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Generation of gravitational radiation in dusty plasmas and supernovae

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We present a novel nonlinear mechanism for exciting a gravitational radiation pulse (or a gravitational wave) by dust magnetohydrodynamic (DMHD) waves in dusty astrophysical plasmas. We derive the relevant equations governing the dynamics of nonlinearly coupled DMHD waves and a gravitational wave (GW). The system of equations is used to investigate the generation of a GW by compressional Alfvén waves in a type II supernova. The growth rate of our nonlinear process is estimated, and the results are discussed in the context of the gravitational radiation accompanying supernova explosions.

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It is well known that there exist numerous mechanisms for the conversion between gravitational waves (GWs) and electromagnetic waves [1 – 19].For example, the propagation of GWs across an external magnetic field gives rise to a linear coupling to the electromagnetic field [1] , which may lead to the gravitational wave excitation of ordinary electromagnetic waves in vacuum, or of magnetohydrodynamic (MHD) waves in a plasma [2 – 4]. Furthermore, various nonlinear coupling mechanisms give rise to three-wave couplings between GWs and electromagnetic waves in matters. We also note that four-wave processes may cause graviton-photon conversion even in the absence of external matters or fields [5]. Moreover, GWs can couple to other types of waves, e.g. sound waves, also in neutral media [6]. There are numerous motives for considering wave couplings involving GWs. In some cases, the emphasis is on the basic theory [5 – 10]. In other works, the focus is on GW detectors [11 – 13], on cosmology [14 – 16] or on astrophysical applications such as binary mergers [17], gamma ray bursts [18] or pulsars [19]. Here supenovae and neutron star formation, giving rise to GWs as considered in, e.g., Refs. [20, 21], will be of special interest, since the possibility of dust formation in supernova remnants is of current astrophysical interest [22, 23]. Many of the previous works have concentrated on the conversion from GWs to electromagnetic waves, which can be analysed within a test matter approach which neglects the back reaction on the gravitational field. We note that such an approach can be justified if the background energy density is low.

In this Letter, we consider the three-wave coupling between two dust magnetohydrodynamic (DMHD) waves and a GW, including the effects of dust particles [24] in a dense medium such as the supernova where electrons, protons, and charged dust macroparticles are abundant. For this purpose, we derive the dust Hall MHD equations [24], i.e. equations describing the dust MHD waves, including the effect of a GW. We emphasize that for a low-beta plasma, the system of equations has a structure which can describe both a dust-dominated plasma, as well as an ordinary Hall-MHD plasma (if we replace the dust mass density by the ion mass density). Using the normal mode approach [25], the three-wave coupling equations are derived, including the back reaction on a GW from the Einstein equations. The system is shown to fulfil the Manley-Rowe relations [25] (which means that the interaction process can be viewed quantum mechanically) and to be energy conserving. The three-wave equations are then used to analyse the generation of a GW by the compressional Alfvén waves in the iron core of the type II supernova [26, 27]. It turns out that the characteristic timescale for the Alfvén-GW conversion can be less than a millisecond, which implies that the mechanism is potentially relevant for the high-frequency part (> 1MHz) of the supernova GW spectrum.

The plasma dynamics, due to the response to a gravitational wave

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$$ds^2 = -dt^2 + (1+h_+)\,dx^2 + + (1-h_+)\,dy^2 + 2h_\times dx\,dy + dz^2 \quad (1)$$

can be formulated according to

$$\partial_t n_s + \nabla\cdot(n_s \mathbf{v}_s) = 0, \quad (2)$$

and

$$m_s n_s(\partial_t + \mathbf{v}\cdot\nabla)\mathbf{v} = = -\nabla p_s + q_s n_s(\mathbf{E} + \mathbf{v}\times\mathbf{B}) + m_s n_s \mathbf{g}_s, \quad (3)$$

where we have introduced the tetrad $\mathbf{e}_0 = \partial_t$, $\mathbf{e}_1 = = (1-h_+/2)\partial_x - h_\times/2\partial_y$, $\mathbf{e}_2 = (1+h_+/2)\partial_y - h_\times/2\partial_x$, $\mathbf{e}_3 = \partial_z$, and $\nabla = (\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3)$. Moreover,

$$\mathbf{g}_s = -\tfrac{1}{2}(1-v_{sz})(v_{sx}\partial_t h_+ + v_{sy}\partial_t h_\times)\mathbf{e}_1 - -\tfrac{1}{2}(1-v_{sz})(v_{sx}\partial_t h_\times - v_{sy}\partial_t h_+)\mathbf{e}_2 - -\tfrac{1}{2}\left[(v_{sx}^2 - v_{sy}^2)\partial_t h_+ + 2v_{sx}v_{sy}\partial_t h_\times\right]\mathbf{e}_3, \quad (4)$$

represents the gravitational acceleration of the particle species s due to the GWs. We have assumed that $\partial_z \approx -\partial_t$ holds for the GWs.

The electromagnetic field is determined through the gravity modified Maxwell's equations. Using the same notation as above, they take the form

$$\partial_t \mathbf{E} = \nabla\times\mathbf{B} - \sum_s q_s n_s \mathbf{v}_s - \mathbf{j}_E, \quad (5)$$

$$\partial_t \mathbf{B} = -\nabla\times\mathbf{E} - \mathbf{j}_B, \quad (6)$$

with the constraints $\nabla\cdot\mathbf{E} = \sum_s q_s n_s$ and $\nabla\cdot\mathbf{B} = 0$. Here, the effects of the GWs (1) are represented by the effective currents

$$\mathbf{j}_E = -\tfrac{1}{2}\left[(E_x - B_y)\partial_t h_+ + (E_y + B_x)\partial_t h_\times\right]\mathbf{e}_1 - -\tfrac{1}{2}\left[-(E_y + B_x)\partial_t h_+ - (E_x - B_y)\partial_t h_\times\right]\mathbf{e}_2, \quad (7)$$

and

$$\mathbf{j}_B = -\tfrac{1}{2}\left[(E_y + B_x)\partial_t h_+ - (E_x - B_y)\partial_t h_\times\right]\mathbf{e}_1 - -\tfrac{1}{2}\left[(E_x - B_y)\partial_t h_+ + (E_y + B_x)\partial_t h_\times\right]\mathbf{e}_2. \quad (8)$$

With the general setting established above, we will from now on focus on the case of a three-component dusty plasma, for which we have the equation of state $p_s = k_B T_s n_s$. Thus, the plasma is composed of electrons (e), ions (i), and dust particles (d). The mass m_d of the dust particles is assumed to be much larger than the electron and ion masses, viz. m_e and m_i, respectively. We will assume that the plasma is approximately quasi-neutral, i.e. $q_i n_i = e n_e - q_d n_d$. Moreover, the waves under consideration are supposed to propagate with phase velocities much smaller than the speed of light c. Thus, we may neglect the displacement current in Ampére's law (5), i.e.

$$\nabla\times\mathbf{B} = \sum_s q_s n_s \mathbf{v}_s + \mathbf{j}_E. \quad (9)$$

Due to the constraint $m_e, m_i \ll m_d$ the momentum conservation equation (3) for the inertialess electrons and ions becomes

$$0 = -k_B T_e \nabla n_e - e n_e(\mathbf{E} + \mathbf{v}_e\times\mathbf{B}) + m_e n_e \mathbf{g}_e, \quad (10)$$

and

$$0 = -k_B T_i \nabla n_i + q_i n_i(\mathbf{E} + \mathbf{v}_i\times\mathbf{B}) + m_i n_i \mathbf{g}_i, \quad (11)$$

respectively. Adding Eqs. (10) and (11), using the quasineutrality condition, assuming that the number densities of the electrons and ions are not much larger than the number density of the dust, and using the heavy dust approximation, the dust momentum equation takes the form

$$\rho_d\,(\partial_t + \mathbf{v}_d\cdot\nabla)\,\mathbf{v}_d = -k_B\left(T_d - \frac{q_d}{q_i}T_i\right)\nabla n_d + +(\nabla\times\mathbf{B})\times\mathbf{B} - \mathbf{j}_E\times\mathbf{B} + \rho_d \mathbf{g}_d \quad (12)$$

where $\rho_d = m_d n_d$. In Eq. (12) we have also used the approximation $[T_e + (e/q_i)T_i]n_e \ll [T_d - (q_d/q_i)T_i]n_d$.

Again using Eqs. (10) and (11) to eliminate the electric field, Faraday's law (6) becomes

$$\partial_t \mathbf{B} = \nabla\times(\mathbf{v}_d\times\mathbf{B}) - -\frac{m_d}{q_d}\nabla\times\left[(\partial_t + \mathbf{v}_d\cdot\nabla)\,\mathbf{v}_d - \mathbf{g}_d\right] - \mathbf{j}_B, \quad (13)$$

where we have used the dust momentum equation (12).

Thus, Eqs. (12) and (13) together with the dust continuity equation

$$\partial_t \rho_d + \nabla\cdot(\rho_d \mathbf{v}_d) = 0, \quad (14)$$

constitute the dust MHD equations in the presence of a GW. For a low-beta plasma, the pressure term in (12) is negligible, which means that the structure of Equations (12)-(14) is the same as in an ordinary Hall-MHD plasma without dust. Henceforth, we will consider a low-beta plasma, drop the index d on all quantities and thus let q/m be either the ion charge to mass ratio or the considerably smaller charge to mass ratio of the dust particles. As a result, our mathematical analysis below will then apply either to a dust Hall-MHD plasma, or to an ordinary Hall-MHD plasma without dust.

To simplify the problem, we consider the case when the dust-acoustic speed $c_s = k_B T/m$ is much smaller than the dust Alfvén velocity $C_A = \left(B_0^2/\mu_0\rho\right)^{1/2}$ such that the pressure term in (12) can be neglected. As a prerequisite for the nonlinear calculations, we first study the linear modes of the system (12)–(14) omitting the gravitational contributions. Letting $B = B_0\hat{\mathbf{z}} + \mathbf{B}_1$, $\rho = \rho_0 + \rho_1$, where the index 1 denotes the perturbation of the equilibrium part, and linearizing Eqs. (12)–(14) and Fourier analysing, we readily obtain the dispersion relation

$$\left(\omega^2 - k_z^2C_A^2\right)\left(\omega^2 - k^2C_A^2\right) - \frac{\omega^2k_z^2k^2C_A^2}{\omega_c^2} = 0, \quad (15)$$

where $\omega_c = qB_0/m$ is the gyrofrequency. For frequencies much smaller than the gyrofrequency, we note that the modes separate into the shear Alfvén wave, $\omega^2 - k_z^2C_A^2 \approx 0$, and the compressional Alfvén wave, $\omega^2 - k^2C_A^2 \approx 0$. Below we will consider the more general case described by (15), however. For later applications it is convenient to use the linear equations to express all quantities in terms of a single variable. Thus, we let the wavevector of the dust MHD waves lie in the $x-z$-plane, and write

$$v_y = i\frac{\omega}{\omega_c}\frac{k_z^2C_A^2}{\left(\omega^2 - k_z^2C_A^2\right)}v_x,\ v_z = 0,\quad \rho_1 = \rho_0\frac{k_xv_x}{\omega}, \quad (16)$$

$$B_x = -B_0\frac{\omega k_z}{k^2C_A^2}v_x, \quad (17)$$

$$B_y = -iB_0\frac{\omega^2}{\omega_c}\frac{k_z}{\left(\omega^2 - k_z^2C_A^2\right)}v_x, \quad (18)$$

$$B_z = B_0\frac{\omega k_x}{k^2C_A^2}v_x. \quad (19)$$

Next, we consider a system of three weakly interacting waves. Two dust MHD waves with frequencies and wave-numbers $(\omega_1, \mathbf{k}_1)$ and $(\omega_2, \mathbf{k}_2)$ respectively, and an arbitrarily polarized gravitational wave propagating along the z-direction with the frequency and wavenumber $(\omega_g, k_g\hat{\mathbf{z}})$. Noting that the gravitational dispersion relation reads $\omega_g = k_gc$ and that $C_A \ll c$, the frequency and wavenumber matchings (energy and momentum conservation) can be approximated

$$\omega_g = \omega_1 + \omega_2,\ \mathbf{k}_g = \mathbf{k}_1 + \mathbf{k}_2 \Rightarrow 0 \approx \mathbf{k}_1 + \mathbf{k}_2. \quad (20)$$

We will thus use $\mathbf{k}_1 \approx -\mathbf{k}_2$ below, and we define $k_{1x} = -k_{2x} \equiv k_x$ as well as $k_{1z} = -k_{2z} \equiv k_z$ (letting $k_y = 0$ for convenience). All quantities are now assumed to be superpositions of two dust MHD waves waves whose amplitudes are weakly varying functions of time, i.e. $\rho = \rho_0 + \sum_{j=1}^{2}\rho_j(t)\exp[i(\mathbf{k}_j\cdot\mathbf{r} - \omega t)] + \text{c.c.}$, where c.c stands for the complex conjugate. In principle, the gravitational wave should also contribute, but we note that within a fluid model the gravitational wave contribution to all plasma perturbations (velocity, magnetic field and density) are second order in the gravitational wave amplitudes, provided that the GW propagates parallel to $\mathbf{B}_0$, as we have assumed. Thus, the only linear perturbations due to the gravitational wave are those of the metric as described by Eq. (1).

Next, in order to simplify the algebra, we introduce the normal mode a_j defined by

$$\begin{aligned} a_j = {} & \frac{\omega_j}{k_{xj}}v_{xj} - i\frac{\omega_j^2k_{zj}^2C_A^2}{\omega_c\left(\omega_j^2 - k_{zj}^2C_A^2\right)k_{xj}}v_{yj} - \\ & - \frac{\omega_j^2}{k_j^2}\frac{k_{zj}}{k_{xj}B_0}B_{xj} + \frac{\omega_j^2k_{zj}^2C_A^2}{\omega_c^2\left(\omega_j^2 - k_{zj}^2C_A^2\right)}\frac{C_A^2}{B_0}B_z + \\ & + i\frac{\left(\omega_j^2 - k_j^2C_A^2\right)\omega_c^2 - k_j^2C_A^2\omega_j^2}{k_j^2\omega_j}\frac{k_{zj}}{\omega_ck_{xj}B_0}B_y. \end{aligned} \quad (21)$$

Returning to Eqs. (12)–(14) and including the nonlinear terms[2)], we can, by keeping the part varying as $\exp[i(\mathbf{k}_1\cdot\mathbf{r} - \omega_1\mathbf{t})]$, derive

$$\begin{aligned} \frac{\partial a_1}{\partial t} = {} & -\frac{\omega_1}{2k_x}\left(v_{2x}^*h_+ + v_{2y}^*h_\times\right) - \\ & - i\frac{\omega_1^2k_z^2C_A^2\left(v_{2x}^*h_\times - v_{2y}^*h_+\right)}{2\omega_c\left(\omega_1^2 - k_z^2C_A^2\right)k_x} - \\ & - \frac{\omega_1^2}{2k^2}\frac{k_z}{k_xB_0}\left(B_{2x}^*h_+ - B_{2y}^*h_\times\right) + \\ & + i\frac{k_z\left[\left(\omega_1^2 - k^2C_A^2\right)\omega_c^2 - k^2C_A^2\omega_1^2\right]}{k^2\omega_1\omega_ck_xB_0}\times \\ & \times\left(B_{2x}^*h_\times - B_{2y}^*h_+\right), \end{aligned} \quad (22)$$

and we obtain a similar result for $\partial a_2/\partial t$ by letting $1 \leftrightarrow 2$. After some algebra, using Eqs. (16)-(19) and (21), we find that Eq. (22) reduces to

$$\frac{dv_{x1}}{dt} = \frac{\rho_0\omega_1}{W_1}v_{x2}^*\left(V_+h_+ + V_\times h_\times\right), \quad (23)$$

and similarly for mode 2

$$\frac{dv_{x2}}{dt} = \frac{\rho_0\omega_2}{W_2}v_{x1}^*\left(V_+h_+ + V_\times h_\times\right), \quad (24)$$

where

$$\begin{aligned} W_{1,2} = {} & \frac{\rho_0}{2}\left[1 + \frac{\omega_{1,2}^2}{k_{1,2}^2C_A^2} + \right. \\ & \left. + \frac{\omega_{1,2}^2k_{z1,2}^4C_A^2}{\omega_c^2\left(\omega_{1,2}^2 - k_{z1,2}^2C_A^2\right)^2}\left(1 + \frac{\omega_{1,2}^2}{k_{z1,2}^2C_A^2}\right)\right], \end{aligned} \quad (25)$$

[2)]When including the nonlinear GW-coupling, it is in principle important to separate the coordinate components (indices x, y, z) from the tetrad components (indices $1, 2, 3$), since the difference is first order in $h_+, h_\times$. However, for notational convenience we let indices x, y, z denote tetrad components $1, 2, 3$ in equation (22) and henceforth.

$$V_\times = i\left[\frac{\omega_1 k_z^2 C_A^2 k^2 - \omega_2\omega_1^2 k_z^2}{\omega_c\left(\omega_1^2 - k_{z1}^2 C_A^2\right)k^2} + 1 \leftrightarrow 2\right], \qquad (26)$$

and

$$V_+ = i\left[1 + \frac{\omega_1\omega_2}{\omega_c^2}\frac{k_z^2 C_A^2\left(k_z^2 C_A^2 + \omega_1\omega_2\right)}{\left(\omega_1^2 - k_z^2 C_A^2\right)\left(\omega_2^2 - k_z^2 C_A^2\right)} + \frac{\omega_1\omega_2 k_z^2}{k^4 C_A^2}\right]. \qquad (27)$$

Next using the Einstein equations, linearized in $h_+, h_\times$, keeping only the resonantly varying part of $T^{\mu\nu}$ we obtain for the $\times$- and $+$-polarization, respectively

$$i\omega_g\frac{dh_\times}{dt} = \kappa\left[\rho_0 v_{x1}v_{y2} + \frac{B_{x2}B_{y1}}{\mu_0}\right], \qquad (28)$$

and

$$i\omega_g\frac{dh_\times}{dt} = \frac{\kappa}{2}\left[\rho_0\left(v_{x1}v_{x2} - v_{y1}v_{y2}\right) + \frac{B_{x1}B_{x2} - B_{y1}B_{y2}}{\mu_0}\right], \qquad (29)$$

which is reduced to

$$\frac{dh_\times}{dt} = -\frac{\rho_0\omega_g}{W_g}V_\times v_{x1}v_{x2}, \qquad (30)$$

and

$$\frac{dh_+}{dt} = -\frac{\rho_0\omega_g}{W_g}V_+ v_{x1}v_{x2}, \qquad (31)$$

where $W_g = \omega_g^2/2\kappa$. The total wave energy is $W_{\rm tot} = W_1\left|v_{x1}\right|^2 + W_2\left|v_{x2}\right|^2 + W_g(\left|h_\times\right|^2 + \left|h_+\right|^2)$ and it is easily verified from (23), (24) together with (30), (31) that $W_{\rm tot}$ is conserved. Furthermore, the appearance of the same coupling coefficients $V_\times$, V_+ in Eqs. (23), (24) as well as in (30), (31) assures that the Manley-Rowe relations are fulfilled, which implies that each mode changes energy in direct proportion to its frequency, i.e. $(dW_1/dt)/(dW_2/dt) = \omega_1/\omega_2$ etc. The system of (23), (24) together with (30), (31) describing the energy conversion between DMHD waves and GWs is one of the main results of the present letter. A more elaborate calculation scheme, including effects such as inhomogeneity and background curvature, is a project for future research.

We now apply our results to the gravitational radiation arising from the iron core of a type II supernova where the densities can be of the order $10^{17}\,\mathrm{kg/m^3}$ [27]. The large neutrino outflow (which can reach powers of $10^{33}\,\mathrm{W/cm^2}$ see e.g. Ref. [26]) can generate MHD waves described by (15). From the flux conservation, we expect the iron core to be strongly magnetized (comparable to pulsars), and for magnetic field strengths $B_0 \sim 10^8\,\mathrm{T}$, the gyrofrequency will be much larger than all other frequencies of the problem. The dispersion relation then separate into the shear Alfvén waves $\omega^2 - k_z^2 C_A^2 \approx 0$ and the compressional Alfvén waves $\omega^2 - k^2 C_A^2 \approx 0$. Assuming that the pump MHD wave is a compressional mode with a frequency 5 MHz, the matching conditions (20) can be fulfilled for a GW with a typical frequency 3MHz and a shear Alfvén wave with the frequency 2 MHz[3]. For the assumed geometry, the MHD waves couple only to the $h_\times$-polarization (to a good approximation), and by combining (24) and (30), we obtain

$$\frac{d^2 h_\times}{dt^2} = -h_\times\frac{\omega_2}{\omega_g}\frac{\left|V_\times\right|^2}{W_2}\frac{16\pi G}{c^2}\rho_0\left|v_{x1}\right|^2. \qquad (32)$$

Thus, noting that the factor $\omega_2\left|V_\times\right|^2/\omega_g W_2$ is negative[3] and has a magnitude of order unity[4] for the given parameters, the growth rate is

$$\gamma \sim \sqrt{16\pi G\rho_0}\frac{\left|v_{x1}\right|}{c}, \qquad (33)$$

which for a weakly relativistic pump quiver speed, $\left|v_{x1}\right|/c \sim 1/10$, implies $\gamma \sim 10\,\mathrm{kHz}$. Thus, we deduce that excitation of a GW by MHD waves is a reasonably fast process in a dense medium such as the supernova iron core. It would be of interest to quantatively estimate the energy that can be converted to GW:s through this process. Clearly the growth rate is fast enough, such that the time available for conversion is not a limiting factor. However, eventually the MHD wave source will be depleted, which give an upper limit for the energy available through this mechanism. How large fraction of the MHD wave source that can be converted to GW:s depend on the influence from competing processs, for example excitation of other MHD waves by the MHD pump wave and/or wave particle interaction. Thus, further research is necessary before a definite estimate of the GW energy levels due to the present mechanism can be given. However, we emphasize that the present process can give rise to GWs of higher frequencies than many of the previously considered excitation mechanisms, see e.g. [20, 21, 28]. Thus, our model contributes to the

[3]Note that from the Manley-Rowe relations the pump wave must have the highest frequency for the growth rates to become real and positive, Thus using the matching condition (20) we see that the shear Alfvén mode formally has a negative frequency. Furthermore, we note that the dispersion relations together with the matching conditions means that the pump mode cannot propagate purely along the magnetic field. Specifically, the wavevector matching implies $k_x C_A \approx 4.6\mathrm{MHZ}$, where $C_A \approx 300\mathrm{m/s}$ for the given parameters.

[4]This can be seen by taking the limit $\omega_c \to \infty$ of the expression, noting that $\left(\omega_2^2 - k_{z2}^2 C_A^2\right)$ scales as $1/\omega_c^2$ from the dispersion relation (15).

understanding of gravitational radiation emissions accompanying supernova explosions [29].

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