

Novel probe of the vacuum of the lattice gluodynamics

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We introduce a notion of minimal number of negative links on the lattice for a given original configuration of $SU(2)$ fields. Negative links correspond to a large potential, not necessarily large action. The idea is that the minimal number of negative links is a gauge invariant notion. To check this hypothesis we measure correlator of two negative links, averaged over all the directions, as function of the distance between the links. The inverse correlation length coincides within the error bars with the lightest glueball mass.

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A traditional way to study spectrum of excitations is to measure correlators of various sources. For example, the correlator¹⁾:

$$D_{gl}(r) \equiv \langle 0 | (G_{\mu\nu}^a(r))^2, (G_{\alpha\beta}^a(0))^2 | 0 \rangle, \quad (1)$$

where $G_{\mu\nu}^a$ is the gluonic field strength tensor, is sensitive to the glueball mass m_{gl} at large (Euclidean) distances r :

$$\lim_{r \rightarrow \infty} D_{gl}(r) = \text{const} + (\text{const})' \exp(-m_{gl}r). \quad (2)$$

On the other hand, one can also study gauge dependent correlators, such as

$$D_{\mu\nu}^{a,b}(r) = \langle 0 | A_\mu^a(r), A_\nu^b(0) | 0 \rangle, \quad (3)$$

where A_μ^a is the gauge field. Such correlators are not unitary, generally speaking, and are not controlled by glueball masses. In particular, the gluon propagator (3) could even grow at large r . More realistically, i.e., as indicated by the lattice measurements, the gluon propagator falls off at large r but exhibits some spurious mass scales, for review see, e.g., [1, 2]. For us, it is important that these spurious mass scales are, as a rule, lower than the lightest glueball mass.

In this note we introduce a new type of correlators which, as we hypothesize, might be unitary although they are not explicitly gauge invariant (like (1)) and check our hypothesis through lattice simulations of $SU(2)$ gluodynamics.

To explain the basic idea behind our measurements it is useful first to remind to the reader how one can introduce a gauge invariant condensate of dimension two in

gauge theories [3]. One starts with the vacuum expectation value $\langle (A_\mu^a)^2 \rangle$ which is obviously gauge dependent. One can, however, minimize this vacuum expectation value on the gauge orbits and the results $\langle (A_\mu^a)^2 \rangle_{\min}$ is gauge invariant by construction. To ensure that the minimum exists, the Euclidean signature is used²⁾.

We generalize this idea to the case of a $Z(2)$ projection of the original $SU(2)$ fields. In this projection the link variables take the values ± 1 (for review see, e.g., [5]). Upon the $Z(2)$ projection there is still remaining $Z(2)$ gauge invariance. We fix this gauge freedom by minimizing the number of negative links over the whole lattice. We speculate, furthermore, that the density of these negative links might well be gauge invariant, in analogy with $\langle (A_\mu^a)^2 \rangle_{\min}$.

Moreover, we expect that the correlator of the negative links is unitary, like (1). This speculation goes apparently beyond the ideas discussed so far. Indeed, the negative links in the continuum limit correspond to singular potentials, $A_\mu^a \sim 1/a$, where a is the lattice spacing. Naively, one could argue that correlator of such strong fields dies off on distances of order a . In particular, this is true for pure perturbative fluctuations.

This argument might not work only if there exist fine tuned vacuum fluctuations which are sensitive both to the lattice spacing and Λ_{QCD} . Although the idea of existence of fine tuned fluctuations may look too exotic at first sight, such fluctuations exist. In particular, the so called P -vortices are closed surfaces whose total area scales in the physical units, for review see, e.g., [5]. On the other hand, the P -vortices are defined as unification of all the negative plaquettes in the $Z(2)$ projection. One can expect, therefore, that they are associated with ac-

¹⁾For simplicity of notations, we do not account for the anomalous dimension.

²⁾A modification suitable for the Minkowski space is to work in the Hamiltonian formalism [4].

tion density which is ultraviolet divergent. And, indeed, the fact that the P-vortices are associated with the ultraviolet divergent non-Abelian action was revealed directly through lattice measurements [6]. It is in analogy with these observations that we expect the “propagator of negative links” to scale in the physical units as well. This is of course a daring possibility which cannot be proven a priori but can be only supported or rejected by measurements.

In more detail, we perform measurements both in the Direct and Indirect Maximal Center Projections (DMCP and IMCP). The details of calculations are given in the Appendix. As a result of $SU(2) \rightarrow Z(2)$ projection, the original $SU(2)$ field configurations get projected into the closest configuration of $Z(2)$ gauge fields. The remaining $Z(2)$ gauge freedom is then fixed by maximizing the functional

$$F(Z) = \sum_{x,\mu} Z_{x,\mu} \quad (4)$$

with respect to $Z(2)$ gauge transformations ($Z_{x,\mu} \rightarrow z_x Z_{x,\mu} z_{x+\hat{\mu}}, z_x = \pm 1$).

After gauge fixing only the positions of the negative links are relevant and it is reasonable to change the variables:

$$\hat{Z}_{x,\mu} = \{1, \text{if } Z_{x,\mu} = -1; 0 \text{ if } Z_{x,\mu} = 1\}. \quad (5)$$

Moreover, to imitate a scalar correlator on the discrete variables, we consider here the isotropic correlator defined as

$$G_{\mu\nu}(r) \equiv \frac{1}{N_r} \sum_{r < |x| < r + \frac{a}{2}} \langle \hat{Z}_{0\mu} \hat{Z}_{x,\nu} \rangle, \quad (6)$$

where the summation is over all links $Z_{x,\mu}$ for x lying in the spherical layer $r < |x| < r + \frac{a}{2}$; N_r is the total number of links in this layer.

The correlator (6) tends to a non-vanishing constant $G(\infty) = \langle \hat{Z}_{x,\nu} \rangle^2$ as $r \rightarrow \infty$ ($\langle \hat{Z}_{x,\nu} \rangle$ is the average density of the negative links) and we fit the data by the expression (see (2)):

$$G(r) = G(\infty) + C \exp\{-mr\}. \quad (7)$$

We thus get the mass parameter m for various values of the lattice spacing a . Logarithmic plots for the isotropic correlator $G(r)$ are presented in Fig.1 for IMCP, while the corresponding values of the mass m are depicted in Fig.2 for IMCP and DMCP.

As is seen from Figs.1,2 the correlator $G_{\mu\nu}(r)$ scales in the physical units (within the error bars) and the mass parameter is close to the scalar 0^{++} glueball mass

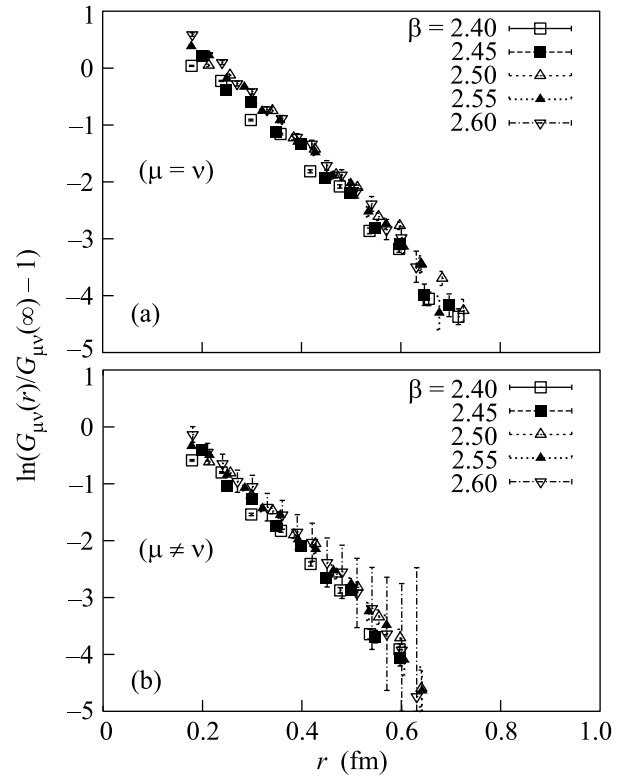


Fig.1. $\ln(\frac{G_{\mu\nu}(r)}{G_{\mu\nu}(\infty)} - 1)$ vs lattice spacing for $\mu = \nu$ (a) and for $\mu \neq \nu$ (b), results obtained for IMCP

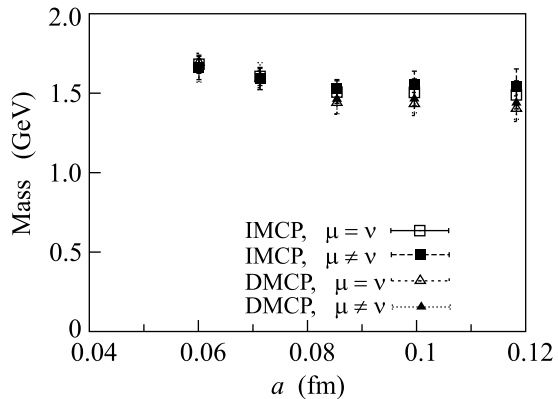


Fig.2. Mass parameters for IMCP and DMCP for $\mu = \nu$ and for $\mu \neq \nu$

($m(0^{++}) = 1.65 \pm 0.05$ GeV [7]). Thus, our measurements support the idea that the correlator (6) is in fact gauge invariant and unitary. Of course the results are not analytical but pure numerical and, in principle, the picture can change at smaller values of the lattice spacing. The non-triviality of this observation is that it is a correlator of potentials, which are not explicitly gauge invariant. Moreover, in the continuum limit the negative links correspond to singular potentials. The scaling of

the correlation length observed here gives further support to the existence of fine tuned vacuum fluctuations.

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Appendix

We perform our calculations both in the Direct [8] and the Indirect [9] Maximal Center Projections (DMCP and IMCP). The DMCP in $SU(2)$ lattice gauge theory is defined by the maximization of the functional

$$F_1(U) = \sum_{x,\mu} (\text{Tr } U_{x,\mu})^2, \tag{8}$$

with respect to gauge transformations, $U_{x,\mu}$ is the lattice gauge field. Maximization of (8) fixes the gauge up to $Z(2)$ gauge transformations and the corresponding $Z(2)$ gauge field is defined as: $Z_{x,\mu} = \text{signTr } U_{x,\mu}$. To get IMCP we first fix the maximally Abelian gauge maximizing the functional

$$F_2(U) = \sum_{x,\mu} \text{Tr} (U_{x,\mu} \sigma_3 U_{x,\mu}^+ \sigma_3), \tag{9}$$

with respect to gauge transformations. We project gauge degrees of freedom $U(1) \rightarrow Z(2)$ by the procedure completely analogous to the DMCP case, that is we maximize $F_1(U)$ (8) with respect to $U(1)$ gauge transformations.

To fix the maximally Abelian and direct maximal center gauge we create 20 randomly gauge transformed copies of the gauge field configuration and apply the Simulated Annealing algorithm to fix gauges. We use in calculations that copy which correspond to the maximal value of the gauge fixing functional. To fix the indi-

rect maximal center gauge from configuration fixed to maximally Abelian gauge and to fix the $Z(2)$ degrees of freedom one gauge copy is enough to work with our accuracy. We work at various lattice spacings to check the existence of the continuum limit. The parameters of our gauge field configurations are listed in Table. To

Parameters of configurations

β	Size	N_{IMCP}	N_{DMCP}
2.35	16^4	20	20
2.40	24^4	50	20
2.45	24^4	20	20
2.50	24^4	50	20
2.55	28^4	37	17
2.60	28^4	50	20

fix the physical scale we use the string tension in lattice units [10], $\sqrt{\sigma} = 440 \text{ MeV}$.

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