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Chiral symmetry breaking and monopole condensation in QCD

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We show that in the dual superconductor picture of the QCD vacuum the presence of the monopole condensate inevitably leads to the breaking of the chiral symmetry.

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Origin of chiral symmetry breaking and nature of confinement of color charges are still unresolved puzzles in the Quantum Chromodynamics (QCD). Analytical methods, based on the perturbation theory, can not describe these phenomena starting from the first principles of the theory. The confinement of quarks is realized as a linear dependence of the interaction energy between a test quark and an antiquark, $V_{q\bar{q}} \sim \sigma R$, on the separation distance R. The coefficient of proportionality, $\sigma \approx 1\,\mathrm{GeV/fm}$, is called "the string tension". The confinement phenomena indicates its presence at relatively large separations, $R \gtrsim 0.2 \sim 0.3\,\mathrm{fm}$, where the QCD coupling constant is large and the perturbation theory is not applicable.

On the other hand, the effect of the chiral symmetry breaking involves a formation of the quark condensate, $\langle\bar{\psi}\,\psi\rangle \approx (250\,\mathrm{MeV})^3$, which is not invariant under the chiral transformations of the quark fields, $\psi \to \exp\{i\gamma_5\alpha\}\,\psi$. Both the chiral condensate and the string tension are dimensional quantities, which remain non–zero in the infinitely heavy quark limit ("quenched QCD"). The last fact also illustrates the non–perturbative nature of both confinement and chiral symmetry breaking phenomena.

There are indications that the confinement and the chiral symmetry breaking phenomena are closely related to each other. For example, at sufficiently high temperatures, $T > T_c$, confinement is lost and the QCD goes from the confinement phase into the deconfinement phase. The remarkable fact is that the chiral symmetry is restored exactly at the same temperatures, $T > T_c$. Below we discuss the relation between these phenomena within the so called dual superconductor model of confinement.

The dual superconductor mechanism [1] of the quark confinement is based on specific monopole–like configurations of the gluonic fields. The configurations – called "Abelian monopoles" – can be identified in appropriate Abelian gauges [2]. In an Abelian gauge the non–Abelian gauge symmetry of QCD, given by the $SU(3)$ gauge group, is fixed up to an Abelian subgroup. Since the original $SU(3)$ gauge group is compact, then the residual Abelian symmetry group is compact as well. The compactness of the Abelian group guarantees the appearance of the monopoles in the vacuum of the theory.

According to the dual superconductor mechanism the quark confinement in the low temperature phase, $T < T_c$, of the four dimensional $SU(3)$ gauge model appears due to the condensation of the Abelian monopoles. The condensation of magnetic charges leads to the dual Meissner effect and, as a result, to the formation of the chromoelectric string between the quarks. Consequently, the quarks get confined by the string. The condensation of the monopoles was established in various numerical simulations of non–Abelian models [3]. Moreover, the Abelian monopoles make to make a dominant contribution to the zero temperature string tension [4] (for a review, see Ref. [5]).

At the critical temperature the monopole condensate disappears and at higher temperatures the quarks are no more confined. The restoration of the chiral symmetry at $T = T_c$ suggests that the chiral condensate and the monopole condensate are tightly related to each other. Numerical simulations suggest that the monopoles provide a dominant contribution to the chiral condensate in various models [6, 7]. Moreover, the monopoles are correlated with topological charge[2)] [8], which is known to be related to the chiral symmetry breaking.

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2)Note that the ensemble of instantons (*i.e.*, of classical objects having non–zero topological charge) can be used to explain the chiral symmetry breaking. Since the QCD instantons fail to explain the quark confinement we do not discuss them in this paper.

The dual superconductor model for QCD is described by the Lagrangian [9]

$$L_{DGL} = -\frac{1}{2}\Big([n\cdot(\partial\wedge\mathbf{A})]^{\nu}[n\cdot{}^{*}(\partial\wedge\mathbf{B})]_{\nu} + [n\cdot(\partial\wedge\mathbf{B})]^{\nu}[n\cdot{}^{*}(\partial\wedge\mathbf{A})]_{\nu} - [n\cdot(\partial\wedge\mathbf{A})]^2 - [n\cdot(\partial\wedge\mathbf{B})]^2\Big) + \sum_{\alpha=1}^{3}\Big[\Big|(i\partial_\mu - g\epsilon_\alpha\mathbf{B}_\mu)\chi_\alpha\Big|^2 - \lambda(|\chi_\alpha|^2 - v^2)^2\Big] + \bar\psi(i\not\partial - e\not\mathbf{A}\cdot\mathbf{H} - m)\psi, \quad (1)$$

where three monopole fields χ_α with $\alpha = 1,2,3$, are interacting with the *dual* gluon fields $\mathbf{B} = (B^3, B^8)$, by the covariant derivative (the root vectors $\boldsymbol{\epsilon}_\alpha$ of the $SU(3)$ group are: $\boldsymbol{\epsilon}_1 = (1,0)$, $\boldsymbol{\epsilon}_2 = (-1/2, -\sqrt{3}/2)$ and $\boldsymbol{\epsilon}_3 = (-1/2, \sqrt{3}/2)$). The Abelian components of the original gluons, $\mathbf{A} = (A^3, A^8)$, are interacting with the dynamical quark fields ψ. The diagonal generators of the $SU(3)$ gauge group are denoted as $\mathbf{H} = (H^3, H^8)$. The magnetic (electric) charge of the monopole (quark) is e (g).

The interaction of the dual gauge field with original gauge field is described by the Zwanziger lagrangian [10] (the first line of Eq. (1)). The constant vector n_μ (with $n_\mu^2 = 1$) does not break the Lorentz symmetry on quantum level provided the Dirac quantization condition, $eg = 2\pi$, is satisfied. Note that in the Lagrangian (1) the off-diagonal components of the gluon field were ignored due to Abelian dominance observed both for the confining [4] and the chiral properties of the vacuum [7].

In the confinement phase the monopole condensate, $|\langle\chi_\alpha\rangle| = v$, is non–zero. Consequently, the dual gauge field $\mathbf{B}_\mu$ gets the mass, $M_B = vg$, due to spontaneous breaking of the dual $U(1)\times U(1)$ symmetry. In order to simplify the calculations let us consider the mean field approximation by setting $\chi_\alpha = v$. Then the Lagrangian becomes quadratic in the gauge fields, and the integration over the dual gauge field $\mathbf{B}_\mu$ in the corresponding partition function can be performed exactly:

$$L_{DGL-mf} = -\frac{1}{4}\mathbf{f}_{\mu\nu}\mathbf{f}^{\mu\nu} + \frac{1}{2}\mathbf{A}^{\mu}K_{\mu\nu}\mathbf{A}^{\nu} + \bar\psi(i\not\partial - e\not\mathbf{A}\cdot\mathbf{H} - m)\psi, \quad (2)$$

where $K^{\mu\nu} = M_B^2 X^{\mu\nu}/[(n\partial)^2 + M_B^2]$ and $X^{\mu\nu} = \epsilon_\lambda{}^{\mu\alpha\beta}\epsilon^{\lambda\nu\gamma\delta}n_\alpha n_\gamma\partial_\beta\partial_\delta$. One can see that if the monopole condensate is absent, $v = 0$, then $M = 0$ and the model becomes equivalent to $U(1)\times U(1)$ electrodynamics.

In the Lorentz–type gauge with the gauge fixing Lagrangian $L_{gf}(A) = (\partial_\mu\mathbf{A}^\mu)^2/2\xi$, the propagator of the gauge field A_μ is:

$$D_{\mu\nu} = \frac{1}{\partial^2}\{g_{\mu\nu} + (\xi - 1)\frac{\partial_\mu\partial_\nu}{\partial^2}\} - \frac{1}{\partial^2}\frac{M_B^2}{\partial^2 + M_B^2}\frac{1}{(n\partial)^2}X_{\mu\nu}. \quad (3)$$

The interaction Lagrangian of the fermion fields can be calculated exactly from Eq. (2) by the Gaussian integration of the gauge field $\mathbf{A}_\mu$:

$$L_{\mathrm{ferm}}(\bar\psi,\psi) = \bar\psi(i\not\partial - m)\psi - \tfrac{1}{2}\mathbf{j}_\mu D^{\mu\nu}\mathbf{j}_\nu, \qquad \mathbf{j}_\mu = ie\bar\psi\gamma_\mu\mathbf{H}\psi. \quad (4)$$

This Lagrangian contains a free (quadratic) part and the four-fermion interaction term similarly to the Nambu–Jona-Lasinio (NJL) model. In the NJL-type models the chiral (quark) condensate appears naturally. However, the presence or absence of the quark condensate depends on the particular form of the four-fermion interaction. In our model (4), the interaction term, and, consequently, the properties of the chiral condensate, depend on the value of the monopole condensate v.

Another simplification of our considerations comes from the fact that the Lagrangian (4) is invariant under the global color rotations of the the fermionic triplet $\psi \equiv (\psi_1, \psi_2, \psi_3)$, which implies that $\langle\bar\psi_i\psi_j\rangle = \langle\bar\psi\psi\rangle\cdot\delta_{ij}/3$. Thus below we consider only one component of the quark field, the dynamics of which is described by the Lagrangian (4) – in which ψ has only one component – with a simple redefinition $e^2 \to e^2\mathbf{H}^2 \equiv e^2/3$.

The chiral symmetry breaking can be conveniently investigated with the help of the generating functional:

$$Z[\bar\eta,\eta] = \frac{\int \mathrm{D}\bar\psi\,\mathrm{D}\psi\, e^{-S[\bar\psi,\psi] + \int d^4x(\bar\eta\psi + \bar\psi\eta)}}{\int \mathrm{D}\bar\psi\,\mathrm{D}\psi\, e^{-S[\bar\psi,\psi]}}, \quad (5)$$

where η and $\bar\eta$ are external fermionic fields, and the quark action S corresponds to Lagrangian (4).

Thus the knowledge of the generating functional (5) allows us to find the existence of the chiral condensate, $\langle\bar\psi(x)\psi(x)\rangle = \delta^2/(\delta\eta(x)\delta\bar\eta(x))\log Z[\bar\eta,\eta]|_{\bar\eta=\eta=0}$, and, as a result, the existence of the chiral symmetry breaking.

In order to integrate over the fermionic fields in (5) we use the method proposed in Ref. [11]. First, we use the Fierz identities to write

$$(\gamma_\mu)_{rs}D_{\mu\nu}(x;\xi)(\gamma_\nu)_{tu} = [\delta_{ru}\delta_{ts} + (i\gamma_5)_{ru}(i\gamma_5)_{ts}]\delta_{\mu\nu}D_{\nu\mu}(x;\xi)/4 + [(i\gamma_\mu)_{ru}(i\gamma_\nu)_{ts} + (i\gamma_\mu\gamma_5)_{ru}(i\gamma_\nu\gamma_5)_{ts}]\tilde D_{\mu\nu}(x;\xi) + [(\Sigma_{\rho\mu})_{ru}(\Sigma_{\rho\nu})_{ts} + (i\gamma_5\Sigma_{\rho\mu})_{ru}(i\gamma_5\Sigma_{\rho\nu})_{ts}]\tilde D_{\mu\nu}(x;\xi) \equiv \sum\nolimits_\Delta\Big((K^\Delta)_{ru}, D^\Delta(x-y;\xi)(K^\Delta)_{ts}\Big). \quad (6)$$

The definitions of the basis forms, K^Δ, and propagators, $\tilde{D}$, are obvious. The indices $\Delta = S,P,V,A,T,AT$ correspond, respectively, to scalar,pseudoscalar, vector, tensor, pseudotensor: $(K^S)_{ru} = \delta_{ru}$, $(K^P)_{ru} = (i\gamma_5)_{ru}$ etc.

Introducing the non-local hermitian variables, $[\beta(x,y)]^+ = [\beta(y,x)]$, we multiply the nominator and denominator of Eq. (5) by $\int \mathrm{D}\beta \exp\{-\frac{1}{2}e^2 \sum_\Delta \left(\beta^\Delta(y,x), D^\Delta(x-y;\xi)\,\beta^\Delta(x,y)\right)\}$. Next, we perform the shift $\beta^\Delta(x,y) \to \beta^\Delta(x,y) + \bar{\psi}(y) K^\Delta \psi(x)$ in the denominator of the obtained expression. The Jacobean of such a shift is unity. After the shift is preformed, the integration over the fermionic fields ψ can be taken in the explicit form:

$$Z[\bar\eta,\eta] = \left[\int \mathrm{D}\beta\ \exp(-S[\beta])\right]^{-1} \times \\ \times \int \mathrm{D}\beta\ \exp\Big\{-S_{\mathrm{eff}}[\beta] + \\ + \int \mathrm{d}^4x\, \mathrm{d}^4y\, \bar\eta(x)\, G(x,y;[\beta])\, \eta(y)\Big\}, \qquad (7)$$

where the action for the variables β reads as follows:

$$S_{\mathrm{eff}}[\beta] = -\mathrm{Tr}\,\mathrm{Log}[(G^{-1})(x,y;[\beta])] + \\ + \frac{e^2}{2} \sum\nolimits_\Delta \Big(\beta^\Delta(y,x), D^\Delta(x-y;\xi)\,\beta^\Delta(x,y)\Big). \qquad (8)$$

The definition of $G(x,y;[\beta])$ is $(G^{-1})(x,y;[\beta]) = (\gamma\cdot\partial)\delta^4(x-y) + \Sigma(x,y;[\beta])$, where the quantity $\Sigma(x,y;\beta]) = \sum_\Delta \Big(\beta^\Delta(y,x), D^\Delta(x-y;\xi)\, K^\Delta(x,y)\Big)$ can be interpreted as a self–energy because $\langle\bar\psi(x)\psi(y)\rangle \equiv \langle G(x,y;[\beta])\rangle$.

We evaluate the functional (7), (8) by the saddle–point method. The stationary point is given by the equations,

$$\beta^{S,P}(x,y) = \mathrm{tr}[K^{S,P}\, G(x,y;[\beta])], \\ \beta^{V,A}_\nu(x,y) = \mathrm{tr}[K^{V,A}_\nu\, G(x,y;[\beta])], \qquad (9) \\ \beta^{T}_{\mu\nu}(x,y) = \mathrm{tr}[K^{T}_{\mu\nu}\, G(x,y;[\beta])].$$

which are then multiplied by the corresponding matrices K^Δ and functions $\tilde{D}$. Summing them up over Δ and vector indices, we get an equation (in momentum space) for the self-energy Σ in the matrix form:

$$\Sigma(\bar p,[\beta]) = \\ = e^2 \int \frac{d^4\bar q}{(2\pi)^4}\, D_{\mu\nu}(\bar p - \bar q)\gamma_\mu \frac{1}{i(\gamma\bar q) + \Sigma(\bar q,[\beta])}\gamma_\nu\,. \qquad (10)$$

We choose the following anzatz for the solution,

$$\Sigma(\bar p,[\beta]) = i[A(\bar p) - 1](\gamma\bar p) + iC(\bar p)(\gamma\bar n) + e^{i\gamma_5\theta}B(\bar p). \qquad (11)$$

Here $\bar p = (p_0,p_1,p_2,p_3)$ and the arbitrary functions A, B and C, depend on $\bar p^2$ and $(\bar n,\bar p)$: $f(\bar p) \equiv f(\bar p^2, (\bar p,\bar n))$.

In the parameterization (11) the first two terms correspond to the chirally invariant vacuum while the third term violates the chiral symmetry. As we show below there exists a non-trivial solution of Eq. (10) with $B \neq 0$, such that the function B does not depend on the angle θ. Therefore there exists an infinite set of solutions (for which the effective functional of the theory is the same!) connected to each other by the transformation $B \to B'\, e^{i\gamma^5\varphi}$. This corresponds to the spontaneous breaking of the chiral symmetry.

It is convenient to set $n_\mu = \delta_{\mu,0}$ and separate the integrations in Eq. (10) into "time", q_0, and "space", $\mathbf{q}$, parts. The integral over the angle part of the space momentum can be taken explicitly. The integral over the temporal variable contains a singularity $1/(p_0-q_0)^2$, which can not be removed by a choice of the gauge. The singularity appears to be related to the (gauge-variant!) direction of the Dirac line and, therefore, this singularity is non-physical in a sense that it should not contribute to any gauge–invariant quantity.

In order to solve equations (10) we regularize the double-pole by the quantity ϵ which has the dimension of mass, $(p_0-q_0)^{-2} \to [(p_0-q_0)^2+\epsilon^2]^{-1}$. This regularization corresponds [6] to the quark–anti-quark pair creation from the vacuum and subsequent flattening of the quark–anti-quark potential at distances $R \gtrsim R_{\mathrm{flat}} \sim \epsilon^{-1}$, which are small compared to the dual penetration depth, M_B^{-1}, of the vacuum. Below we study the problem in the leading order of the smallness parameter, $M_B/\epsilon \ll 1$.

In the Landau gauge, $\xi = 0$, the saddle-point equations are (all regions of integration are $(0,\infty)$):

$$\bar C(p_0,p) - p_0 = \frac{\tilde e^2}{(2\pi)^3} \times \\ \times \int dq\, q^2\, \frac{\bar C(p_0,q)}{q^2 + B^2(p_0,q)\, p_0, (\mathbf{p})^2 + \bar C^2(p_0,q)} \times \\ \times \frac{M_B^2}{pq} \log\frac{M_B^2 + (p-q)^2}{M_B^2 + (p+q)^2}, \qquad (12)$$

$$B(p_0,p) = \\ = -\frac{\tilde e^2}{(2\pi)^3} \int dq\, q^2\, \frac{B(p_0,p)}{q^2 + B^2(p_0,p^2) + \bar C^2(p_0,q)} \times \\ \times \frac{M_B^2}{pq} \log\frac{M_B^2 + (p+q)^2}{M_B^2 + (p-q)^2}, \qquad (13)$$

where $\tilde e^2 \equiv \pi e^2/\epsilon$ and $\bar C(p_0,p) \equiv A(p_0,p)\,p_0 + C(p_0,p)$. We also used the fact $A(p_0,p) = 1 + O((M_B/\epsilon)^4)$ which follows from Eqs. (10), (11).

Equations (12), (13) possess two types of the solutions: the chirally invariant solution with $B=0$ and the chirally broken solution with $B\neq 0$. To discriminate between them one should compare the corresponding values of the action. The action of a classical solution is given by Eq. (8) evaluated on the anzatz (11)

$$S[A,B,C] = -\mathrm{Tr}\,\mathrm{Ln}[(\gamma\cdot\partial)A(x-y) + \\ + e^{i\gamma_5\theta}B(x-y) + i(\bar\gamma\cdot\bar n)C(x-y)] + \frac{e^2}{2}\times \\ \int d^4x d^4y \Big\{ \Big[\beta^S(y,x)\beta^S(x,y) + \beta^P(y,x)\beta^P(x,y)\Big] \times \\ \times \tilde D(x-y;\xi) + \beta^V_\mu(y,x)\beta^V_\nu(x,y)\tilde D_{\mu\nu}(x-y;\xi)\Big\}, \quad (14)$$

where β^Δ are given by Eqs. (9): $e^{i\gamma_5\theta} = e^2\tilde D(\beta^S + +i\gamma_5\beta^P)$ and $(\gamma\cdot\partial)(A-\delta^{(4)})+i(\bar\gamma\cdot\bar n)C = e^2\tilde D_{\mu\nu}(i\gamma_\mu)\beta^V_\nu$. The action (14) does not depend on the angle θ.

Up to $O((M_B/\epsilon)^2)$ corrections the difference between the actions of $B=0$ and $B\neq 0$ vacua is

$$\Delta S \equiv S[A_{B=0},0,C_{B=0}] - S[A_{B\neq 0},B,C_{B\neq 0}] = \\ = \int \mathrm{d}^4x \int \frac{\mathrm{d}^4 p}{(2\pi)^4}\Big\{ 2\log(1+\frac{B^2}{\bar p^2}) - \frac{2B^2}{\bar p^2+B^2} + \\ + \frac{4C^{'}_{B=0}}{p_0} - \frac{4C^{'}_{B\neq 0}}{p_0}\frac{1}{1+\bar g^2B^2} + \frac{2p_0C^{'}_{B=0}}{\bar p^2} - \\ - \frac{2p_0C^{'}_{B\neq 0}}{\bar p^2+B^2} + \frac{4p_0C^{'}_{B\neq 0}B^2}{(\bar p^2+B^2)^2}\Big\}\Big[1+O\Big((M_B/\epsilon)^2\Big)\Big], \quad (15)$$

where $\bar C = p_0 + C' M_B^2$, and $C_{B=0}$ is a solution of Eq. (12) with $B=0$, while $C_{B\neq 0}$ is solution of Eqs. (12), (13). In the absence of the monopole condensate, $M_B^2 = 0$, the difference (15) coincides with the result of QED [11].

The important fact is that Eq. (12) implies that $C_{B=0} > C_{B\neq 0}$. Therefore the action difference ΔS is *always* positive due to presence of the monopole condensate! Moreover, the concrete form of solutions of Eqs. (12), (13) does not influence this conclusion. One can show [12] with the help of the both analytical and numerical tools that the $B\neq 0$ solutions of Eqs.(12,13) do exist. Thus the presence of the monopole condensate makes the vacuum which is degenerate in the chiral angle θ more energetically preferable with respect to the chirally non–degenerate vacuum. Therefore, in scope of the dual superconductor model of the QCD vacuum, the presence of the monopole condensate implies the chiral symmetry breaking in QCD.

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