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Quantum teleportation of entanglement using four-particle entangled states

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We present a model to realise a probabilistic quantum teleportation of two-particle mode entangled state through the four-photon quantum channel. Four modes of the two-photon mode entangled state are directly transferred to other spatial four modes of the quantum channel with success probability of 50%. The quantum protocol operates in space of photon number states. A Bell state measurement with four beam splitters and four pairs of detectors in the teleportation protocol is accomplished in the fourfold coincidence basis.

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Quantum teleportation, proposed by Bennett et al. [1], is the process that transmits an unknown two-state particle from a sender (Alice) to a receiver (Bob) via a quantum channel with help of sending some classical information to Bob. In original scheme [1], such a quantum channel has been represented by a Bell maximally entangled state or Einstein- Podolsky-Rosen (EPR) pair. Experimental realization of the quantum teleportation protocol by a partial set of Bell measurements has been performed in [2] with success probability of 50%. The main difficulty with any optical approach is that nonlinear interactions between individual photons are required in order to implement the quantum teleportation protocol that operates with 100% efficiency [3]. The Bell-state measurement being inherently nonlinear is required for the 100% teleportation [4].

Quantum teleportation of a qubit occupying only one optical mode via one-photon quantum channel was studied in [5]. Experimental realisation of the quantum teleportation of one mode qubit through the one-photon quantum channel is reported in [6]. Problems of quantum teleportation by employing Greenberg–Horne–Zeilinger (GHZ) quantum channel [7] have been studied in [8]. More general questions of quantum teleportation of two qubits involving noisy quantum channels are involved in [9]. One should be noted the work [10], in which quantum teleportation protocol of N – particle entangled state via $N+1$ – particle quantum channel has been developed.

As pointed out by Bennett et al. [1] in their original proposal for quantum teleportation, entanglement can be transferred through teleportation of two modes one of the particles forming the entangled state. This method, known as entanglement swapping [11], provides only partial teleportation of entanglement. We propose an alternative method in which the two-photon mode entangled state is directly transferred from one place to another. We use four-photon quantum channel to perform the teleportation protocol for the two-photon mode entangled state unlike [8 – 10]. We make use of the number state representation to perform the total teleportation of the two-photon mode entangled state (all four modes of the state are teleported). One should be noted, it was recently recognised in [12] that spatial encoding is easier, for example, to manipulate and construct universal quantum gates unlike the standard method for encoding qubits in optics in which polarization degrees of freedom of single photon are used. The source of the mode entanglement may consist of two non-collinear degenerated on frequency spontaneous parametric down converters with type-I phase matching (SPDCI) [13]. Quantum teleportation utilizing such four-photon quantum channel is not required special detectors distinguishing between one- and two-photon number states. The teleportation scheme of the entangled state through the four-photon quantum channel may be really performed in practice unlike teleportation schemes based on the GHZ quantum channel [8 – 10].

We employ setup shown in Figure for the teleportation of entanglement. As in the standard teleportation scheme [1], our scheme (Figure) consists of three distinct parts: the source station that generates a quantum channel (it is not shown in Figure), Alice's station where a Bell state measurement is performed and its result is sent away through the classical communication channel, and Bob's station where the signal from Alice is read and a suitable unitary transformation with output state is performed. The four-photon mode entangled state

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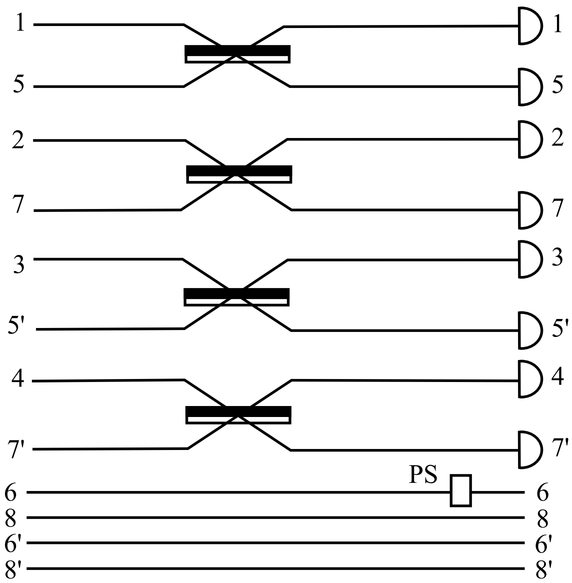

The experimental scheme realizes quantum state teleportation of unknown two-photon qubit by the four-photon quantum channel (1). White side of the beam splitter indicates the surface from which a sign change in exiting mode occurs upon reflection. The use of this beam splitter phase convention is convenient but not essential. PS means π-phase shifter in the output sixth mode. Bob's modes are shown at lower part

$$|\Xi_1^{(56785'6'7'8')}\rangle = = \frac{1}{\sqrt{2}}\{|11110000\rangle + |00001111\rangle\}_{56785'6'7'8'} \quad (1)$$

is utilized as a quantum channel which is shared by Alice (modes 5, 7, $5'$ and $7'$ in Figure belong to Alice) and Bob (modes 6, 8, $6'$ and $8'$ in Figure). Here, the numbers in the subscripts of the states (Eq. (1)) are referred to the optical modes of the photons [14]. For example, the state $|11110000\rangle_{56785'6'7'8'}$ in Eq. (1) is a tensor product of one-photon number states where the modes 5, 6, 7, and 8 are occupied by four photons while other residuary modes $5', 6', 7'$ and $8'$ have zero photons. An unknown two-photon mode entangled state $|\Psi_1^{(1234)}\rangle = \{\alpha|1100\rangle + \beta|0011\rangle\}_{1234}$ with amplitudes α and β satisfying condition $|\alpha|^2 + |\beta|^2 = 1$ must be teleported to Bob. According to Figure, the first, second, third and fourth modes of the teleported state $|\Psi_1^{(1234)}\rangle$ are mixed with fifth, seventh, fifth prime, and seventh prime modes of the quantum channel (Figure). Then, the input tensor product of six photons is given by

$$|\Psi_1^{(1234)}\rangle|\Xi_1^{(56785'6'7'8')}\rangle = = \frac{1}{2}\{|\Xi_1^{(1257345'7')}\rangle \times \times\{\alpha|1100\rangle + \beta|0011\rangle\}_{686'8'} + |\Xi_2^{(1257345'7')}\rangle, \{\alpha|1100\rangle - \beta|0011\rangle\}_{686'8'} + |\Xi_3^{(1257345'7')}\rangle \times \times\{\alpha|0011\rangle + \beta|1100\rangle\}_{686'8'} + |\Xi_4^{1257345'7')}\rangle, \quad (2a)$$
$$\{\alpha|0011\rangle - \beta|1100\rangle\}_{686'8'}$$

where we introduce the following four-photon mode entangled states

$$|\Xi_1^{(1257345'7')}\rangle = \frac{1}{\sqrt{2}}\{|11110000\rangle + |00001111\rangle\}_{1257345'7'}, \quad (2b)$$

$$|\Xi_2^{(1257345'7')}\rangle = \frac{1}{\sqrt{2}}\{|11110000\rangle - |00001111\rangle\}_{1257345'7'}, \quad (2c)$$

$$|\Xi_3^{(1257345'7')}\rangle = \frac{1}{\sqrt{2}}\{|11000011\rangle + |00111100\rangle\}_{1257345'7'}, \quad (2d)$$

$$|\Xi_4^{(1257345'7')}\rangle = \frac{1}{\sqrt{2}}\{|11000011\rangle - |00111100\rangle\}_{1257345'7'}. \quad (2e)$$

The states in the modes $6, 8, 6'$ and $8'$ is the teleported state while the states $|\Xi_i^{(1257345'7')}\rangle$ ($i = 1$–4) of the ancillary photons must be subjected to the Bell state measurement to end up the teleportation protocol [1]. Straightforward calculations based on the quantum theory of the beam splitter yield the following outcomes of the states (2b)–(2e)

$$|\Xi_1^{(1257345'7')}\rangle \to \frac{1}{2\sqrt{2}}\{|22000000\rangle + |00220000\rangle - -|20020000\rangle - |02200000\rangle + |00002200\rangle + +|00000022\rangle - |00002002\rangle - |00000220\rangle\}_{1257345'7'}, \quad (3a)$$

$$|\Xi_2^{(1257345'7')}\rangle \to \frac{1}{2\sqrt{2}}\{|22000000\rangle + |00220000\rangle - -|20020000\rangle - |02200000\rangle + |00002200\rangle - -|00000022\rangle + |00002002\rangle + |00000220\rangle\}_{1257345'7'}, \quad (3b)$$

$$|\Xi_3^{(1257345'7')}\rangle \to \frac{1}{2\sqrt{2}}\{|11001100\rangle + |11000011\rangle + +|00111100\rangle + |00110011\rangle - |10011001\rangle - -|10010110\rangle - |01101001\rangle - |01100110\rangle\}_{1257345'7'}, \quad (3c)$$

$$|\Xi_4^{(1257345'7')}\rangle \to -\frac{1}{2\sqrt{2}}\{|11001001\rangle + |11000110\rangle + +|00111001\rangle + |00110110\rangle - |10011100\rangle - -|10010011\rangle - |01101100\rangle - |01100011\rangle\}_{1257345'7'}. \quad (3d)$$

We have four different outcomes that can be grouped into two separate groups, namely, first group involves the outcomes characterized by simultaneous clicks in two different detectors (two bits of information Eqs. (3a), (3b)). The states $|\Xi_1^{(1257345'7')}\rangle$ and $|\Xi_2^{(1257345'7')}\rangle$ (first group) result in the outcomes different only by sign from each other. Thus, the teleportation protocol in Figure fails when two detectors simultaneously register four photons as the outcomes of the first group (Eqs. (3a), (3b)) are not distinguishable from each other. Unlike the outcomes of the first group, a Bell state measurement of the states of the second group $|\Xi_3^{(1257345'7')}\rangle$ and $|\Xi_4^{(1257345'7')}\rangle$ projects the states on coincidence registration of four photons in four detectors (four bits, Eqs. (3c), (3d)). It is easy to check the outcomes (3c), (3d) are distinguishable from each other. Thus, when two pairs of detectors register a particular combination of four coincidences, Alice knows which state ($|\Xi_3^{(1257345'7')}\rangle$ or $|\Xi_4^{(1257345'7')}\rangle$) correspond to it and, consequently, which state Bob has in his hands and Alice must inform him about it using classical communication [1].

To end up the teleportation protocol, one should define what state is considered to be successfully teleported. Let us consider the quantum teleportation performance to be successful if the following state $\{\alpha|0011\rangle+\beta|1100\rangle\}_{686'8'}$ occurs at Bob's station. If Bob is informed about outcome corresponding to the state $|\Xi_3^{(1257345'7')}\rangle$, he must do nothing. Other possible case corresponds to the outcomes following from the input state $|\Xi_4^{(1257345'7')}\rangle$. To restore the teleported state in the case, Bob must apply corresponding unitary transformations in one of the output modes. In our case, it is sufficient to make use of the π-phase shifter, for example, in the mode 6 to restore the teleported state. Since the states forming second group represents one second of the entire initial photon state (2a), the success probability for the entanglement teleportation using the quantum channel (1) is 50% also as in original proposal [1].

One should say some words about a possibility to generate quantum channel (1) for the quantum teleportation. From the "no cloning theorem" [15], we know the conversion $(\alpha|0\rangle+\beta|1\rangle)\rightarrow(\alpha|0\rangle+\beta|1\rangle)(\alpha|0\rangle+\beta|1\rangle)$ is forbidden while the following operation $(\alpha|0\rangle+\beta|1\rangle)\rightarrow$ $\rightarrow(\alpha|00\rangle+\beta|11\rangle)$ is allowed. To construct the quantum channel (1), we must make use of a quantum encoder operating on the mode entangled states [13] and whose output in a probabilistic way $(\alpha|00\rangle+\beta|11\rangle)\rightarrow$ $\rightarrow(\alpha|0000\rangle+\beta|1111\rangle))$ will be given by the state (1) in the case of quantum optics.

In conclusion, we have proposed a method to realize a probabilistic quantum teleportation of two-photon mode entangled state through the four-photon quantum channel (1). This method is based only on a few linear optics elements, namely four balanced beam splitters, eight photodetectors and postselection. The maximum success probability of the teleportation protocol is 50% . To achieve the quantum teleportation protocol, fourfold coincidences are required to be performed that are fully available with the present technology.

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