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Two-terminal conductance of a fractional quantum Hall edge

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We have found solution to a model of tunneling between a multi-channel Fermi liqiud reservoir and an edge of the principal fractional quantum Hall liquid (FQHL) in the strong coupling limit. The solution explains how the chiral edge propagation makes the universal two-terminal conductance of the FQHL fractionally quantized and different from that of a 1D Tomonaga-Luttinger liquid wire, where a similar model but preserving the time reversal symmetry predicts unsuppressed free-electron conductance.

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Low energy transport through an incompressible quantum Hall liquid with gapped bulk excitations is carried by gapless edge modes [1, 2]. For principal fractional quantum Hall liquid (FQHL) of the filling factor $\nu = 1/odd$ these modes are described as a single branch of a chiral Luttinger liquid (χLL) [3]. In presence of the right and left chiral edges, the model of the FQHL transport [4] appears to be equivalent to that of a metallic phase of a 1D interacting electron gas [5] known as a Tomonaga-Luttinger liquid (TLL) [6, 7]. To describe the two-terminal transport experiments, the external reservoirs have to be added to the model [8, 2] so that the full transport process includes transformation of the reservoir electrons into the FQHL/TLL quasiparticles in the junctions. The transformation process makes the two-terminal conductance of both the TLL wire [9, 10] and the narrow FQHL junctions [11, 12] equal to the free electron conductance σ_0. The standard experimental observation, however, is that the two-terminal FQHL conductance is equal to the Hall conductivity $\nu\sigma_0$ (see, e.g., [13]) but not σ_0, the fact that implies equilibration between the chemical potentials of the reservoirs and the outgoing edges [14]. This problem was recently studied [15, 12] for the junction modeled as a sequence of point-like contacts between the edge and different channels of a multi-channel Fermi liquid reservoir under an additional assumption [15, 12] of suppressed quantum interference between electron tunneling at different contacts. The purpose of this work is to construct quantum solution of the model of tunneling between a multi-channel Fermi-liquid reservoir and the χLL edge. Our solution shows how both the standard fractional quantization of the FQHL conductance and the free electron conductance can be obtained not from the additional assumption of decoherence but from the appropriate account of different patterns of quantum interference depending on the junction structure. The solution explains (in obvious agreement with experiment) the difference between the universal two-terminal conductance $\nu\sigma_0$ of the 1D FQHL egde and conductance σ_0 of the 1D TLL wire.

The model we consider represents n scattering channels of the spinless FL reservoir as free chiral fermions. Tunneling from the channels (labeled by $j = 1, ..., n$) into the edge (labeled by 0) is assumed to be localized on the scale of magnetic length at the points x_j along the edge, where $x_j < x_i$ for $1 \le i < j \le n$. It is described by a tunnel Lagrangian:

$$\mathcal{L}_{tunn} = \sum_{j=1}^{n} [U_j \psi_0^+(x_j,t)\psi_j(x_j,t) + \text{h.c.}] , \qquad (1)$$

where U_j are chosen real and positive. Bosonization expresses the operators of free electrons $\psi_j(x,t) = (2\pi\alpha)^{-1/2}\xi_j e^{i\phi_j(x,t)}$ in the reservoir channels and the operator of electrons propagating along the edge $\psi_0 = (2\pi\alpha)^{-1/2}\xi_0 e^{i\phi_0(x,t)/\sqrt{\nu}}$ through their associate bosonic fields ϕ_l, the Majorana fermions ξ_l accounting for their mutual statistics, and a common factor $1/\alpha$ denoting momentum cut-off of the edge excitations. Since the spatial dynamics of the reservoir channels ($l > 0$) does not affect the tunneling currents, velocities of these channels are irrelevant, and we take them equal to the velocity v of the edge excitations. Free dynamics of the bosonic fields is governed then by the Lagrangian $\mathcal{L}_O = \sum_{l=0}^{n} (\phi_l \hat{K}^{-1} \phi_l)/2$, where the differential operator is:

$$\hat{K}^{-1}\phi_l(x,t) = \frac{1}{2\pi}\partial_x(\partial_t + v\partial_x)\phi_l(x,t). \qquad (2)$$

The full Lagrangian $\mathcal{L} = \mathcal{L}_O + \mathcal{L}_\xi + \mathcal{L}_{tunn}$ also includes an additional kinematic part $\mathcal{L}_\xi = (1/4)\xi\partial_t\xi$ describing a pure statistical dynamics of the Majorana fermions

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(time-ordering). A finite voltage applied to the reservoir is accounted for by the opposite-sign shift μ of the electrochemical potential of the edge, which can be introduced by an additional non-equilibrium part of the Lagrangian $\mathcal{L}_V = \sqrt{\nu}/2\pi \int dx \phi_0(x,t)\partial_x V(x)$, where $V(x)$ depends on the modeled physics and in our case can be chosen as follows. In the absence of tunneling, when the evolution of the edge is governed by the retarded Green function $K(x,t) = \pi\theta(t)\mathrm{sgn}(x-vt)$ of the operator $\hat{K}^{-1}$ in Eq. (2), the Lagrangian $\mathcal{L}_V$ shifts ϕ_0 and results in its non-zero average

$$\overline{\phi}_0(x,t) = -(\sqrt{\nu}/2\pi)\int dt' \int dy K(x-y,t-t')\partial_y V(y)$$

satisfying

$$-\partial_t\overline{\phi}_0 = v\partial_x\overline{\phi}_0 + \sqrt{\nu}[V(x) - \frac{1}{2}\sum_{\pm} V(\pm\infty)]. \qquad (3)$$

This equation shows that $\mathcal{L}_V$ describes two physical processes. The first is current injection into the edge due to $V(x)$: $\partial_t\rho_0 + \partial_x j_0 = -\nu\partial V(x)/2\pi$ (chiral anomaly), where the chiral edge density $\rho_0 = \sqrt{\nu}\partial_x\phi_0/2\pi$ is related to the current as $j_0 = v\rho_0$. The second is an additional shift of electrochemical potential of the edge equal to $[V(x) - \sum_\pm V(\pm\infty)/2]$. By choosing $V(x) = -\mu\ \mathrm{sgn}(x - y_X)$, with $y_X \to \infty$, we reduce the whole effect to the shift of the edge potential relative to the reservoir by in general time-dependent potential $\mu(t)$ without producing any additional edge current at $x < y_X$. With this choice, in presence of tunneling, the edge current caused by $\mathcal{L}_V$ is just the opposite of the total tunneling current.

We start by considering the strong coupling limit of a one-point contact ($n = 1$). The tunneling Lagrangian reduces to

$$\mathcal{L}_{tunn} = U_1/2\pi\alpha\cos(\phi_0(x_1,t)/\sqrt{\nu} - \phi_1),$$

and in the limit $U_1 \to \infty$ fixes the argument of the cos-term at one of the cosine maxima, e.g., $\phi_0(x_1,t)/\sqrt{\nu} = \phi_1(x_1,t)$. Then, introducing the vector $\phi(x,t) = [\phi_0,\phi_1]^T$, one can find its two-component average $\overline{\phi}(x,t) \equiv \langle\phi(x,t)\rangle$ as

$$\overline{\phi}(x,t) = -i\frac{\sqrt{\nu}}{\pi}\int\frac{d\omega}{\omega}e^{-i\omega t}g(x,\omega)\mu(\omega), \qquad (4)$$

where $-2\pi i g/\omega$ is the first column of the (2×2) matrix Green function. The two components $g_{0,1}(x)$ of the function g do not depend on $y_X \to \infty$ for $x < y_X$, and satisfy the homogeneous differential equation (2) at $x \neq x_1$, and therefore can be written as $g_{0,1} = a_{0,1} + b_{0,1}\exp[-i\omega x/v]$. The coefficients a and b take different values $a_{0,1}^{\wp}, b_{0,1}^{\wp}$ for x smaller and larger than x_1 ($\wp$ denotes $<$ or $>$, respectively). They are related among themselves by four conditions: contituinty of g_0 and g_1; continuity of the current flow: $\sqrt{\nu}b_0^< + b_1^< = \sqrt{\nu}b_0^> + b_1^>$; and maximum of the tunneling term: $g_0(x_1) - \sqrt{\nu}g_1(x_1) = 0$. The solution g is a linear combination of the four independent functions:

$$f_c^- = [\sqrt{\nu},1]^T,\ f_b^- = e^{i\omega x/v}f_c^-,$$

$$f_1^{\langle,\rangle} = \theta(\mp(x-x_1))(e^{i\omega x/v}-1)[1,-\sqrt{\nu}]^T,$$

which are constructed to satisfy these conditions. Since propagation of tunneling electrons are governed by the free matrix Green function, which is diagonal and equal to $K \times 1$, where

$$K(x-y,\omega) = -\frac{2\pi i}{\omega}\left[\frac{1}{2} + \theta(x-y)(e^{\frac{-i\omega(x-y)}{v}} - 1)\right], \qquad (5)$$

we can find more restrictions on the coefficients: $b_{0,1}^< = 0$, $a_1^< = -a_1^>$, $a_0^{\wp} = 1/2 - a_1^{\wp}/\sqrt{\nu}$. They uniquely specify $g(x,\omega) = [\sqrt{\nu}f_c^-/2 - f^>]/(1+\nu)$.

The currents follow then from Eq. (4) as

$$j_1(x,t) = -j_0 = \frac{2\nu}{1+\nu}\theta(x-x_1)\sigma_0\mu(t - [x-x_1]/v).$$

The tunneling conductance is equal to $G_1 = 2\nu\sigma_0/(1+\nu)$ in agreement with the result of application [12] of the chirally symmetric solution developed for a point scatterer in TLL [5].

To extend this approach to the multi-channel contact, we notice that although the statistical factors $\pm\xi_0\xi_j$ attributed to annihilation/creation of electrons in the jth channel can not be ignored for more than one j involved, they can be substituted [7] by the exponents $\exp\{\pm i\sqrt{\gamma}\eta_j\}$ of the zero-energy bosonic fields satisfying $[\eta_i,\eta_j] = i\pi\mathrm{sgn}(i-j)$ with an odd integer γ that specifies a phase branch of the fermionic statistics. These fields can be readily constructed from the standard creation and annihilation operators of $n-1$ independent zero-energy bosonic modes. Since any non-vanishing term of the perturbative expansion in $\mathcal{L}_{tunn}$ contains $\pm$ exponents in pairs, a proper interchange cancels all exponents and leaves only the statistical sign, the same one would get directly from the Majorana fermions. The substitution of the Majorana fermions by bosonic modes transforms $\mathcal{L}_{tunn}$ into:

$$\mathcal{L}_{tunn} \equiv \sum_{j=1}^{n}\mathcal{L}_j = \sum\frac{U_j}{2\pi\alpha}\cos\{\frac{\phi_0(x_1,t)}{\sqrt{\nu}} - \phi_j - \sqrt{\gamma}\eta_j\}. \qquad (6)$$

Equation (6) posseses the initial commutation symmetry between the different parts of the tunneling Lagrangian, since permutation of $\mathcal{L}_i$ and $\mathcal{L}_j$ results in appearance of the phase factors $\exp\{\pm i\pi \mathrm{sgn}(i-j)(\gamma-1/\nu)\}$ equal to 1 for any odd γ. As all U_j in (6) tend to ∞, all the cosine arguments are simultaneously fixed. One can notice, however, that this strong coupling limit depends on the choice of γ. Indeed, in this limit, each $\mathcal{L}_j$ can be approximated as $-\overline{U}_j/(2\pi\alpha)(\phi_0(x_1,t)/\sqrt{\nu}-\phi_j-\sqrt{\gamma}\eta_j)^2$ with sufficiently large $\overline{U}_j$, the form that clearly puts γ (and not exponent of γ) in the commutator between $\mathcal{L}_i$ and $\mathcal{L}_j$. Moreover, if $|x_i-x_j|\gg\alpha$ for all i,j, there is only one choice of γ, $\gamma=1/\nu$, which does not violate the commutativity of limiting forms of $\mathcal{L}_j$. Relevance of the different choices of γ in the strong coupling limits can be understood from their effect on the energy of the system [14]. Here, however, we chose a more heuristic physical argument. We prove that only the symmetric strong coupling limit can be relevant, since all others choices of γ lead to solutions which do not satisfy the condition of causality.

To show this, we calculate the current flow in the strong coupling limit of Eq. (6) keeping γ as a free parameter. The calculation generalizes the one for the single-point contact. The average $\overline{\phi}(x,t)$ in Eq. (4) becomes the $(n+1)$-component vector $<[\phi_0,...,\phi_j+\sqrt{\gamma}\eta_j,...]^T>$, and $-2\pi i g/\omega$ is the first column of the corresponding $(n+1)\times(n+1)$ matrix Green function. The coefficients $a_j, b_j, j=1\div n$ take different values $a_j^{\wp}, b_j^{\wp}$ for x smaller and larger than x_j, where $\wp$ denotes $<$ and $>$ as before. The edge channel coefficients a_0, b_0 take $(n+1)$ different values, changing at each tunneling contact $x=x_j$ in a way that relates them to a_j, b_j by the four matching conditions derived above for the single-contact case. We denote with $a_0^{\wp}$ and $b_0^{\wp}$ their values for x smaller than x_n ($\wp=<$) and larger than x_1 ($\wp=>$). A set of $2(n+1)$ independent vector-functions satisfying all these conditions may be chosen as:

$$f_c^- = [\sqrt{\nu},1,1,1,\ldots]^T,\ f_b^- = e^{i\omega x/v} f_c^-,$$
$$f_j = (e^{i\omega x/v} - e^{i\omega x_j/v})e_j,$$
$$f_j^> = (e_0/\sqrt{\nu}-e_j)\theta(x-x_j)(e^{i\omega(x-x_j)/v}-1) + \sum_{l=1}^{j-1} e_l(e^{i\omega(x_l-x_j)/v}-1)/\nu,$$

where a vector e_l has the only non-zero l-th component equal to 1. Since all coefficients $b_l^<$ of the function g are zero, it can be expanded in this basis as

$$g = s_c f_c^- + \sum_{j=1}^{n} s_j f_j^> \qquad (7)$$

with $(n+1)$ unknown coefficients s_l. The non-zero s_j lead to finite jumps of a_j and b_j at $x=x_j$, and therefore, to the non-vanishing $b_j^> = -s_j e^{-i\omega x_j/v}$. Then, in accordance with Eq. (4), the reservoir channel currents arising at x_j can be found as $j_j(\omega,x)=2\sqrt{\nu}\mu\sigma_0 b_j^>\theta(x-x_j)e^{i\omega x/v}$. Jumps of the coefficients a_j and b_j are caused by the charge tunneling at the contact points x_j, with further propagation of charge governed by the free retarded Green function. This means that this function determines both the coninuous parts of the a, b coefficients and the relations between their discontinuous parts and the coefficients s_j. The Green function is $(n+1)\times(n+1)$ matrix, and can be written as $K\times\mathbf{1}-\gamma\pi i\mathbf{C}/\omega$, where $\mathbf{C}$ is the antisymmetric matrix with all elements above the diagonal, except the first row, equal to 1. From this form one can find that $b_l^<=0$ (the fact already used in Eq. (7)), and that the coefficients a_l are related to s_j. In particular: $a_0^< = 1/2+\sum_{p=1}^n s_p/2\sqrt{\nu}$, $a_j^< = -s_j/2+\gamma/2\sum_{p\neq j}\mathrm{sgn}(j-p)s_p$. From comparison of these relations to those obtained by direct substitution of the f-vectors into Eq.(7) we get n equations:

$$\frac{\sqrt{\nu}}{1+\nu} + s_n = -\frac{1-\gamma\nu}{1+\nu}\sum_1^{n-1} s_i,$$
$$s_p = s_n + \sum_{j=p+1}^n \frac{2s_j}{(1+\gamma)}[(1-e^{i\omega(x_p-x_j)/v})/\nu-\gamma], \qquad (8)$$

where $p=1\div(n-1)$. Equations (8) allow us to determine all unknown coefficients s_j.

For the two-point contact these equations reduce to:

$$s_1 = (1-\gamma+\frac{2}{\nu}[1-e^{i\omega(x_1-x_2)/v}])\frac{s_2}{1+\gamma},$$
$$s_2 = -\frac{\nu^{3/2}(1+\gamma)}{2R}, \qquad (9)$$
$$R\equiv 1+\nu(1-\gamma)+\frac{\nu^2}{2}(1+\gamma^2)-(1-\nu\gamma)e^{i\omega(x_1-x_2)/v}.$$

The part of the denominator R proportional to $(1-\nu\gamma)$ signals the appearence of an interference structure in the currents. Substituting $s_{1,2}$ (9) into $j_0(x,t) = -\int d\omega e^{-i\omega t}\sum_{1,2} j_j(\omega,x)$ one can see that, indeed, the time dependence of charge propagation along the edge exhibits multiple backscattering at x_2 and x_1: A charge wave started by the tunneling into the edge propagates from the point x_2 to x_1 with the velocity v and then instantly recoils back to x_2 from x_1 with a finite reflection coefficient proportional to $(1-\nu\gamma)$. The formal possibility of the charge propagation with infinite velocity in the direction opposite to the edge chirality is a combined effect of x-independent solutions of the operator $\hat{K}^{-1}$ from (2) and the matching conditions at the tunneling points. However, the instant "counter-propagation" violates causality of the edge response to external perturbations and can not appear in the final physical results. This makes $\gamma = 1/\nu$ the only relevant strong coupling limit for $x_1 - x_2 \gg \alpha$ and clarifies the consequences of breaking the commutational symmetry of the initial tunneling Lagrangian for other choices of γ.

When the two tunneling points practically coincide, $x_1 - x_2 \leq \alpha$, the Lagrangian symmetry is preserved for any γ. To make a physical choice of γ in this case we look at the tunneling conductance $G = \sigma_0 \frac{4\nu}{2+\nu[1+\gamma^2]}$ that follows from Eq. (9) at zero frequency. If $\gamma = 1$ (corresponding to the minimal phase of the fermionic statistics), then $G = G_1$. In the tunneling model (1) with $n = 2$, this value of conductance represents the situation when the chiral dynamics of the edge does not play any role, and the two reservoir channels are reduced to one tunneling mode. The choice of γ can be also confirmed by consideration of the tunneling energy [14], which for $x_1 \simeq x_2$ is minimized by the smallest γ consistent with the statistics of the tunneling operators. In particular, $\gamma = 0$ gives the strong tunneling conductance in the model of an impurity scatterer in TLL of 2 spin-degenerate channels with the spin coupling constant $g_s = 2$ and the charge constant $g_c = 1/(1/\nu + 1/2)$.

For spatially separated tunneling points, the symmetry preserving solution with $\gamma = 1/\nu$ reproduces equilibration between the reservoir and the edge. To see this, we substitute $\gamma = 1/\nu$ into the first of Eqs. (8) and find that $s_n = -\sqrt{\nu}/(1+\nu)$ for any n. This shows that the tunneling into the nth channel is described by the one-point tunneling conductance G_1 for all frequencies ω, since it can not be affected by other contacts down the edge. The zero-frequency solution of the second of Eqs. (8) is: $s_{p-1} = q s_p, p = 2 \div n$, with $q = (1-\gamma)/(1+\gamma)$ equal to $1 - G_1/\sigma_0\nu$ for $\gamma = 1/\nu$. It means that in the strong coupling limit the tunneling current Δj out of the edge results in the $\Delta j/\sigma_0\nu$ drop of the edge chemical potential. The zero-frequency n point tunneling conductance follows from $b_0^{>} = \sum s_j/\sqrt{\nu} = s_n(1+\nu)(1-q^n)/2\sqrt{\nu}$ as $\sigma_0\nu(1-q^n)$ and saturates at $\nu\sigma_0$, when $n \to \infty$ and the outgoing edge is equilibrated with the reservoir.

In conclusion, we have found the strong-coupling solution of the model of tunneling between the multimode Fermi-liquid reservoir and an edge of the principal FQHL. The solution depends on the choice of the statistical phase branch of different reservoir modes with the physically relevant choice of the phase preserving the initial commutation symmetry of the tunneling Lagrangian. The statistical phase accounts for an even number of fluxes absorbed/emitted by tunneling electrons. The results explain the difference between transport through a 1D FQHL edge and a TLL wire: the two-terminal universal conductance of the edge is renormalized by the flux attachment, while direct electron-electron interaction in the wire does not change its universal free-electron conductance.

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