Dissymmetrical tunnelling in heavy fermion metals

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A tunnelling conductivity between a heavy fermion metal and a simple metallic point is considered. We show that at low temperatures this conductivity can be noticeably dissymmetrical with respect to the change of voltage bias. The dissymmetry can be observed in experiments on the heavy fermion metals whose electronic system has undergone the fermion condensation quantum phase transition.

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Understanding the unusual quantum critical properties of heavy-fermion (HF) metals at low temperatures T remains challenging. It is a common belief that quantum phase transitions developing in the HF metals at $T = 0$, which have ability to influence the finite temperature properties, are responsible for the anomalous behavior. Experiments on the HF metals explore mainly their thermodynamic properties which proved to be quite different from that of ordinary metals described by the Landau Fermi liquid (LFL) theory. In the LFL theory, considered as the main instrument when investigating quantum many electron physics, the effective mass M^* of quasiparticle excitations controlling the density of states determines the thermodynamic properties of electronic systems. It is possible to explain the observed thermodynamic properties of the HF metals on the basis of the fermion condensation quantum phase transition (FCQPT) which allows the existence of the Landau quasiparticles down to the lowest temperatures [1, 2]. In contrast to the Landau quasiparticles, these are characterized by the effective mass which strongly depends on temperature T, applied magnetic field B and the number density x of the heavy electron liquid of HF metal. Thus, we come back again to the key role of the density of state. It would be desirable to probe the other properties of the heavy electron liquid such as the probabilities of quasiparticle occupations which are not directly linked to the density of states or to the behavior of M^*. Scanning tunnelling microscopy being sensitive to both the density of states and the probabilities of quasiparticle occupations is an ideal technique for the study of such effects at quantum level.

The tunnelling current I through the point contact between two ordinary metals is proportional to the driving voltage V and to the squared modulus of the quantum mechanical transition amplitude t multiplied by the difference $N_1(0)N_2(0)(n_1(p,T) - n_2(p,T))$, see e.g. [3]. Here $n(p,T)$ is the quasiparticle distribution function and $N(0)$ is the density of states of the corresponding metal. On the other hand, the wave function calculated in the WKB approximation and defining t is proportional to $(N_1(0)N_2(0))^{-1/2}$. As a result, the density of states is dropped out and the tunnelling current does not depend on $N_1(0)N_2(0)$. Upon taking into account that at $T \to 0$ the distribution $n(p, T \to 0) \to n_F(p)$, where $n_F(p)$ is the step function $\theta(p - p_F)$ with p_F being the Fermi momentum, one can check that within the LFL theory the differential tunnelling conductivity $\sigma_d(V) = dI/dV$ is a symmetric function of the voltage V. In fact, the symmetry of $\sigma_d(V)$ holds provided that so called particle-hole symmetry is preserved as it is within the LFL theory, but the relation $n(p, T \to 0) \to \theta(p - p_F)$ will do. Therefore, the existence of the $\sigma_d(V)$ symmetry is quite obvious and common in the case of metal-to-metal contacts when these metals are in the normal state or in the superconducting one.

In this letter we show that the situation can be different when one of the two metals is a HF metal whose electronic system is represented by the heavy electron liquid. When the heavy electron liquid has undergone FCQPT its distribution function is no longer the step function as soon as the temperature tends to zero [4]. As a result, both the differential tunnelling conductivity $\sigma_d(V)$ and the tunnelling conductivity $\sigma(V)$ become dissymmetrical as a function of voltage V. While the application of magnetic field destroying the non-Fermi liquid behavior of the heavy electron liquid restores the symmetry.

At first, we briefly describe the heavy electron liquid with the fermion condensate (FC) [4 – 6]. When the number density x of the liquid approaches some density

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x_{FC} the effective mass diverges. Because the kinetic energy near the Fermi surface is proportional to the inverse effective mass, FCQPT is triggered by the frustrated kinetic energy. Behind the critical point x_{FC}, the quasiparticle distribution function represented by $n_F(p)$ does not deliver the minimum to the Landau functional $E[n(\mathbf{p})]$. As a result, at $x < x_{FC}$ the quasiparticle distribution is determined by the standard equation to search the minimum of a functional [4]

$$\frac{\delta E[n(\mathbf{p})]}{\delta n(\mathbf{p},T=0)} = \varepsilon(\mathbf{p}) = \mu;\ p_i \le p \le p_f. \qquad (1)$$

Equation (1) determines the quasiparticle distribution function $n_0(\mathbf{p})$ which delivers the minimum value to the ground state energy E. Being determined by Eq. (1), the function $n_0(\mathbf{p})$ does not coincide with the step function $n_F(p)$ in the region $(p_f - p_i)$, so that $0 < n_0(\mathbf{p}) < 1$, while outside the region it coincides with $n_F(p)$. It follows from Eq. (1) that the single particle spectrum is completely flat over the region. Such a state was called the state with FC because quasiparticles located in the region $(p_f - p_i)$ of momentum space are pinned to the chemical potential μ. We note that the behavior obtained as observed within exactly solvable models [7, 8] and represents a new state of Fermi liquid [9]. We can conclude that the relevant order parameter $\kappa(\mathbf{p}) = \sqrt{n_0(\mathbf{p})(1-n_0(\mathbf{p}))}$ is the order parameter of the superconducting state with the infinitely small value of the superconducting gap Δ [5]. Thus this state cannot exist at any finite temperatures and driven by the parameter x: at $x > x_{FC}$ the system is on the disordered side of FCQPT; at $x = x_{FC}$, Eq. (1) possesses the non-trivial solutions $n_0(\mathbf{p})$ with $p_i = p_F = p_f$; at $x < x_{FC}$, the system is on the ordered side. At $T > 0$, the quasiparticle distribution is given by

$$n(\mathbf{p},T) = \left\{1 + \exp\left[\frac{(\varepsilon(\mathbf{p},T)-\mu)}{T}\right]\right\}^{-1}, \qquad (2)$$

where $\varepsilon(\mathbf{p},T)$ is the single-particle spectrum, or dispersion, of the quasiparticle excitations and μ is the chemical potential. Equation (2) can be recast as

$$\varepsilon(\mathbf{p},T) - \mu(T) = T \ln \frac{1-n(\mathbf{p},T)}{n(\mathbf{p},T)}. \qquad (3)$$

As $T \to 0$, the logarithm on the right hand side of Eq. (3) is finite when p belongs to the region $(p_f - p_i)$, therefore $T\ln(...) \to 0$, and we again arrive at Eq. (1). Near the Fermi level the single particle spectrum can be approximated as

$$\varepsilon(p \simeq p_F, T) - \mu \simeq \frac{p_F(p-p_F)}{M^*}. \qquad (4)$$

It follows from Eq. (2) that $n(p, T \to 0) \to n_F(p)$ provided that M^* is finite at $T \to 0$. Thus at low temperatures, the left hand side of Eq. (3) determines the behavior of the right hand side. In contrast to this case, the right hand side of Eq. (3) determines the behavior of M^* when FC is set in at the liquid. Indeed, it follows from Eq. (1) that $n(\mathbf{p}, T \to 0) = n_0(\mathbf{p})$. Therefore at low temperatures, as seen from Eq. (3), the effective mass diverges as [10]

$$M^*(T) \simeq p_F \frac{p_f - p_i}{4T}. \qquad (5)$$

At $T \ll T_f$, Eq. (5) is valid and determines quasiparticles with the energy z and characterized by the distribution function $n_0(p)$. Here T_f is the temperature at which the influence of FCQPT vanishes [5]. The energy z belongs to the interval

$$\mu - 2T \le z \le \mu + 2T. \qquad (6)$$

Now we turn to a consideration of the tunnelling current at low temperatures which in the case of ordinary metals is given by [3]

$$I(V) = 2|t|^2 \int [n_F(z-\mu) - n_F(z-\mu+V)]\, dz. \qquad (7)$$

We use an atomic system of units: $e = m = \hbar = 1$, where e and m are electron charge and mass, respectively. Since temperatures are low we approximate the distribution function of ordinary metal by the step function n_F. It follows from Eq. (7) that quasiparticles with the energy z, $\mu - V \le z \le \mu$, contribute to the current, while $\sigma_d(V) \simeq 2|t|^2$ is a symmetrical function of V. In the case of the heavy electron liquid with FC, the tunnelling current are found to be of the form

$$I(V) = 2 \int [n_0(z-\mu) - n_F(z-\mu+V)]\, dz. \qquad (8)$$

Here we have replaced the distribution function of ordinary metal by n_0 being the solution of Eq. (1). We have also taken units such that $|t|^2 = 1$. Assume that V satisfies the condition, $|V| \le 2T$, while the current flows from the HF metal to ordinary one. Quasiparticles of the energy z, $\mu - V \le z$, contribute to $I(V)$, and the differential conductivity $\sigma_d(V) \simeq 2n_0(z \simeq \mu - V)$. If the sign of the voltage is changed, the direction of the current is also changed. In that case, quasiparticles of the energy z, $\mu + V \ge z$, contribute to $I(V)$, and the differential conductivity $\sigma_d(-V) \simeq 2(1 - n_0(z \simeq \mu + V))$. The dissymmetrical part $\Delta\sigma_d(V) = (\sigma_d(-V) - \sigma_d(V))$ of the differential conductivity is of the form

$$\Delta\sigma_d(V) \simeq 2[1 - (n_0(z-\mu \simeq V) + n_0(z-\mu \simeq -V))]. \qquad (9)$$

It is worth noting that it follows from Eq. (9) that $\Delta\sigma_d(V) = 0$ if the HF metal in question is replaced by an ordinary metal. Indeed, the effective mass is finite at $T \to 0$, then $n_0(T \to 0) \to n_F$ being given by Eq. (2), and $1 - n(z - \mu \simeq V) = n(z - \mu \simeq -V)$. One might say that the dissymmetrical part vanishes due to the particle-hole symmetry. On the other hand, there are no reasons to expect that $(1 - n_0(z-\mu \simeq V) - n_0(z-\mu \simeq \simeq -V)) = 0$. Thus, we are led to the conclusion that the differential conductivity becomes a dissymmetrical function of the voltage. To estimate $\Delta\sigma_d(V)$, we observe that this is zero when $V = 0$, because $n_0(p = p_F) = 1/2$ as it should be and it follows from Eq. (3) as well. It is seen from Eq. (9) that $\Delta\sigma_d(V)$ is an even function of V. Therefore we can assume that at low values of the voltage V the dissymmetrical part behaves as $\Delta\sigma_d(V) \propto V^2$. Then, the natural scale to measure the voltage is $2T$ as it is seen from Eq. (6). In fact, the dissymmetrical part is to be proportional to $(p_f - p_i)/p_F$. As a result, we obtain

$$\Delta\sigma_d(V) \simeq c\left(\frac{V}{2T}\right)^2 \frac{p_f - p_i}{p_F}. \tag{10}$$

Here c is a constant which is expected to be of the order of unit. This constant can be evaluated by using analytical solvable models. For example, calculations of c within a simple model, when the Landau functional $E[n(p)]$ is of the form [4]

$$E[n(p)] = \int \frac{p^2}{2M}\frac{d\mathbf{p}}{(2\pi)^3} + V_1 \int n(p)n(p)\frac{d\mathbf{p}}{(2\pi)^3}, \tag{11}$$

give that $c \simeq 1/2$. It follows from Eq. (10) that when $V \simeq 2T$ and FC occupies a noticeable part of the Fermi volume, $(p_f - p_i)/p_F \simeq 1$, the dissymmetrical part becomes comparable with differential tunnelling conductivity, $\Delta\sigma_d(V) \sim V_d(V)$.

The dissymmetrical behavior of the tunnelling conductivity can be observed in measurements on the heavy fermion metals, for example, such as $YbRh_2(Si_{0.95}Ge_{0.05})_2$ or $YbRh_2Si_2$ which are expected to have undergone FCQPT. In that case, upon the application of magnetic field B the effective mass is to diverge as [1, 11]

$$M^*(B) \propto (B - B_{c0})^{\alpha}. \tag{12}$$

Here B_{c0} is the critical magnetic field which drives the HF metal to its magnetic field tuned quantum critical point. The value of the critical exponent $\alpha = -1/2$ is in good agreement with experimental observations collected on these metals [12, 13]. The measurements of $\Delta\sigma_d(V)$ have to be carried out applying magnetic field B_{c0} at temperatures $T \le T_f$. In the case of these metals, T_f is of the order of few Kelvin [11]. We note that at sufficiently low temperatures, the application of magnetic field $B > B_{c0}$ leads to the restoration of the Landau Fermi liquid with $M^*(B)$ given by Eq. (12) [1, 11]. As a result, the dissymmetrical behavior of the tunnelling conductivity vanishes.

The dissymmetrical differential conductivity $\Delta\sigma_d(V)$ can also be observed when the HF metal in question goes from normal to superconducting. The reason is that $n_0(p)$ is again responsible for the dissymmetrical part of $\sigma_d(V)$. This $n_0(p)$ is not appreciably disturbed by the pairing interaction which is relatively weak as compared to the Landau interaction forming the distribution function $n_0(p)$ [10, 14]. In the case of superconductivity, we have to take into account that the density of states,

$$\frac{N_s(E)}{N(0)} = \frac{|E|}{\sqrt{E^2 - \Delta^2}}, \tag{13}$$

comes into the play because N_s is zero in the gap, that is when $|E| \le |\Delta|$. Here E is the quasiparticle energy, while the normal state quasiparticle energy is $\varepsilon - \mu = \sqrt{E^2 - \Delta^2}$. Now we can arrange Eq. (9) for the case of superconducting HF metal by multiplying the right hand side of Eq. (9) by $N_s/N(0)$ and replacing the quasiparticle energy $z - \mu$ by $\sqrt{E^2 - \Delta^2}$ with E being represented by the voltage V. As a result, Eq. (10) can be cast into the following form

$$\Delta\sigma_d(V) \simeq \frac{\left(\sqrt{V^2 - \Delta^2}\right)^2}{|\Delta|\sqrt{V^2 - \Delta^2}}\frac{p_f - p_i}{p_F} = \\ = \sqrt{\left[\frac{V}{\Delta}\right]^2 - 1}\,\frac{p_f - p_i}{p_F}. \tag{14}$$

Note that the scale $2T$ entering Eq. (10) is replaced by the scale Δ in Eq. (14). In the same way, as Eq. (10) is valid up to $V \simeq 2T$, Eq. (14) is valid up to $V \simeq 2|\Delta|$. It is seen from Eq. (14) that the dissymmetrical part of the differential tunnelling conductivity becomes as large as the differential tunnelling conductivity at $V \simeq 2|\Delta|$ provided that FC occupies a large part of the Fermi volume, $(p_f - p_i)/p_F \simeq 1$. In the case of a d-wave gap, the right hand side of Eq. (14) has to be integrated over the gap distribution. As a result, $\Delta\sigma_d(V)$ is expected to be finite even at $V = \Delta_1$, where Δ_1 is the maximum value of the d-wave gap. A detailed consideration of the superconducting case will be published elsewhere.

In summary, we have shown that the differential tunnelling conductivity between metallic point and an ordinary metal which is commonly symmetric as a function of the voltage becomes noticeably dissymmetrical when the ordinary metal is replaced by a HF metal the electronic system of which has undergone FCQPT. This dissymmetry can be observed when the HF metal is both normal and superconducting. We have also discussed possible experiments to study the dissymmetry.

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