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Large Amplitude Oscillations Sustained by Stochastic Plasma Density Fluctuations in Plasma Sheaths

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We have examined the dynamics of a dust grain immersed in a plasma sheath. It is shown that the presence of stochastic plasma density variations can sustain a large amplitude dust grain oscillations once these have been induced by a slow plasma number density variation. Such dust oscillations have been observed in the sheath region of an radio-frequency or a dc plasma discharge at very low pressures. A physical mechanism for the excitation and maintenance of large amplitude grain oscillations is discussed.

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About seven years ago, several groups [1 – 5] reported experimental observations of dust crystals in radio-frequency (rf) and dc plasma discharges. It has been observed that charged dust grains may in fact not only levitate in the sheath of an rf or a dc plasma discharge, but they also "crystallize" due to a strong intergrain coupling. In laboratory experiments on the earth, the negatively charged dust grains of a plasma crystal are suspended/levitated over a negatively biased electrode due to a balance between the sheath electric and gravity forces. Numerous properties [6 – 8] of these new plasma states as well as their phase transitions [9, 10] have been investigated since the individual grains can be visualized and followed kinetically. The presence of gravity restricts the experiments that can be carried out on the earth. However, under microgravity conditions [11] there appear new features of the dust grain dynamics and collective interactions involving heart beat instability and the formation of dust voids and vortices.

The dynamics of a dust particle, at high pressure, is dominated by the neutral drag which damps its motion. The situation changes when the pressure is reduced to a few mTorr. Several experiments have now been carried out and careful observations have been made under very low gas pressures [12 – 15]. Under low pressures, the dust crystal is organized in a single layer. Nunomura et al. [14] have reported that in the 1–10 mTorr range a reduction of the plasma number density induces a large amplitude vertical grain oscillations. Such oscillations have been observed both in one- and two-dimensional dust crystals [12, 13]. It has been reported that the neighboring grains do not necessarily oscillate coherently; some do not oscillate at all, while others, with oscillation amplitude larger than a critical value ($\sim 1\,\mathrm{mm}$), drop onto the negative electrode. The typical oscillation frequency is in the range 10–14 Hz.

When a dust grain is immersed in an electron-ion plasma, it needs a finite time to acquire an equilibrium charge that is defined to be the charge that results from a zero net current onto the dust grain surface. Assuming that the electron current is dominant, we can estimate the charging time of a dust particle when it has reached an equilibrium potential. Considering a plasma electron temperature T_e =1 eV, an electron number density $n_0 \sim 1 \cdot 10^8\,\mathrm{cm}^{-3}$ and a particle radius $R = 2.5\,\mu\mathrm{m}$, we obtain a charging time $\tau \sim 6 \cdot 10^{-4}\,\mathrm{s}$ [16]. The charge delay of a dust particle can be important in the dust dynamics and give rise to many effects that have been studied by several authors [17, 18]. An index of the importance of the charge delay for an oscillating dust particle is the ratio between the characteristic grain charging time and the grain oscillation time. In our case, this ratio is $N_D \sim 6 \cdot 10^{-3}$. As pointed out by Nitter et al. [19], when N_D is small the effect of the charge delay can be neglected. Accordingly, in the following, we will not consider the damping of dust oscillations due to the charge delay.

A negatively charged dust grain levitated over an electrode in a plasma chamber is trapped in a potential well. In this Letter, we show that under low pressure conditions the potential well is strongly dependent on the plasma number density n_0. Small variations in the

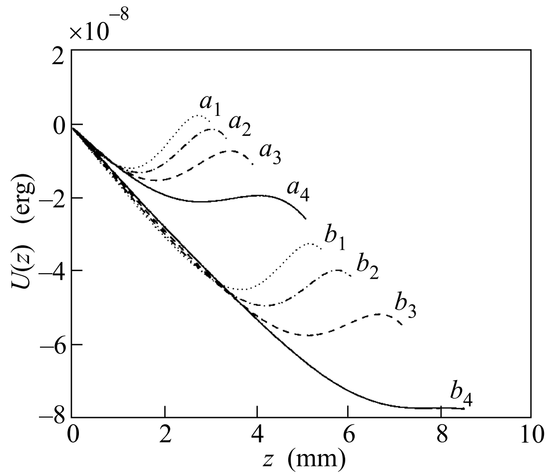

Fig.1. Profiles of the potential energy of a dust grain in the sheath region: a) curves correspond to $P = 20\,\mathrm{mTorr}$, and b) curves to $P = 1\,\mathrm{mTorr}$. The plasma number densities are: a_1, b_1) $4 \cdot 10^8$ cm^{-3}, a_2, b_2) $3 \cdot 10^8\,\mathrm{cm}^{-3}$, a_3, b_3) $2 \cdot 10^8\,\mathrm{cm}^{-3}$ and a_4, b_4) $1 \cdot 10^8$ cm^{-3}. The grain radius is $R = 2.9\,\mu$m

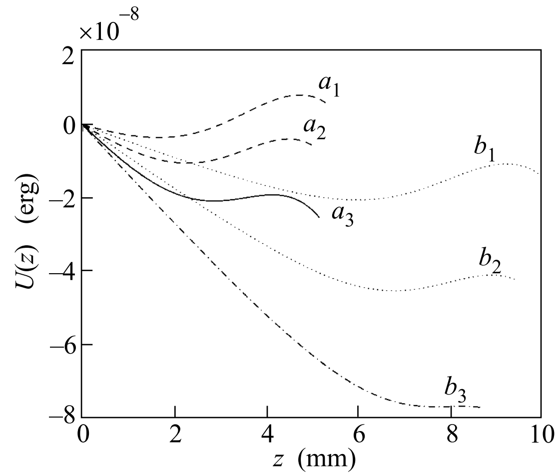

Fig.2. Profiles of the potential energy as a function of position: a) curves are calculated at $P = 20$ mTorr, and b) curves at $P = 1$ mTorr. a_1, b_1) $R = 2\,\mu$m, a_2, b_2) $R = 2.5\,\mu$m, a_3, b_3) $R = 2.9\,\mu$m. The parameters are the same as in Fig.1 except for the plasma number density which is fixed at $n_0 = 1 \cdot 10^8\,\mathrm{cm}^{-3}$

plasma number density ($\sim 1\%\ n_0$) modify the shape of the potential well as well as position of its stable equilibrium point leading to a large amplitude vertical oscillations of the dust grains. Under appropriate circumstances, the sensitivity of the potential well to the plasma number density increases as the plasma number density decreases. The lower the plasma density the greater the importance of small variations that can create and sustain oscillations. If the plasma number density is decreased below a critical value (n_{0C}) that depends on the grain radius R and the background pressure P, the grain will eventually fall onto the electrode; its kinetic energy being larger than the potential well confinement. This point will be discussed further in connection with Fig.1 and 2.

In the following, we have examined the dynamics of an isolated dust grain, but the results of the investigation are expected to be applicable to linear chains and sheets. In a 2D crystal, if we consider an electron Debye radius $\simeq 430\,\mu$m (equal to the intergrain separation distance) and an average grain charge $|Q/e| \approx 10^4$ (with Q and e the grain and elementary charges respectively), the interaction energy is $U_{int} = (Q^2/R)\exp(-R/\lambda_{D_e}) \sim 10^{-10}$ erg. As will become clear, the horizontal interaction energy is two orders of magnitude smaller than the external vertical interaction energy.

We consider the one-dimensional, source free, weakly collisional glow discharge model [20] to describe the unmagnetized sheath region. As the dust particles do not contribute significantly to the total space charge, since $|Qn_d/en_0| \ll 1$, we assume an 'empty sheath' [19]. In order to focus our attention on the importance of stochastic plasma number density fluctuations, we have kept the pressure constant ($P = 1{-}20\,\mathrm{mTorr}$) and no pressure oscillations are considered.

In the sheath, the electrons are thermalized and their number density is $n_e = n_0 \exp(e\phi_s/k_B T_e)$, where k_B is the Boltzmann constant, ϕ_s is the electrostatic potential, and T_e is the electron temperature. The ions, which are accelerated in the sheath region, experience a drag force $\mathbf{F}_c = m_i \nu_{in} \mathbf{v}_i$ due to collisions with neutrals. Consequently, an ion, on average, looses its momentum while traveling through the sheath. Here, m_i is the ion mass, $\nu_{in} = n_n \sigma_s v_i$ is the ion-neutral collision frequency, n_n is the neutral gas number density, $\mathbf{v}_i$ is the ion velocity, and σ_s is the momentum transfer cross section for collisions between ions and neutrals. Elastic and charge exchange are the main collision mechanisms in the sheath, and its cross section is almost constant over the energy range of interest (1–100 eV). The cross-section for collisions between ions and neutrals, σ_s, is typically $5 \cdot 10^{-15}\,\mathrm{cm}^2$ [21] over the energy range that is appropriate for dusty plasma laboratory experiments. The ions obey the continuity equation $\nabla \cdot (n_i \mathbf{v}_i) = 0$ and the steady state equation of motion $m_i (\mathbf{v}_i \cdot \boldsymbol{\nabla} \mathbf{v}_i) = -e\boldsymbol{\nabla}\phi_s - F_c$. Here, n_i is the ion number density. Introducing Poisson's equation, we obtain a complete set of differential equations describing the plasma sheath region. In the Gaussian unit they are

$$v_i \frac{dv_i}{dz} = -\frac{e}{m_i}\frac{d\phi_s}{dz} - n_n \sigma_s v_i^2, \qquad (1)$$

and

$$\frac{d^2\phi_s}{dz^2} = -4\pi e n_0 \left[\frac{v_{i0}}{v_i} - \exp\left(\frac{e\phi_s}{k_B T_e}\right)\right], \qquad (2)$$

where z is the particle distance from the sheath edge ($z = 0$). The electrode is at $z = D$. The ion velocity v_{i0} at the sheath edge, following the Bohm criterion, is the ion sound velocity. The electrostatic potential and the ion acceleration are chosen to be zero at the sheath edge [22]. The latter condition leads to $d\phi/dx|_{z=0} = -m_i n_n \sigma_s v_i^2/e$.

The sheath model gives the ion and electron number densities n_i, n_e, the ion velocity v_i, and the electrostatic potential ϕ_s. Thus one can calculate the forces that act on a dust particle and that are responsible for its equilibrium. For our parameters, the main forces are the electrostatic force acting vertically upwards and gravity and ion drag acting vertically downwards. Since the damping due to charge delay is negligible the friction is due only to the neutral drag. We consider a dust particle of size $2.9\,\mu$m and the mass density $\rho = 1.3\,\mathrm{g/cm^3}$.

As we already pointed out, the charge delay is so small that the grain is always at the equilibrium potential relative to its position in the sheath. Since the grain radius R is much smaller than the Debye radius ($\approx 430\,\mu$m) and the Debye length is much less than the neutral collision mean free path ($\sim 10\,$cm), we can use the orbital motion limited (OML) theory [23] to obtain the collection currents. For the ion current, we replace the thermal velocity and energy terms in the OLM expression by the mean speed $v_s = \sqrt{8k_BT_i/\pi m_i + \overline{v}_i^2}$. Here $\overline{v}_i = v_i - v$ is the ion speed relative to a dust grain moving with velocity v. The ion current thus may be regarded as a mono-energetic current, if the ion velocity is large compared to the ion thermal velocity, and as a spherically symmetric current in the opposite limit [24]. The normalized potential $y(z) \equiv e(\phi_g - \phi_s)/k_BT_e$ can now be calculated equating the sum of the electron and ion currents to zero. The electric force, considering a conducting dust grain, can be expressed as $F_e(z,v) = Q(z,v)E(z)$ where $Q(z,v) = k_BT_eRy(z,v)/e$.

In the sheath region, there is a continuous flow of ions towards the electrode driven by the electric field. The momentum transferred by the ions to the dust grain consists of two components: the collection and the orbit forces [24, 25]. It is found that the latter dominates since the collection radius is much smaller than the impact radius, ($b_\pi/2$). The orbit force is [25]

$$F_{io}(z,v) = 4\pi n_i v_{th_i} \tilde{v}_s m_i \overline{v}_i b_{\frac{\pi}{2}}^2 \Gamma, \qquad (3)$$

where $b_{\frac{\pi}{2}}^2 = (eQ)^2/(m_i\overline{v}_i^2)^2$ and $\Gamma = \frac{1}{2}\ln(1 + b_{\max}^2/b_{\min}^2)$. The maximum and minimum impact parameters, corresponding to maximum and minimum deflection angles, are $b_{\max} = \overline{\lambda} = (2n_e\lambda_{D_e} + n_i\lambda_{D_i})/(n_e + n_i)$ and $b_{\min} = r_p$, respectively. Here the ion and electron Debye radii are $\lambda_{D_i} = [4\pi e^2 n_i/((1/2)m_i\overline{v}_i^2 + k_BT_i)]^{-1/2}$ and $\lambda_{D_e} = (4\pi e^2 n_e/k_BT_e)^{-1/2}$, respectively.

In the low pressure regime, the molecular mean free path ($\ell \sim 10\,$cm) is larger then the grain dimension (i.e. $K_n = \ell/R \gg 1$). In such a "free molecular regime", taking into consideration that the dust mass m_d is 12 orders of magnitude larger than the gas molecules, the drag force for specular reflection is [24, 26]

$$F_{n_d}(v) = -(8/3)\sqrt{2\pi} r_p^2 n_n k_B T_n (v/v_{th_n}),$$

where $v_{th_n} = \sqrt{T_n/m_n}$ is the neutral thermal speed.

The total force F_t that acts on a dust particle is $F_t = F_e + F_g + F_{io} + F_{nd}$, where $F_g = \frac{4}{3}\pi R^3 \rho g$ and g represents gravity. The potential energy obtained for zero grain speed $U_t(z) = -\int_0^z dz' F_t(z')$ results from the total force as a function of z, and its shape and equilibrium points are strongly dependent on the discharge parameters.

The equations of motion for a dust grain, in normalized units, are

$$dz/dt = v, \qquad (4)$$

and

$$dv/dt = \tau_0 F_t/m_p v_0. \qquad (5)$$

In (4) and (5), although the same symbols are being used, they are to be understood as normalized quantities. Thus z is normalized by the ion Debye length in the plasma, the time is normalized by $\tau_0 = \sqrt{m_i/32n_0e^2}$, the total force F_t is normalized by $(k_BT_e/e)^2$, and the velocity is normalized by $v_0 = v_{thi} = \sqrt{8k_BT_i/\pi m_i}$.

Fig.1 shows the potential energy $U(z)$ for different plasma densities ($n_0 = 4\cdot 10^8 - 3\cdot 10^8 - 2\cdot 10^8 - 1\cdot 10^8\,\mathrm{cm^{-3}}$) at two different pressures $P = 1\,$mTorr and $P = 20\,$mTorr. The grain radius is $R = 2.9\,\mu$m. The Argon plasma parameters are: $T_e = 1\,$eV, $T_i = 0.05\,$eV and $T_n = 0.05\,$eV. At $P = 1$ mTorr the potential energy strongly depends on the plasma density. As the plasma density decreases the potential well becomes wider and the wall on electrode side becomes lower. When the plasma density is $n_0 = 1\cdot 10^8\,\mathrm{cm^{-3}}$ the confining potential almost disappears. Below the critical plasma number density $n_{0c}(P,R) = 0.96\cdot 10^8\,\mathrm{cm^{-3}}$ the grain confinement vanishes and the particle falls onto the electrode. It is important to observe that the energy a particle would gain from a jump in the plasma number density from $2\cdot 10^8\,\mathrm{cm^{-3}}$ to $1\cdot 10^8\,\mathrm{cm^{-3}}$ is $1.2\cdot 10^{-8}$ erg and is 10 times larger than the energy that the particle would gain with the same jump but from $4\cdot 10^8\,\mathrm{cm}^{-3}$ to $3\cdot 10^8\,\mathrm{cm}^{-3}$. Further, even a small plasma density fluctuation ($\sim 0.1\%$) near $1\cdot 10^8\,\mathrm{cm}^{-3}$ is able to offset the neutral damping. As an example we can consider a grain velocity of $v = 4\,\mathrm{cm/s}$ (conservative hypothesis) at 1 mTorr pressure (with $n_n = 1.16\cdot 10^{13}\,\mathrm{cm}^{-3}$). The neutral drag force resulting from these values is $F_{nd} = 4.3\cdot 10^{-11}$ dyne. If we consider an average 0.1% plasma number density fluctuation near $n_0 = 1\cdot 10^8\,\mathrm{cm}^{-3}$ the vertical energy gain would be $\sim 1.2\cdot 10^{-11}$ erg. Considering an oscillation amplitude of 1 mm the resulting force would be $\sim 1.2\cdot 10^{-10}$ dyne, more than enough to offset the neutral damping and sustain the oscillations. From figure 1 (with $P = 1\,\mathrm{mTorr}$), it can be argued that when a grain gains enough energy, it can fall onto the electrode. In other words, if the amplitude of the oscillations is larger than a critical value, the grain simply falls. On the other hand, when the pressure is higher (viz. $P = 20\,\mathrm{mTorr}$), the neutral friction is large and the energy gain for the same step described above (at $P = 1\,\mathrm{mTorr}$) is 5 times smaller. In the high pressure regimes the stochastic plasma number density fluctuations described in this letter do not influence the grain dynamics.

In Fig.2 the potential energy profiles are represented as a function of position for different grain radii ($R = 2.9 - 2.5 - 2\,\mu\mathrm{m}$) and for different values of pressures ($P = 1$ mTorr and $P = 20\,\mathrm{mTorr}$). The parameters are the same as in Fig.1 except for the plasma number density which is now $n_0 = 1\cdot 10^8\,\mathrm{cm}^{-3}$. Different grains have different equilibrium positions, and the heavier the grain the lower is the electrode side of the potential well. Therefore, the influence of plasma number density variations changes with the grain radius with the light particles being more stable.

In a real experiment, plasma density fluctuations are always present. In our numerical simulation the amplitude of the fluctuation is random with a maximum value of 2% n_0. The plasma density variations have a random time frequency with a maximum time step of 0.3 s. Since the fluctuations are random in amplitude and frequency, they can actually either excite or damp the dust oscillations. Thus some grains would gain energy and will have a large amplitude oscillation, some will stand still and others may actually fall onto the electrode.

Fig.3 shows the position of a dust grain as a function of time for a particle of radius $R = 2.9\,\mu\mathrm{m}$ immersed in a plasma sheath at a constant background pressure $P = 1\,\mathrm{mTorr}$. The simulation stops when the grain falls onto the electrode. The plasma number density decreases from $n_0 = 1.5\cdot 10^8\,\mathrm{cm}^{-3}$ to $n_0 = 0.96\cdot 10^8\,\mathrm{cm}^{-3}$. We presume that stochastic density fluctuations are always present in the plasma sheath [8], and in order to have a plasma number density reduction, we impose negative fluctuations 55% of the time. This is done to simulate experimental procedures whereby the plasma number density is decreased very slowly. At the beginning of the simulation, the plasma number density oscillates around a quasi fixed value (at $P = 1.5\,\mathrm{mTorr}$). As the plasma number density is slowly decreased on average, at some point the grain starts to oscillate with a high amplitude displaying a threshold behaviour. For the simulation run presented in Fig.3, the grain oscillation amplitude suddenly increases after roughly 15 seconds corresponding to a plasma density of $1.2\cdot 10^8\,\mathrm{cm}^{-3}$.

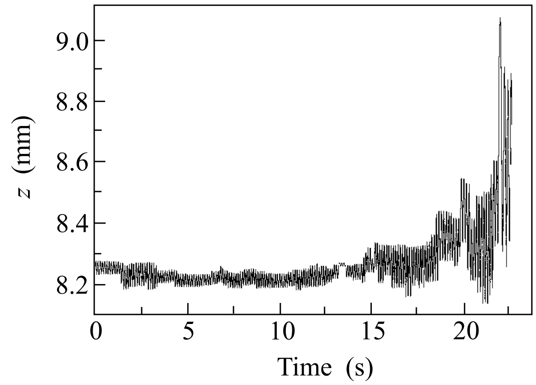

Fig.3. Position of an oscillating dust grain in the sheath as a function of time. The parameters are the same as in figure 1 except for $P = 1\,\mathrm{mTorr}$. The plasma number density decreases from $1.5\cdot 10^8\,\mathrm{cm}^{-3}$ to $0.96\cdot 10^8\,\mathrm{cm}^{-3}$ over a time interval of 23 s

To summarize, we have presented a novel mechanism that explains the salient features of dust grain oscillations which are observed in several dusty plasma experiments performed at low pressures [12–15]. The grain gains energy from the plasma number density reduction and stochastic plasma density fluctuations which are always present; but they are relevant only in the 1-10 mTorr pressure range when the neutral drag is weak. Depending on the final value of the plasma number density, there appear either a large amplitude dust grain oscillations, which are subsequently sustained due to random density fluctuations, or the grains actually fall onto the electrode. The dust oscillation frequencies that result from our simulation (9–11 Hz) are comparable with the frequencies that are observed experimentally [14].

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