

The bound state corrections to the leptonic decays of the B -meson. The light-front approach versus ACM model

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A generalization of the parton-like formula is used for the first time to find the differential distributions in the inclusive semileptonic weak decays of the B -meson. The main features of this new approach are the treatment of the b -quark as an on-mass-shell particle and the inclusion of the effects arising from the b -quark transverse motion in the B -meson. Using the b -quark light-front (LF) distribution function related to the equal time momentum wave function taken from the ACM model we compute the electron energy spectra and the total semileptonic widths of the B -meson. We find an impressive agreement between the electron energy spectra calculated in the LF approach and the ones obtained in the ACM model, provided the b -quark mass is identified with the average of the floating b -quark mass in the ACM model. In spite of the simplicity of the model we obtain a fair good description of the CLEO data for $|V_{cb}| = 0.042$.

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1. Introduction. Studying of the leptonic spectra in the weak semileptonic $B \rightarrow X_c(X_u)\ell\nu_\ell$ decays²⁾ is important to extract $|V_{cb}|$ and $|V_{ub}|$. The latter plays an important role in the determination of the unitarity triangle. Since the b -quark is heavy compared to the QCD scale, the inclusive semileptonic B -decays can be treated with the help of an operator product expansion (OPE) combined with the heavy quark expansion (HQE) [1]. The result (away from the endpoint of the spectrum) is that the inclusive differential decay width $d\Gamma/dE$ may be expanded in Λ/m_b , where Λ is a QCD related scale of order 500 MeV, and m_b is the mass of the heavy quark. The leading term (zeroth order in Λ/m_b) is the free quark decay spectrum, the subleading term vanishes, and the subsubleading term involves parameters from the heavy quark theory, but should be rather small, as it is of order $(\Lambda/m_b)^2$. However, near the endpoint the $1/m_b$ expansion has to be replaced by an expansion in twist. To describe this region one has to introduce a so-called ‘shape function’, which in principle introduces a large hadronic uncertainty. This is quite analogous to what happens for the structure function in deep-inelastic scattering in the region where the Bjorken variable $x_B \rightarrow 1$. A model independent determination of the shape function is not available at the present time, therefore a certain model dependence in this region seems to be unavoidable, unless lattice data become reasonably precise.

As to phenomenological analyzes of the photon spectra up to now they have been solely based on the ACM model [2, 3]. The various light-front (LF) approaches to consideration of the inclusive semileptonic transitions were suggested in Refs.[4–7]. In Refs.[4, 5] the Infinite Momentum Frame prescription $p_b = \xi P_B$, and, correspondingly, the floating b -quark mass $m_b^2(\xi) = \xi^2 M_B^2$ have been used. The transverse b -quark momenta were consequently neglected. In Ref.[7] the b -quark was considered as an on-mass-shell particle with the definite mass m_b and the effects arising from the b -quark transverse motion in the B -meson were included. The corresponding ansatz of Ref.[7] reduces to a specific choice of the primordial LF distribution function $|\psi(\xi, p_\perp^2)|^2$, which represents the probability to find the b -quark carrying a LF fraction ξ and a transverse momentum squared $p_\perp^2 = |\mathbf{p}_\perp|^2$. As a result, a new parton-like formula for the inclusive semileptonic $b \rightarrow c, u$ width has been derived [7], which is similar to the one obtained by Bjorken et al. [8] in case of infinitely heavy b - and c -quarks.

In this paper, we use the techniques developed in Ref. [7] to evaluate the non-perturbative corrections to the lepton spectrum in the inclusive $B \rightarrow X_c\ell\nu_\ell$ decays. We strive to implement the binding and the B -meson wave function effects on the lepton energy spectrum. The main purpose of the paper is to confront the lepton spectra and the total semileptonic decay widths calculated in the LF and ACM approaches. We find that the discrepancy between the two is very small numerically.

2. ACM versus LF. The decay spectrum in the ACM model is determined by the kinematics constrains

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²⁾Throughout this paper we use the charge conjugated notations.

on the b -quark. It incorporates some of the corrections related to the fact that the decaying b -quark is not free, but in a bound state. The model treats the B -meson with the mass m_B as consisting of the heavy b -quark plus a spectator with fixed mass m_{sp} ; the latter usually represents a fit parameter. The spectator quark has a momentum distribution $\phi(|\mathbf{p}|)$ where \mathbf{p} is its three-dimensional momentum. The momentum distribution is usually taken to be Gaussian:

$$\phi(|\mathbf{p}|) = \frac{4}{\sqrt{\pi} p_f^3} \exp\left(-\frac{|\mathbf{p}|^2}{p_f^2}\right), \quad (1)$$

which is normalized so that the integral over all momenta of $\phi(|\mathbf{p}|) p^2$ is 1. The energy-momentum conservation in the B -meson vertex implies that the b -quark energy $E_b = m_B - \sqrt{\mathbf{p}^2 + m_{sp}^2}$, where m_B is the mass of the B -meson; thus the b -quark cannot possess a definite mass. Instead, one obtains a ‘‘floating’’ b -quark mass $(m_b^f)^2 = m_B^2 + m_{sp}^2 - 2m_B \sqrt{\mathbf{p}^2 + m_{sp}^2}$, which depends on $|\mathbf{p}|$. The lepton spectrum is first obtained from the spectrum $d\Gamma_b^{(0)}(m_f, E')/dE'$ of the b -quark of invariant mass m_b^f (in the b -quark rest frame) and then boosting back to the rest frame of the B -meson and averaging over the weight function $\phi(|\mathbf{p}|)$ (further details can be found in the original work [3]).

The LF approach differs from that of the ACM model in two respects. First, similar to the ACM model the LF quark model treats the beauty meson B as consisting of the heavy b -quark plus a spectator quark. Both quarks have fixed masses, m_b and m_{sp} , though. This is at variance with the ACM model, that has been introduced in order to avoid the notion of the heavy quark mass at all. Secondly, the calculation of the distribution over lepton energy in the LF approach does not require any boosting procedure but instead is based on the standard Lorentz-invariant kinematical analysis (see e.g. Ref.[9]), which we now briefly discuss.

There are three independent kinematical variables in the inclusive phenomenology, for which we choose the lepton energy E_ℓ, q^2 , where $q = p_\ell + p_{\nu_\ell}$, and the invariant mass $M_X^2 = (p_B - q)^2$ of a hadronic state. Introducing the dimensionless variables $y = 2E_\ell/m_B$, $t = q^2/m_B^2$, and $s = M_X^2/m_B^2$, the differential decay rate for semileptonic (SL) B -decay can be written as

$$\begin{aligned} \frac{d\Gamma_{\text{SL}}}{dy} &= \frac{G_F^2 m_B^5}{64\pi^3} |V_{cb}|^2 \int_0^{t_{\max}} dt \int_{s_0}^{s_{\max}} ds \times \\ &\times \left\{ tW_1 + \frac{1}{2}[y(1+t-s) - y^2 - t]W_2 + \right. \\ &\quad \left. + t\left[\frac{1+t-s}{2} - y\right]W_3 + \dots \right\}, \quad (2) \end{aligned}$$

where the structure functions $W_i = W_i(s, t)$ appear in the decomposition of the hadronic tensor $W_{\alpha\beta}$ in Lorentz covariants [9]. The ellipsis in (2) denote the terms proportional to the lepton mass squared. The kinematical limits of integration can be found from equation

$$\frac{s}{1-y} + \frac{t}{y} \leq 1. \quad (3)$$

They are given by $0 \leq y \leq y_{\max} = 1 - (m_{X_c}^{(\text{min})}/m_B)^2$, where $m_{X_c}^{(\text{min})}$ is the minimal mass of the hadronic state X_c , $s_{\max} = 1 + t - (y + t/y)$, and $t_{\max} = y(1 - (1 - y_{\max})/(1 - y))$.

3. Light-cone distribution functions. In a parton model we treat inclusive semileptonic $B \rightarrow X_c \ell \nu_\ell$ decay in a direct analogy to deep-inelastic scattering. Specifically, we assume that the sum over all possible charm final states X_c can be modeled by the decay width of an on-shell b -quark into on-shell c -quark weighted with the b -quark distribution. Following the above assumption, the hadronic tensor $W_{\alpha\beta}$ is written as

$$\begin{aligned} W_{\alpha\beta} &= \int w_{\alpha\beta}^{(cb)}(p_c, p_b) \delta[(p_b - q)^2 - m_c^2] \times \\ &\times \frac{|\psi(\xi, p_\perp^2)|^2}{\xi} \theta(\varepsilon_c) d\xi d^2 p_\perp, \quad (4) \end{aligned}$$

where

$$w_{\alpha\beta}^{(cb)}(p_c, p_b) = \frac{1}{2} \sum_{\text{spins}} \bar{u}_c O_\alpha u_b \cdot \bar{u}_b O_\beta^+ u_c, \quad (5)$$

with $O_\alpha = \gamma_\alpha(1 - \gamma_5)$. The factor $1/\xi$ in Eq.(4) comes from the normalization of the B -meson vertex [10].

Eq. (4) amounts to averaging the perturbative decay distribution over motion of heavy quark governed by the distribution function $f(x, p_\perp^2) = |\psi(x, p_\perp^2)|^2$. In this respect our approach is similar to the parton model in deep inelastic scattering, although it is not really a parton model in its standard definition. The normalization condition reads

$$\pi \int_0^1 d\xi \int dp_\perp^2 f(\xi, p_\perp^2) = 1. \quad (6)$$

The function $\theta(\varepsilon_c)$ where ε_c is the c -quark energy is inserted in Eq. (4) for consistency with the use of valence LF wave function to calculate the b -quark distribution in the B -meson.

Since we do not have an explicit representation for the B -meson Fock expansion in QCD, we shall proceed by making an ansatz for $\psi(\xi, p_\perp^2)$. This is model dependent enterprise but has its close equivalent in studies of

electron spectra using the ACM model. We choose the momentum space structure of an equal time (ET) wave function $\phi(|\mathbf{p}|)$ as in Eq. (1). We convert from ET to LF momenta by leaving the transverse momenta unchanged and letting

$$p_{iz} = \frac{1}{2}(p_i^+ - p_i^-) = \frac{1}{2}(p_i^+ - \frac{p_{i\perp}^2 + m_i^2}{p_i^+}) \quad (7)$$

for both the b -quark ($i = b$) and the quark-spectator ($i = sp$). The longitudinal LF momentum fractions ξ_i are defined as $\xi_{sp} = p_{sp}^+/P_B^+$, $\xi_b = p_b^+/P_B^+$, with $\xi_b + \xi_{sp} = 1$. In the B -meson rest frame $P_B^+ = m_B$. Then for the distribution function $|\psi(\xi, p_\perp^2)|^2$ ($\xi = \xi_b$) normalized according to (6) one obtains³⁾

$$|\psi(\xi, p_\perp^2)|^2 = \frac{4}{\sqrt{\pi}p_f^3} \exp\left(-\frac{p_\perp^2 + p_z^2}{p_f^2}\right) \left|\frac{\partial p_z}{\partial \xi}\right|, \quad (8)$$

where

$$p_z^2(\xi, p_\perp^2) = \frac{1}{2} \left((1 - \xi)m_B - \frac{p_\perp^2 + m_{sp}^2}{(1 - \xi)m_B} \right), \quad (9)$$

$$\left|\frac{\partial p_z}{\partial \xi}\right| = \frac{1}{2} \left(m_B + \frac{p_\perp^2 + m_{sp}^2}{(1 - \xi)^2 m_B} \right).$$

The calculation of the structure functions $W_i(t, s)$ in the LF parton approximation (4) is straightforward. The result is

$$W_i(t, s) = \int w_i(s, t, \xi, p_\perp^2) \delta[(p_b - q)^2 - m_c^2] \times \\ \times \frac{|\psi(\xi, p_\perp^2)|^2}{\xi} \theta(\varepsilon_c) d\xi d^2 p_\perp, \quad (10)$$

where $w_i(s, t, \xi, p_\perp^2)$ are the structure functions for the free quark decay. For further details see Appendix of [7]. Eq. (10) differs from the corresponding expressions of Refs. [4] and [5] by the non-trivial dependence on p_\perp^2 which enters both $|\psi(\xi, p_\perp^2)|^2$ and argument of the δ -function. For further details see [7]⁴⁾.

4. Results. Having specified the non-perturbative aspects of our calculations, we proceed to present numerical results for the lepton spectrum in the decay

³⁾On the same footing one can consider $|\mathbf{p}|$ in (1) as the relative momentum between heavy and light quarks. In this case it is more convenient to use the quark-antiquark rest frame instead of the B -meson rest frame. Recall that in the LF formalism these two frames are different. Then the longitudinal LF momentum fractions ξ_i are defined as $\xi_{sp} = p_{sp}^+/M_0$, $\xi_b = p_b^+/M_0$, where the free mass M_0 is $M_0 = \sqrt{m_b^2 + \mathbf{p}^2} + \sqrt{m_{sp}^2 + \mathbf{p}^2}$ with $\mathbf{p}^2 = p_\perp^2 + p_z^2$, $p_z = (\xi - \frac{1}{2})M_0 - (m_b^2 - m_{sp}^2)/2M_0$. In this case the explicit form of $|\partial p_z / \partial \xi|$ is given e.g. by Eq. (10) of Ref. [6]. We have checked that *numerically* both approaches yield identical results for the electron spectra.

⁴⁾Note that the expressions for w_i in Ref. [7] miss an extra factor of 2.

$B \rightarrow X_c e \nu_e$. Our main computation refers to the case $m_{sp} = 0.15$ GeV, $m_c = 1.5$ GeV as chosen in Ref. [11].

The choice of m_b in our approach deserves some comments. In the ACM model, it was shown [11, 12] (see also [13]) that the corrections to first order in $1/m_b$ both to the inclusive semileptonic width and to the regular part of the lepton spectrum can be absorbed into the definition of the quark mass: $m_b^{\text{ACM}} = \langle m_b^f \rangle$, where $\langle m_b^f \rangle$ is the value of the floating mass, averaged over the distribution $\phi(|\mathbf{p}|)$.

The choice of m_b in the LF approach was first addressed in the context of the LF model for $b \rightarrow s \gamma$ transitions in Ref. [14]. Using the scaling feature of the photon spectrum in the LF model, it was suggested that m_b^{LF} can be defined from the requirement of the vanishing of the first moment of the distribution function. This condition coincides with that used in HQE to define the pole mass of the b -quark. In this way one avoids an otherwise large (and model dependent) correction of order $1/m_b$ but at expense of introducing the shift in the constituent quark mass which largely compensates the bound state effects. It has been also demonstrated that the values of m_b^{LF} found by this procedure agree well with the average values $\langle m_b^f \rangle$ in the ACM model. The photon energy spectra calculated in the LF approach were found to agree well with the ones obtained in the ACM model.

Accepting the identification $m_b^{\text{LF}} = m_b^{\text{ACM}}$ we want now to check whether this result holds for the description of the other channels, like $b \rightarrow c$. We find again a good agreement between the LF and ACM results but now for the semileptonic $b \rightarrow c$ decays. The results of our computations of electron spectra and the semileptonic widths are reported in Fig.1 and Table using $|V_{cb}| = 0.04$. The different curves in Fig.1 correspond to the different values of p_f . For each case we show separately the inclusive differential semileptonic decay widths for the LF and ACM models and the free quark decay. The quark decay spectra vanish for $y > (m_b/m_B)(1 - m_c^2/m_b^2)$, whereas the physical endpoint is $y_{\text{max}} = 1 - m_D^2/m_B^2$, where m_D is the mass of the D -meson. In the LF approach the endpoint for the electron spectrum is in fact not y_{max} but $y_{\text{max}}^{\text{LF}} = 1 - m_c^2/m_B^2$. This is the direct consequence of the p_\perp^2 integration in Eq. (10) [7]. Note that $y_{\text{max}}^{\text{LF}}$ coincides with $y_{\text{max}}^{\text{ACM}}$ with accuracy $\sim m_{sp}/m_B$. For $m_c \sim 1.5$ GeV the difference between $y_{\text{max}}^{\text{LF}} \sim y_{\text{max}}^{\text{ACM}}$ and y_{max} is of the order 10^{-2} . Another possibility advocated in [6] is to sum the electron spectra from the exclusive $B \rightarrow D, D^*$ channels and from the inclusive $B \rightarrow X_c$ channels, where X_c is the hadronic state with the mass $m_{X_c} \geq m_{D^*}$. Such the 'hybrid' approach will be considered elsewhere.

p_f	$\langle m_b^f \rangle$	Γ_{SL}^b	Γ_{SL}^{ACM}	Γ_{SL}^{LF}	δ^{ACM}	δ^{LF}
0.1	5.089	0.1007	0.1005	0.1005	0.2	0.2
0.2	5.004	0.0906	0.0902	0.0901	0.4	0.5
0.3	4.905	0.0799	0.0792	0.0789	0.9	1.2
0.4	4.800	0.0696	0.0688	0.0682	1.1	2.0
0.5	4.692	0.0602	0.0592	0.0584	1.7	3.0

For the values of p_f in column (1) we display the average value of the floating b -quark mass $\langle m_b^f \rangle$ (both in units of GeV) in the second column and the total semileptonic width of the free b -quark (in units of ps^{-1}) in the third column. In the fourth and fifth columns we compare the total semileptonic widths, calculated in the ACM and LF approaches, respectively. In all cases $m_{sp} = 0.15$ GeV and $m_c = 1.5$ GeV and the radiative corrections are neglected. In the sixth and seventh columns, we give the fractional deviation in percent between the semileptonic widths determined in the LF and ACM models and that of the free quark. A momentum distribution of the b -quark is taken in the standard Gaussian form (1) with the Fermi momentum p_f . $|V_{cb}| = 0.04$.

In Table for various values of p_f , we give the corresponding values of the total semileptonic width for the free quark with the mass $m_b = \langle m_b^f \rangle$ and the B -meson semileptonic widths, calculated using the LF and ACM approaches, respectively. In the last two columns, we give the fractional deviation $\delta = \Delta\Gamma_{SL}/\Gamma_{SL}^b$ (in percent) between the semileptonic widths determined in the LF and ACM models and that of the free quark. The agreement between the LF and ACM approaches for the electron spectra is excellent for small p_f as is exhibited in Fig.1. A similar agreement also holds for integrated rates shown in Table. This agreement is seen to be breaking down at $p_f \geq 0.4$ GeV, but even for $p_f \sim 0.5$ GeV the difference between the ACM and LF inclusive widths is still small and is of the order of a percent level.

Finally, we calculate the $b \rightarrow c$ spectrum and compare it with the experimental data from the CLEO collaboration [15]. This is a direct calculation of the spectrum and not a χ^2 fit. We briefly investigated the sensitivity of the electron spectra to other parameters of the models and found that the choice $p_f = 0.4$ GeV, $m_c = 1.5$ GeV $m_{sp} = 0.15$ GeV is quite acceptable.

We display the results in Fig.2, where the three theoretical curves are presented for the LF, ACM and free quark models. In these calculations we have implicitly included the $O(\alpha_s)$ perturbative corrections arising from gluon Bremsstrahlung and one-loop effects which modify an electron energy spectra at the partonic level (see e.g. [16] and references therein). It is customary to define a correction function $G(x)$ to the electron spectrum

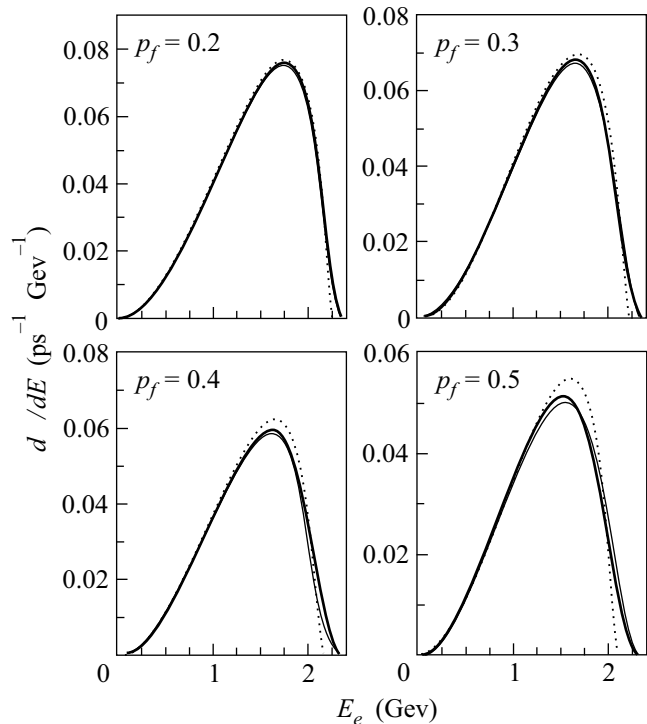


Fig.1. The inclusive differential semileptonic decay widths $d\Gamma_{SL}/dE_e$ in units $\text{ps}^{-1}\cdot\text{GeV}^{-1}$ for the LF model (thick solid lines), ACM model (thin solid lines), and the free quark decays (dashed lines). The parameters are $p_f = 0.2 - 0.5$ GeV, $m_{sp} = 0.15$ GeV, and $m_c = 1.5$ GeV; $|V_{cb}| = 0.04$

$d\Gamma_b^{(0)}$ calculated in the tree approximation for the free quark decay through

$$\frac{d\Gamma_b}{dx} = \frac{d\Gamma_b^{(0)}}{dx} \left(1 - \frac{2\alpha_s}{3\pi} G(x) \right), \quad (11)$$

where $x = 2E/m_b$. The function $G(x)$ contains the logarithmic singularities $\sim \ln^2(1-x)$ which for $m_c = 0$ appear at the quark-level endpoints $x_{max} = 1$. This singular behaviour at the end point is clearly a signal of the inadequacy of the perturbative expansion in this region. The problem is solved by taking into account the bound state effects [4]. Since the radiative corrections must be convoluted with the distribution function the endpoints of the perturbative spectra are extended from the quark level to the hadron level and the logarithmic singularities are eliminated. In actual calculations we neglect the terms $\sim \rho$ in $G(x)$ and take this function from [16] (Eq. (4.10)).

The agreement with the experimental data is good. Using $|V_{cb}| = 0.042$ for the overall normalization⁵⁾ we obtain for the semileptonic branching ratios

⁵⁾This value agrees with the combined average $|V_{cb}| = (40.2 \pm 1.9) \cdot 10^{-3}$ of Ref. [17].

$$\begin{aligned} BR_{LF} &= 10.16\%, \\ BR_{ACM} &= 10.23\%, \\ BR_{free} &= 10.37\%, \end{aligned} \quad (12)$$

in agreement with the experimental finding [15] $BR_{SL} = 10.49 \pm 0.17 \pm 0.43\%$.

4. Conclusions. We have applied a new LF formula [7] to calculate the partial electron spectra in the semileptonic B decays. Using the ET and LF b -quark distribution functions related by a simple kinematical transformation we compared the LF and ACM models by computing the $b \rightarrow c$ decays for $m_b^{LF} = \langle m_b^f \rangle$. A summary of our results is presented in Table and Fig.1 shows a good agreement between the results of the two models. We have also calculated (Fig.2) the $b \rightarrow c$ spectrum including the perturbative corrections

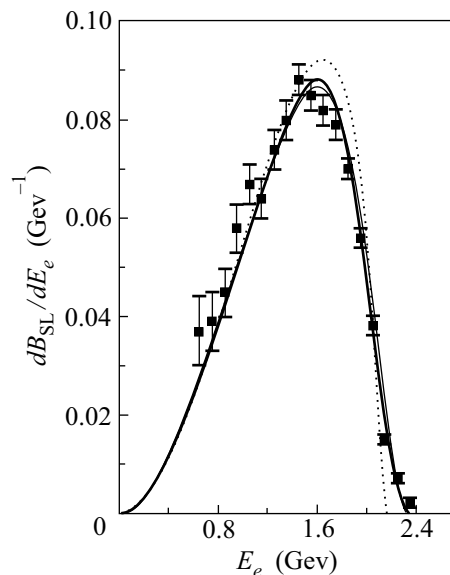


Fig.2. The predicted electron energy spectrum compared with the CLEO data [15]. The calculation uses $p_f = 0.4$ GeV, $m_b = 4.8$ GeV, $m_c = 1.5$ GeV, and $\alpha_s = 0.25$ for the perturbative corrections. Thick solid line is the LF result, thin solid line is the ACM result, dashed line refers to the free quark decay. The spectra normalized to 10.16%, 10.23%, and 10.36%, respectively. $|V_{cb}| = 0.042$

and found an agreement with the experimental data from the CLEO collaboration [15]. A more detailed fit to the measured spectrum can impose constraints on the distribution function and the mass of the charm quark. Such the fit should also account for detector resolution.

The same formulae can be also applied for nonleptonic B -decay widths (corresponding to the underlying

quark decays $b \rightarrow cq_1q_2$) thus making it possible to calculate the B lifetime. A preview of this calculation can be found in [18]. It would be interesting to check whether the effective values of the b -quark mass can appear to be approximately the same for different quark channels and for different beauty hadrons. This work is in progress, and the results will be reported elsewhere.

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