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Do neutrino oscillations allow an extra phenomenological parameter?

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The quantity ξ introduced recently in the phenomenological description of neutrino oscillations is in fact not a free parameter, but a fixed number.

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The literature on phenomenology of neutrino oscillations is vast (see, e.g., [1 – 6] and references therein). In a recent paper [7] Giunti and Kim in the case of two-flavour mixing have introduced a new phenomenological parameter ξ. According to [7], $\xi = 0$ corresponds to the so-called equal momentum assumption [1, 2], while $\xi = 1$ corresponds to equal energy assumption [5, 6]. Authors of [7] emphasize that ξ disappears from final expressions for the neutrino oscillation probability.

The aim of this note is to indicate that parameter ξ is fixed by energy-momentum conservation in the process which is responsible for neutrino emission, as explicitly assumed in ref. [7].

Following ref. [7] we will consider the decay $\pi \to \mu\nu$ in the framework of two-flavour toy model. The parameter ξ is defined in [7] for the pion rest-frame by considering the auxiliary case of absolutely massless neutrinos and denoting the energy of such neutrinos as E,

$$\xi = {}^1/_2(1 + m_\mu^2/m_\pi^2), \qquad (1)$$

where m_μ and m_π are the masses of the muon and the pion. Then for massive (but light!) neutrinos authors of [7] get:

$$E_{1,2} = E + (1 - \xi)m_{1,2}^2/2E, \qquad (2)$$

$$p_{1,2} = E - \xi m_{1,2}^2/2E. \qquad (3)$$

Here $E_{1,2}$, $p_{1,2}$ and $m_{1,2}$ are the energies, momenta and masses of the neutrinos, respectively. Wherefrom the above statement about $\xi = 0, 1$ follows:

$$E_1 = E_2 \ \text{for} \ \xi = 1 \ \text{and} \ p_1 = p_2 \ \text{for} \ \xi = 0. \qquad (4)$$

Thus, the equal energy and equal momentum assumptions in the form $\Delta E \equiv E_1 - E_2 = 0$ and $\Delta p \equiv p_1 - p_2 = 0$, respectively, are treated by authors of ref. [7] as particular cases of the general kinematical relations (1) and (2):

$$\Delta E = (1 - \xi)\Delta m^2/2E = 0 \ \text{for} \ \xi = 1, \qquad (5)$$

$$\Delta p = \xi \Delta m^2/2E = 0 \ \text{for} \ \xi = 0. \qquad (6)$$

Unfortunately, both the treatment and the relations (6) – (8) are erroneous.

On one hand, the quantity ξ is not a free parameter. Indeed, it follows from (5) that ξ has a fixed value ($\simeq 0.8$) for the decay under consideration. On the other hand, it is evident from definitions of E and ξ that

$$E = m_\pi(1 - \xi). \qquad (7)$$

The parameter ξ determines sharing of the decay energy. As seen from (3), the values $\xi = 0$ and $\xi = 1$ are senseless ones because they refer accordingly to the limiting cases of $E_{\text{recoil}} = 0$ and $E = 0$. Therefore one cannot assume that ξ can be equal to 1 or 0. Instead of that, the solution of the equalities (7) and (8) is the vanishing Δm^2, that is absence of the oscillations.

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