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Formation of shocks related to dust particle charging in complex plasmas

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The non-stationary problem of the evolution of perturbation and its transformation into nonlinear wave structure in complex plasmas (multicomponent plasmas containing ions, electrons, charged microspheres or dust grains, and neutral gas) is considered. For this purpose the model, which takes into account the variation of the ion density and the ion momentum dissipation due to dust particle charging as well as the source of plasma particles due to ionization process, is developed. The model is appropriate for the description of laboratory experiments in complex plasmas and contains all basic mechanisms responsible for the formation of a new kind of shock waves which are related to the anomalous dissipation due to dust particle charging process. The consideration on the basis of this model allows us to obtain shock structures as a result of evolution of an initial perturbation and to explain the experimental value of the width of the ion acoustic shock wave front as well as the shock wave speed. The solution of the problem of the evolution of perturbation and its transformation into shock wave in complex plasmas opens up possibilities for description of the real phenomena like supernova explosions as well as of the laboratory and active space and geophysical experiments.

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At present a major portion of the investigations of plasmas is devoted to multicomponent plasmas containing electrons, ions, charged microspheres or dust grains, and neutral particles. The term "complex plasmas" is finding increasing use for such plasmas. Complex (dusty) plasma systems cannot usually survive in the absence of either external sources of electrons and ions or plasma particle fluxes from the regions where there is no dust. The fluxes of electrons and ions are absorbed by dust particles that results in variable charges of the latter. The strong dissipativity of the complex plasma system originating from the dust particle charging processes [1] points to the exceptional role of the dissipative structures (like shock waves) in complex plasmas.

Shock waves often arise in nature because of a balance between wave breaking nonlinear and wave damping dissipative forces. Collisional and collisionless shock waves can appear because of friction between the particles [2] and wave-particle interaction [3], respectively. In complex plasmas an appearance of anomalous dissipation which originates from the charging processes results in a possibility of existence of a new kind of shock waves related to this dissipation. They are collisionless in the sense that they do not involve electron–ion collisions. However, in contrast to the classical collisionless shock waves, the dissipation due to dust charging involves interaction of the electrons and ions with the dust grains in the form of microscopic grain currents. The case when the shock waves related to the dust particle charging process are rather intensive corresponds to the ion acoustic wave propagation. The basic theoretical results on ion acoustic shocks in complex plasmas are obtained in [4 – 8]. Recently the first laboratory experimental results confirming the effect of negatively charged dust on ion acoustic shock formation have been obtained [7, 9]. The problem of shock waves in complex plasmas is considered now in the dusty plasma community as one of the key problems. The importance of shock waves in complex plasmas is associated, in particular, with different astrophysical and geophysical applications [5, 6, 10]. For example, the investigation of such shocks can be important for the description of the process of star and planet formation, shocks in supernova explosions, particle acceleration in shocks, the explanation of the effects in active experiments which involve the release of some gaseous substance in the Earth's ionosphere, etc.

In spite of the importance of shock structures in complex plasmas, the question whether the evolution of an arbitrary perturbation leads to the formation of shocks in a charge–varying complex plasma is still an open question. All previous investigations dealt with the steady–state or steady–state shock–like wave solutions. However, it is the solution of the problem of the evolution of perturbation and the possibility of its transformation into shock wave that can allow us to investigate in detail (with taking into account the charge–varying macro particles) the real phenomena like supernova explosions as well as the laboratory and active space experiments.

Furthermore, the main theoretical results which concern the ion acoustic shocks in complex plasmas are obtainted in two ways:

(1) by using and solving the exact equation for dust particle charge variation (see, e.g., [4]);

(2) by using a Korteweg – de Vries-Burgers equation with the dissipation coefficient (proportional to the frequency of collisions between ions and dust particles) (see, e.g., [7, 8]).

Both these approaches use the ion continuity equation with zero righ–hand side (see [4, 8]). This means that the total ion density is constant. This assumption in complex plasmas can be valid, e.g., if the ions and electrons entering the dust grain recombine into neutral atoms, which then re–enter the plasma and re–ionize, thus preserving the number of ions and electrons. However, in most laboratory complex plasmas this assumption is violated. As it has been mentioned, complex plasma systems cannot usually survive in the absence of either external sources of electrons and ions or plasma particle fluxes from the regions where there is no dust. In laboratory experiments the external source of plasma particles is usually due to the ionization process. Thus, more appropriate model for the description of laboratory experiments has to include the effects of the variation of the ion density and the ion momentum dissipation due to dust particle charging as well as the source of plasma particles due to ionization process.

In this study we develop the model based on a set of fluid equations, Poisson's equation, and a charging equation for dust, which takes into account the variation of the ion density and the ion momentum dissipation due to dust particle charging as well as the ionization process. This model is the nonstationary analog of the model used for the description of dust voids (see [11]). We compare the computational results with the experimental data [9].

We assume that the following simplifying approximations are valid:

1) the plasma can be considered as the uniform unmagnetized one;

2) the time scale corresponds to ion acoustic wave propagation;

3) the dust particle charge variation is solely due to the microscopic electron and ion grain currents originating from the potential difference between the plasma and the grain surface;

4) the average radius a of the dust particles is much smaller than the electron Debye length λ_D, the spatial scale of the perturbations, and the distance between the plasma particles;

5) the dust grains are negatively and heavily charged (with the absolute values which can exceed $10^3 e$, where $-e$ is the electron charge);

6) the dust particles are massive ($m_i Z_d \ll m_d$, where $m_{i,d}$ are the ion and dust masses, $q_d(x) = -Z_d e$ is the dust particle average charge. Then the dust can be considered as stationary and its density n_d is constant in the ion acoustic time scale [12, 13];

7) in the absence of perturbations the quasineutrality condition $n_{i0} = n_{e0} + Z_d n_d$ (where $n_{e(i)}$ is the electron (ion) density, the subscript 0 denotes unperturbed quantities) holds;

8) the ions are singly charged.

9) the electron (T_e) and ion (T_i) temperatures are approximated to be spatially uniform;

10) the orbit–limited probe model [14, 15] is valid;

11) nonlinear waves propagate along the axis x.

Furthermore, we neglect any heat transfer processes, which might influence the propagation and evolution of ion acoustic perturbation. Analogously to [11], the most noteworthy approximation is that we do not include ion–neutral collisions, which would exert a drag force on the ions. They reduce the ion velocity, which affects the ion drag force on a dust particle as well as the dust particle's charge. Neglect of the direct influence of neutrals on dust particles is justified by the consideration of ion acoustic time scales so that the dust can be considered as stationary. Finally, we assume that in the absence of perturbations in plasmas the number of electrons and ions is constant due to competition of the processes of their recombination on dust particles and ionization.

In this case the evolution equations for ion density n_i and velocity v_i take the form (cf. [11, 16])

$$\partial_t n_i + \partial_x (n_i v_i) = -\nu_{\text{ch}} n_i + \nu_i n_e, \tag{1}$$

$$\partial_t (n_i v_i) + \partial_x (n_i v_i^2) = -\frac{e n_i}{m_i} \partial_x \varphi - \tilde{\nu} n_{0i} v_i, \tag{2}$$

where φ is the electrostatic potential, ν_i is the plasma ionization frequency, which increases exponentially with T_e and also depends on the atomic parameters of the neutral gas [17], ν_{ch} is the frequency of ion recombination on dust particles,

$$\nu_{\text{ch}} = \nu_q \frac{Z_{d0} d}{1 + Z_{d0} d} \frac{(T_i/T_e + z_0)}{z_0 (1 + T_i/T_e + z_0)}, \tag{3}$$

$\nu_q = \omega_{pi}^2 a (1 + z_0 + T_i/T_e) / \sqrt{2\pi} v_{Ti}$ is the dust particle charging frequency, $z = Z_d e^2 / a T_e$, $d = n_{d0}/n_{e0}$, $\omega_{pi} = \sqrt{4\pi n_{i0} e^2 / m_i}$ is the ion plasma frequency, $v_{Ti} = \sqrt{T_i/m_i}$ is the ion thermal speed, $\tilde{\nu}$ is the frequency characterizing a loss in ion momentum due to recombination on dust particles and Coulomb elastic collisions between ions and dust,

$$\tilde{\nu} = \nu_q \frac{Z_{d0}d}{(1+Z_{d0}d)\, z_0\, (1+T_i/T_e+z_0)} \times$$
$$\times \left(z_0 + \frac{4T_i}{3T_e} + \frac{2z_0^2 T_e}{3T_i}\Lambda \right), \qquad (4)$$

$\Lambda = \ln(\lambda_{Di}/\max\{a,b\})$ is the Coulomb logarithm, λ_{Di} is the ion Debye length, $b = Z_{d0}e^2/T_i$. The expressions (3) and (4) for the values $\nu_{\rm ch}$ and $\tilde{\nu}$ are valid for $v_i/c_s < 1$.

Analogously to [11], the electron density is taken to be Boltzmann with constant electron temperature

$$n_e = n_{e0} \exp\left(\frac{e\varphi}{T_e}\right). \qquad (5)$$

We also use the equations (see, e.g., [4])

$$\partial^2_{xx}\varphi = 4\pi e\,(n_e + Z_d n_d - n_i) \qquad (6)$$

and

$$\partial_t q_d = I_e(q_d) + I_i(q_d) \qquad (7)$$

for the electrostatic potential and the variation of dust particle charge, respectively. Here the microscopic electron and ion grain currents (for equilibrium electrons and kinetic ions) are

$$I_e \approx -\pi a^2 e \left(\frac{8T_e}{\pi m_e}\right)^{1/2} n_e \exp\left(\frac{eq_d}{aT_e}\right) \qquad (8)$$

and

$$I_i = \sqrt{\frac{\pi}{2}} a^2 v_{Ti} e n_i \left[2\exp\left(-\frac{v_i^2}{2v_{Ti}^2}\right) + \right.$$
$$\left. + \sqrt{2\pi}\frac{v_{Ti}}{v_i}\left(1 + \frac{v_i^2}{v_{Ti}^2} - \frac{2eq_d}{am_i v_{Ti}^2}\right) \operatorname{erf}\left(\frac{v_i}{\sqrt{2}v_{Ti}}\right)\right]; \qquad (9)$$

m_e is the electron mass and $\operatorname{erf}(x)$ is the error function.

The set of equations (1)–(7) describes the evolution of perturbation and its transformation into nonlinear wave structure. The only steady–state solution of this set of equations corresponds to the unperturbed plasma parameters. Thus the evolution of perturbation within the ionization source model can lead only to an appearance of a non–stationary nonlinear ion acoustic wave structure.

Let us consider the evolution of a non–moving region with a constant enhanced ion density within the above (the ionization source) model. In this consideration we apply the data close to those of the University of Iowa laboratory experiment [9].

This experiment was performed in a Q machine device that was modified to allow the introduction of dust grains into the plasma. The experiment was fulfilled with Cs^+ ions. The plasma parameters of the experiment were $T_e \approx T_i \approx 0.2$ eV, $n_{i0} \sim 10^6 - 10^7$ cm^{-3}, $a \sim 0.1 - 1$ μm. The parameter $\epsilon Z_{d0} = n_{d0}Z_{d0}/n_{i0}$ was varied from 0 to 0.95. This corresponds to the variation of the parameter $Z_{d0}d$ from 0 to 19. The evolution of large amplitude density pulses propagating in complex plasmas was investigated. In the presence of a substational component of negatively charged dust (when $\epsilon Z_{d0} \geq 0.75$, i.e. $Z_{d0}d \geq 3$) a sharpeninig up of the leading edge of the pulse as it propagates down the plasma column (shock formation) was observed.

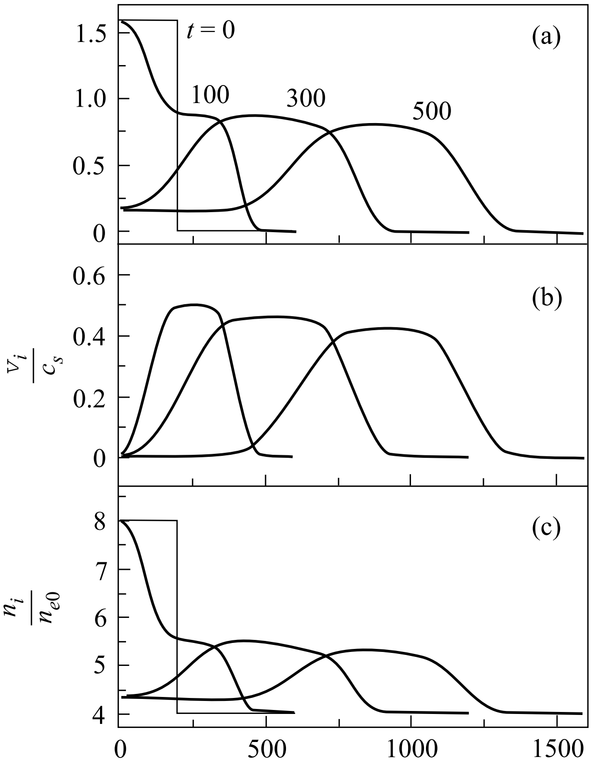

Fig.1. The profiles of $\varphi(x)$, $v_i(x)/c_s$, and $n_i(x)/n_{e0}$ at $t = 100$, 300, and 500 showing the evolution of the initial perturbation obtained within the ionization source model. The parameters are $Z_{d0}d = 3$, $T_e = T_i = 0.2$ eV, $a = 0.1$ μm, $n_{e0} = 2.56 \cdot 10^6$ cm^{-3}. The initial normalized dust particle charge number is $z_0 \approx 3.36$. The initial profiles ($t = 0$) of the potential φ and the normalized ion density n_i/n_{e0} are presented by the thin lines at the left of the figures

The results of the calculations on the basis of the proposed ionization source model, which describe the evolution of the initial non–moving region with a constant enhanced ion density within the set of equations (1)–(7) are given in Figs. 1 and 2. We use the normalization $x/\lambda_D \to x$ for the spatial variable and $tc_s/\lambda_D \to t$ for the time one, where $c_s = \sqrt{T_e/m_i}$ is the ion acoustic speed. The plasma parameters are $Z_{d0}d = 3$, $T_e = T_i = 0.2$ eV, $a = 0.1$ μm, $n_{e0} = 2.56 \cdot 10^6$ cm^{-3}. The initial ion density in the perturbation is two times larger than the background one, that corresponds to the case of Fig.2b of [9]. The initial charge of the dust particles is the equilibrium one in the absence of wave perturbations ($z_0 \approx 3.36$). Figs.1a–c show the profiles of the potential $\varphi(x)$, the ion speed normalized to the ion acoustic speed, and the ion density n_i normalized to the unperturbed electron density n_{e0} at the instants of time $t = 100$, 300, and 500. The initial profiles ($t = 0$) of the potential φ and the normalized ion density n_i/n_{e0} are presented by the thin lines at the left of the corresponding figures. We see that the realationship $v_i/c_s < 1$ remains valid in the process of the evolution of the perturbation. Thus the use of the expressions (3) and (4) for the values ν_{ch} and $\tilde{\nu}$ is justified.

From Figs.1a–c we see that the evolution of an intensive initial non–moving region with a constant enhanced ion density results in an appearance of a shock wave structure. For $t > 200$ the speed of the structure is approximately constant ($M \approx 1.94$). The width of the shock front is of the order of $\Delta\xi \sim 100\lambda_D \sim 20$ cm. This value is in a good agreement with the theoretical estimate [6] $\Delta\xi \sim c_s/\nu_q$ for the width of the front of shocks related to dust particle charging. For comparison with the data of the experiment [9] we have to find the width of the front calculated in terms of time variables $\Delta\xi/Mc_s \sim 0.3$ ms. This value corresponds to the observed one (see Fig.2b of [9]). Thus the proposed ionization source model allows to obtain shock structures as a result of evolution of an initial perturbation and to explain the width of the shock front. This means that the shocks observed in [9] are related to the anomalous dissipation due to dust particle charging process.

Fig.2 is constructed analogously to Fig.2b of [9]. It shows the dependence of ion density on time at various axial positions from the grid of the experimental installation [9] (the grid is used to separate the region of enhanced ion density from the background one in a Q machine device). The profiles of the perturbations are shown by bold lines. The sets of thin lines indicate some steepening (shock formation). The computational results presented in Fig.2 are in a good qualitative and quantitative agreement with the experimental ones shown in Fig.2b of [9]. In particular, this concerns the width of the shock front (as it has been mentioned, the calculated value corresponds to the observed one). Furthermore, the experimental profiles of the perturbations at large distances from the grid (60 cm, 65 cm, 70 cm) are similar to each other (see Fig.2b of [9]). This indicates the fact that at large distances from the grid a quasi–steady–state shock structure is formed. The analogous statement for the theoretical results can be made on the basis of Fig.2. Finally, the comparison of these two figures allows us to conclude that the velocity of perturbation predicted by theory on the basis of the ionization source model is close to the experimentally observed value.

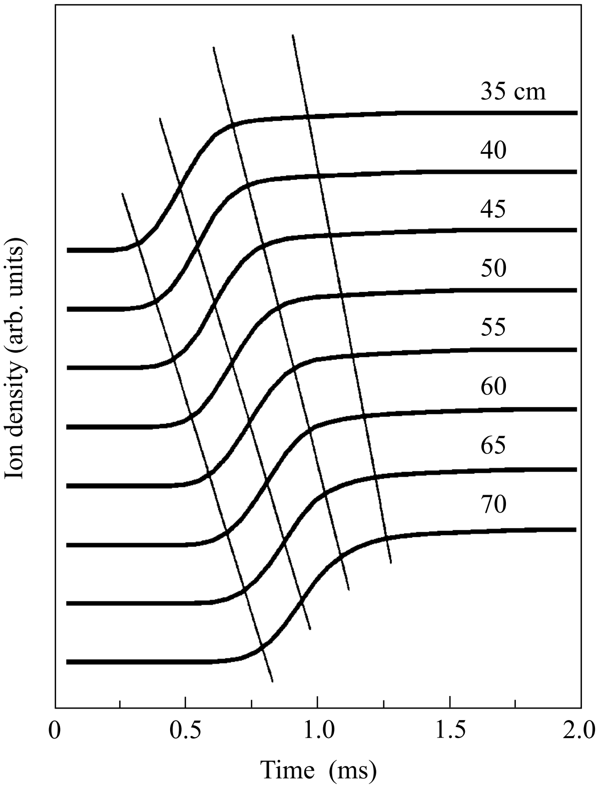

Fig.2. Ion density vs time at various axial positions from the grid. The parameters are the same as in Fig.1. The profiles of the perturbations are shown by bold lines. The sets of thin lines indicate some steepening (shock formation)

To summarize, we have considered the non–stationary problem of the evolution of perturbation and its transformation into nonlinear wave structure. For this consideration we have developed the one–dimensional (ionization source) model which takes into account the variation of the ion density and the ion momentum dissipation due to dust particle charging as well as the source of plasma particles due to ionization process. We have performed the consideration of the evolution of a non–moving region with a constant enhanced ion density within the ionization source model for the data of the laboratory experiment [9]. This consideration has shown that this model allows to obtain shock structures as a result of evolution of an initial perturbation and to explain the experimental value of the width of the shock wave front as well as the shock wave speed. This indicates that the shocks observed in [9] are related to the anomalous dissipation due to dust particle charging process. In future we intend to carry out the description of the experiment [9] in more detail using the modified ionization source model which additionally includes the effects of Landau damping of ion acoustic waves and ion–neutral collisions.

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