum [9, 4].

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