

# The bubbles of matter from multiskyrmions

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Approximate analytical solutions describing the skyrmions given by rational map ansaetze are obtained. At large baryon numbers these solutions are similar to the domain wall, or to spherical bubbles with energy and baryon number density concentrated at their boundary. Rigorous upper bound is obtained for the masses of rational map multiskyrmions which is close to known masses, especially at large  $B$ . The main properties of bubbles of matter are obtained for arbitrary number of flavors.

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1. Among many known soliton models used in different fields of physics the chiral soliton approach, starting with few basic concepts and ingredients incorporated in the model lagrangian [1, 2] provides, probably, the most realistic description of baryons and baryonic systems. The latter appear within this approach as quantized solitonic solutions of equations of motion, characterized by the so called winding number or topological charge which is identified with baryon number  $B$ . Numerical studies have shown that the chiral field configurations of lowest energy possess different topological properties – the shape of the mass and  $B$ -number distribution – for different values of  $B$ . It is a sphere for  $B = 1$  hedgehog [1], a torus for  $B = 2$ , tetrahedron for  $B = 3$ , cube for  $B = 4$ , and higher polyhedrons for greater baryon numbers. The symmetries of various configurations for  $B$  up to 22 and their masses have been determined in [3] (the references to earlier original papers can be found in [3, 4]). These configurations have one-shell structure and for  $B > 6$  all of them, except two cases, are formed from 12 pentagons and  $2B - 14$  hexagons, in carbon chemistry similar structures are known as fullerenes [3c].

A remarkable feature of the whole approach is that the baryons/nucleons individuality is absent in the lowest energy static configurations, and the conventional picture of nuclei should appear when non-zero modes motion – vibration, breathing, etc. – are taken into account (see, e.g. discussion in [4]). The so called rational map (RM) ansatz, proposed for skyrmions in [5] and widely used now allows to simplify the problem of finding the configurations of lowest energy. As will be shown here, many important properties of RM multiskyrmions can be studied analytically, and this allows to make some conclusions for arbitrary large  $B$  and for any number of flavors  $N_F$  independently on presence of numerical calculations.

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So far, the chiral soliton models have been considered as a special class of models. Their connection with other soliton models would be of interest, and this is also an issue of present paper.

2. Here we consider the large  $B$  multiskyrmions described by RM ansaetze mainly in  $SU(2)$  model and also in  $SU(3)$  variant using the projector ansatz in this case [6]. In  $SU(2)$  model the chiral fields are functions of profile  $f$  and unit vector  $\mathbf{n}$ , according to definition of the unitary matrix  $U \in SU(2)$   $U = c_f + is_f \mathbf{n} \tau$ . For the ansatz based on rational maps the profile  $f$  depends only on variable  $r$ , and components of vector  $\mathbf{n}$  – on angular variables  $\theta, \phi$ .  $n_1 = (2 \operatorname{Re} R)/(1 + |R|^2)$ ,  $n_2 = (2 \operatorname{Im} R)/(1 + |R|^2)$ ,  $n_3 = (1 - |R|^2)/(1 + |R|^2)$ , where  $R$  is a rational function of variable  $z = \operatorname{tg}(\theta/2) \exp(i\phi)$  defining the map of degree  $\mathcal{N}$  from  $S^2 \rightarrow S^2$ .

The notations are used [5]

$$\begin{aligned} \mathcal{N} &= \frac{1}{8\pi} \int r^2 (\partial_i \mathbf{n})^2 d\Omega = \frac{1}{4\pi} \int \frac{2idRd\bar{R}}{(1 + |R|^2)^2}, \\ \mathcal{I} &= \frac{1}{4\pi} \int r^4 \frac{[\partial n_1 \partial n_2]^2}{n_3^2} d\Omega = \\ &= \frac{1}{4\pi} \int \left( \frac{(1 + |z|^2) |dR|}{(1 + |R|^2) |dz|} \right)^4 \frac{2idz d\bar{z}}{(1 + |z|^2)^2}, \end{aligned} \quad (1)$$

where  $\Omega$  is a spherical angle. For  $B = 1$  hedgehog  $\mathcal{N} = \mathcal{I} = 1$ .  $\mathcal{N} = B$  for configurations of lowest energy.

The classical mass of skyrmion for RM ansatz in universal units  $3\pi^2 F_\pi/e$  is [5, 6]:

$$M = \frac{1}{3\pi} \int \left\{ A_N r^2 f'^2 + 2B s_f^2 (f'^2 + 1) + \mathcal{I} \frac{s_f^4}{r^2} \right\} dr, \quad (2)$$

$r$  measured in units  $2/F_\pi e$ , where we inserted the coefficient  $A_N = 2(N - 1)/N$  for symmetry group  $SU(N)$  [6] to have a possibility to consider models with arbitrary

number of flavors  $N = N_F$  – essentially nonembeddings of  $SU(2)$  in  $SU(N)$ . To find the minimal energy configuration at fixed  $\mathcal{N} = B$  one minimizes  $\mathcal{I}$ , and then finds the profile  $f(r)$  by minimizing energy (2). The inequality takes place:  $\mathcal{I} \geq B^2$  [5, 6]. Direct numerical calculations have shown, and the analytical treatment here supports, that at large  $B$  and, hence, large  $\mathcal{I}$  multiskyrmion looks like a spherical bubble with profile equal to  $f = \pi$  inside and  $f = 0$  outside. The energy and  $B$ -number density of this configuration is concentrated at its boundary, similar to the domain walls system considered in [7] in connection with cosmological problems.

Denote  $\phi = \cos f$ , then the energy (2) can be presented as

$$M = \frac{1}{3\pi} \int \left\{ \frac{1}{(1-\phi^2)} \left[ A_N r^2 \phi'^2 + 2B(1-\phi^2)^2 \right] + 2B\phi'^2 + \mathcal{I}(1-\phi^2)^2/r^2 \right\} dr, \quad (3)$$

with  $\phi$  changing from  $-1$  at  $r = 0$  to  $1$  at  $r \rightarrow \infty$ . The first half of (3) is the second order term contribution into the mass, the second – the Skyrme term contribution. At fixed  $r = r_0$  the latter coincides exactly with 1-dimensional domain wall energy. It is possible to write the second order contribution in (2) in the form:

$$M^{(2)} = \frac{1}{3\pi} \int \left\{ \frac{A_N r^2}{(1-\phi^2)} \left[ \phi' - \sqrt{\frac{2B}{A_N}}(1-\phi^2)/r \right]^2 + 2r\sqrt{2A_N B} \phi' \right\} dr, \quad (4)$$

and similar for the 4-th order Skyrme term. The equality  $\phi' = \sqrt{2B/A_N}(1-\phi^2)/r$  eliminates considerable part of integrand in (4). Therefore, it is natural to consider function  $\phi$  satisfying the following differential equation:

$$\phi' = \frac{b}{2r}(1-\phi^2) \quad (5)$$

which has solution satisfying boundary conditions  $\phi(0) = -1$  and  $\phi(\infty) = 1$ :

$$\phi = \frac{(r/r_0)^b - 1}{(r/r_0)^b + 1} \quad (6)$$

with arbitrary  $r_0$  – the distance from the origin of the point where  $\phi = 0$  and profile  $f = \pi/2$ .  $r_0$  can be considered as a radius of multiskyrmion.

After substitution of this ansatz one obtains the soliton mass in the form:

$$M(B, b) = \frac{1}{3\pi} \int \left\{ (A_N b^2/4 + 2B)(1-\phi^2) + \right.$$

$$\left. + (Bb^2/2 + \mathcal{I})(1-\phi^2)^2/r^2 \right\} dr. \quad (7)$$

Integrating over  $dr$  using known expressions for the Euler-type integrals, e.g.

$$\int_0^\infty \frac{dr}{1+(r/r_0)^b} = \frac{r_0 \pi}{b \sin(\pi/b)}, \quad b > 1,$$

allows to obtain the mass of multiskyrmion in simple analytical form as a function of parameters  $b$  and  $r_0$ :

$$M(B, r_0, b) = \frac{4}{3b^2 \sin(\pi/b)} \left[ (A_N b^2/4 + 2B)r_0 + \frac{1}{3r_0}(Bb^2 + 2\mathcal{I})(1 - 1/b^2) \right], \quad (8)$$

which gives after simple minimization in  $r_0$

$$r_0^{\min} = 2 \left[ \frac{(Bb^2 + 2\mathcal{I})(1 - 1/b^2)}{3(A_N b^2 + 8B)} \right]^{1/2} \quad (9)$$

and

$$M(B, b)/B = \frac{4}{3b \sin(\pi/b)} \times \left[ (b^2 + 2\mathcal{I}/B)(A_N b^2 + 8B)(1 - 1/b^2)/(3b^2 B) \right]^{1/2}. \quad (10)$$

For any value of the power  $b$  (10) provides an upper bound for the mass of  $RM$  skyrmion. To get better bound we should minimize (10) in  $b$ . At large enough  $B$  when it is possible to neglect the influence of slowly varying factors  $(1 - 1/b^2)$  and  $b \sin(\pi/b)$  we obtain easily that

$$b_{\min} = b_0 = 2(\mathcal{I}/A_N)^{1/4}, \quad r_0^{\min} \simeq \left[ \frac{2}{3} \left( \sqrt{\frac{\mathcal{I}}{A_N}} - \frac{1}{4} \right) \right]^{1/2} \quad (11)$$

and, therefore,

$$\frac{1}{3}(2 + \sqrt{\mathcal{I}A_N}/B) < \frac{M}{B} < \frac{1}{3}(2 + \sqrt{\mathcal{I}A_N}/B) \frac{4}{b_0 \sin(\pi/b_0)} \left[ \frac{2}{3} \left( 1 - \frac{1}{4} \sqrt{A_N \mathcal{I}} \right) \right]^{1/2}. \quad (12)$$

The lower bound in (12) was known previously [5, 6]. The correction to the value  $b_0$  can be found including into minimization procedure the factor  $(1 - 1/b^2)$  and variation of  $b \sin(\pi/b) \approx \pi[1 - \pi^2/6b^2]$ . It provides:

$$\delta b = \frac{B(\pi^2/3 - 1)(2 + \sqrt{A_N \mathcal{I}}/B)^2}{16\mathcal{I}^{3/4} A_N^{1/4}}, \quad (13)$$

and the value  $b = b_0 + \delta b$  should be inserted into (10). This improves the values of  $M/B$  for  $B = 1, 2, 3, \dots$  but

$B$	2	3	4	5	6	7	13	17	22	32	64
$M/B _{\text{RM}}$	1.208	1.184	1.137	1.147	1.137	1.107	1.098	1.092	1.092	—	—
$b(B)$	3.89	4.47	4.85	5.39	5.80	6.03	8.00	9.02	10.23	12.24	17.2
$M/B _{\text{appr}}$	1.229	1.198	1.151	1.158	1.147	1.117	1.106	1.0976	1.098	1.094	1.089
$M/B _{\text{num}}$	1.1791	1.1462	1.1201	1.1172	1.1079	1.0947	1.0834	1.0774	1.0766	—	—
$M/B _{\text{RM}}^{SU_3}$	1.222	1.215	1.184	1.164	1.146	—	—	—	—	—	—
$b(B)^{SU_3}$	3.57	4.08	4.47	4.83	5.13	5.46	7.13	8.06	9.09	10.86	15.19
$M/B _{\text{appr}}^{SU_3}$	1.259	1.231	1.198	1.176	1.156	1.149	1.127	1.121	1.116	1.111	1.106

The skyrmion mass per unit  $B$ -number in universal units  $3\pi^2 F_\pi/e$  for RM configurations, approximate and precise solutions. The approximate values are calculated using formula (10) with the power  $b = b_0 + \delta b$ . The numerical values for  $SU(2)$  model are from the papers [3] and earlier papers. The last 3 lines show the result for  $SU(3)$  projector ansatz [6] and approximation to this case,  $A_N = 4/3$ .

provides negligible effect for  $b \sim 17$  and greater, since  $\delta b \sim 1/\sqrt{B}$ . The comparison of numerical calculation result and analytical upper bound (12) is presented in the Table.

Numerically (10) – (13) provide upper bound which differs from the masses of all known  $RM$  skyrmions within  $\sim 2\%$ , beginning with  $B = 4$ , see Table. Even for  $B = 1$ , where the method evidently should not work well, we obtain  $M = 1.289$  instead of precise value  $M = 1.232$ . For maximal values of  $B$  between 17 and 22 where the value of  $\mathcal{I}$  is calculated, the upper bound exceeds the RM value of mass by 0.5% only. We took here the ratio  $R_{I/B} = \mathcal{I}/B^2$  in the cases where this ratio is not determined yet, the same as for highest  $B$  where it is known, i.e. 1.28 for  $SU(2)$  case[3],  $B = 32$  and 64, and 1.037 for  $B > 6$  in  $SU(3)$  [6]. For  $R_{I/B} = 1$  the numbers in Table decrease by  $\sim 0.1\%$  only in the latter case. Note also, that asymptotically at large  $B$  the ratio of upper and lower bounds

$$\frac{M_{\max}}{M_{\min}} = \frac{4}{\pi} \left(\frac{2}{3}\right)^{1/2} \simeq 1.0396, \quad (14)$$

i.e. the gap between upper and lower bounds is less than 4%, independently on  $B$ , the particular value of  $\mathcal{I}$  and the number of flavors  $N$ . With decreasing  $\mathcal{I}$  the upper bound decreases proportionally to the lower bound. In view of such good quantitative agreement of analytical and numerical results the studies of basic properties of bubbles of matter made in present paper are quite reliable.

The width (or thickness)  $W$  of the bubble shell can be estimated easily. We can define the half-width as a distance between points where  $\phi = \pm 1/2$ , then:

$$W = 4 \left[ \frac{2}{3} \left( 1 - \frac{1}{b_0^2} \right) \right]^{1/2} \ln 3. \quad (15)$$

It is clear that at large  $B$   $W$  is constant, and does not depend on the number of flavors  $N$  as well. The radius of the bubble (11) grows with increasing  $B$  like  $[\mathcal{I}/A_N]^{1/4}$ .

Previously we considered large  $B$  skyrmion within the ‘‘inclined step’’ approximation [4]. Let  $W$  be the width of the step, and  $r_0$  – the radius of the skyrmion where the profile  $f = \pi/2$ .  $f = \pi/2 - (r - r_0)\pi/W$  for  $r_0 - W/2 \leq r \leq r_0 + W/2$ . This approximation describes the usual domain wall energy [7] with accuracy  $\sim 9.5\%$ .

We wrote the energy in terms of  $W$ ,  $r_0$ , then minimized it with respect to both of these parameters, and find the minimal value of energy.

$$M(W, r_0) = \frac{\pi^2}{W} (B + A_N r_0^2) + W \left( B + \frac{3\mathcal{I}}{8r_0^2} \right). \quad (16)$$

This gave

$$W_{\min} = \pi \left[ \frac{B + A_N r_0^2}{B + 3\mathcal{I}/8r_0^2} \right]^{1/2} \quad (17)$$

and, after minimization,  $r_{0\min}^2 = \sqrt{3\mathcal{I}/8A_N} \simeq 0.612\sqrt{\mathcal{I}/A_N}$ , close to the above result  $r_{0\min}^2 \simeq 0.667\sqrt{\mathcal{I}/A_N}$ . In dimensional units  $r_0 = (6\mathcal{I}/A_N)^{1/4}/F_\pi e$ . Since  $\mathcal{I} \geq B^2$ , the radius of minimized configuration grows as  $\sqrt{B}$ , at least.  $W_{\min} = \pi$ , i.e. it does not depend on  $B$  for any  $SU(N)$ , similar to previous result (15) which gives  $W \simeq 3.59$  for large  $B$ , all in units  $2/F_\pi e$ . The energy

$$M_{\min} \simeq (2B + \sqrt{3A_N\mathcal{I}/2})/3. \quad (18)$$

For  $SU(2)$  model  $A_N = 1$  and the energy  $M_{\min} = (2B + \sqrt{3\mathcal{I}/2})/3$ . The formula gives the numbers for  $B = 3, \dots, 22$  in agreement with calculation within RM approximation within 2 – 3% [3, 5].

More detailed analytical calculation made here confirm the results of such ‘‘toy model’’ approximation and both reproduce the picture of RM skyrmions as a two-phase object, a spherical bubble with profile

$f = \pi$  inside and  $f = 0$  outside, and a fixed thickness shell with fixed surface energy density,  $\rho_M^{surf} \simeq (2B + \sqrt{3\mathcal{I}A_N}/2)/12\pi r_0^2$ . The average volume density of mass in the shell is

$$\rho_M^{vol} = \frac{3\pi}{64W} (2B + \sqrt{\mathcal{I}A_N}) \sqrt{A_N/\mathcal{I}} F_\pi^4 e^2 \quad (19)$$

and for  $SU(2)$  model at large  $B$  it is approximately equal to  $\sim 0.3 \text{ GeV}/\text{Fm}^3$  at reasonable choice of model parameters  $F_\pi = 0.186 \text{ GeV}$ ,  $e = 4.12$  [4], i.e. about twice greater than normal density of nuclei.

**3.** Consider also the influence of the chiral symmetry breaking mass term (M.t.) which is described by the lagrangian

$$\text{M.t.} = \tilde{m} \int r^2 (1 - \cos f) dr, \quad \cos f = \phi, \quad (20)$$

$\tilde{m} = 8\mu^2/3\pi F_\pi^2 e^2$ ,  $\mu = m_\pi$ . For strangeness, charm, or bottom the masses  $m_K$ ,  $m_D$  or  $m_B$  can be inserted instead of  $m_\pi$ .

Instead of above expression (8) we obtain now

$$M(B, r_0, b) = \alpha(B, b)r_0 + \beta(B, b)/r_0 + m r_0^3 \quad (21)$$

with  $\alpha, \beta$  given in (8) and  $m = 2\pi\tilde{m}/b \sin(3\pi/b)$ . It is possible to obtain precise minimal value of the mass

$$M(B, b) = \frac{2r_0^{\min}}{3} (\sqrt{\alpha^2 + 12m\beta} + 2\alpha), \quad (22)$$

at the value of  $r_0$

$$r_0^{\min}(B, b) = \left[ \frac{\sqrt{\alpha^2 + 12m\beta} - \alpha}{6m} \right]^{1/2}. \quad (23)$$

When the mass  $m$  is small enough, the expansion in  $12m\beta/\alpha^2$  can be made, and one obtains the following reduction of dimension  $r_0$ :

$$\begin{aligned} r_0 &\rightarrow r_0 - \frac{3m}{2\alpha} \left( \frac{\beta}{\alpha} \right)^{3/2} \simeq \\ &\simeq \sqrt{\frac{2}{3}} \left( \frac{\mathcal{I}}{A_N} \right)^{1/4} \left[ 1 - \frac{3\pi m}{2(2B + \sqrt{\mathcal{I}A_N})} \left( \frac{\mathcal{I}}{A_N} \right)^{3/4} \right], \end{aligned} \quad (24)$$

and increase of the soliton mass

$$\delta M = M \frac{m\beta}{2\alpha^2} \left[ 1 - \frac{9m\beta}{8\alpha^2} \right] \simeq M m \frac{\pi(\mathcal{I}A_N)^{3/4}}{2(2B + \sqrt{\mathcal{I}A_N})}. \quad (25)$$

We used that at large  $B$

$$\alpha \simeq \frac{1}{3\pi} \left( A_N b + \frac{8B}{b} \right), \quad \beta \simeq \frac{4}{9\pi} \left( Bb + \frac{2\mathcal{I}}{b} \right). \quad (26)$$

As it was expected from general grounds, dimensions of the soliton decrease with increasing  $m$ . However, even

for large value of  $m$  the structure of multiskyrmion at large  $B$  remains the same: the chiral symmetry broken phase inside of the spherical wall where the main contribution to the mass and topological charge is concentrated. The value of the mass density inside of the bubble is defined completely by the mass term with  $1 - \phi = 2$ . The baryon number density distribution is quite similar, with only difference that inside the bag it equals to zero. It follows from these results that RM approximated multiskyrmions cannot model real nuclei at large  $B$ , probably  $B > 12 - 20$ , and configurations like skyrmion crystals [8] may be more valid for this purpose.

It is of interest to study what happens at very large values of the mass, when  $12m\beta \gg \alpha^2$ . Making expansion in  $\alpha^2/12m\beta$  we obtain:

$$r_0^{\min} \simeq [\beta/3m]^{1/4}, \quad M_{m \rightarrow \infty}(B, b) \simeq \frac{4}{3} (3m\beta^3)^{1/4}. \quad (27)$$

Minimizing  $M(B, b)$  in  $b$  gives  $b_0^{\min} = \sqrt{2\mathcal{I}/B}$ ,  $\beta^{\min} = 8\sqrt{2\mathcal{I}B}/9\pi$  and

$$M_{m \rightarrow \infty}(B) \simeq \frac{16}{9} [4\sqrt{2}m/3\pi^3]^{1/4} [B\mathcal{I}]^{3/8}. \quad (28)$$

So, in this extreme case  $b \sim \sqrt{B}$ ,  $r_0 \sim (\mu)^{-1/2} B^{3/8}$ , the mass of the soliton increases as  $M \sim \sqrt{\mu} B^{9/8}$ , and the binding should become weaker with increasing  $m$  and baryon number of skyrmion.

It is possible also to calculate analytically the tensors of inertia of multiskyrmion configurations within this approximation, see [4] for definitions and formulas.

**4.** By means of consideration of special class of functions (6) we established the link between topological soliton models in rational maps approximation and the soliton models of "domain wall" or "spherical bubble" type. Of course, it is a domain wall of special kind, similar to honeycomb. The upper bound for the energy of multiskyrmions is obtained which is very close to the known energies of RM multiskyrmions, especially at largest  $B$ , and is higher than the known lower bound by  $\sim 4\%$  only.

The following properties of bubbles of matter from RM multiskyrmions are established analytically, mostly independent on particular values of the quantity  $\mathcal{I}$ .

The dimensions of the bubble grow with  $B$  as  $\sqrt{B}$ , or as  $\mathcal{I}^{1/4}$ , whereas the mass is proportional to  $\sim B$  at large  $B$ . Dimensions of the bubble decrease slightly with increasing  $N$  – the number of flavors,  $r_0 \sim [N/(2(N-1))]^{1/4}$ , see (11), (12).

The thickness of the bubbles envelope (15) is constant at large  $B$  and does not depend on the number of flavors, therefore, the average surface mass density is constant at large  $B$ , as well as average volume density of the shell. Both densities increase slightly with

increasing  $N$ . At the same time the mass and  $B$ -number densities of the whole bubble  $\rightarrow 0$  when  $B \rightarrow \infty$ , and this is in contradiction with nuclear physics data.

It follows from the above consideration that the spherical bubble or bag configuration can be obtained from the lagrangian written in terms of chiral degrees of freedom only, i.e. the Skyrme model lagrangian leads at large baryon numbers to the formation of spherical bubbles of matter. This picture of mass distribution in RM multiskyrmions contradicts at first sight to what is known about nuclei, however, it emphasises the role of periphery of the nucleus and could be an argument in favour of shell-type models. The skyrmion crystals [8] are believed to be more adequate for modelling nuclear matter.

It would be of interest to perform the investigation of the dynamics of bubbles in chiral soliton models similar to that performed recently for the simplified two-component sigma model, or sine-Gordon model in (3+1) dimensions [9]. Observations concerning the structure of large  $B$  multiskyrmions made here can be useful in view of possible cosmological applications of Skyrme-type models, see e.g. [10]. The large scale structure of the mass distribution in the Universe [11] is similar to that in topological soliton models, and it can be the consequence of the similarity of the laws in micro- and macroworld.

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