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Damping effects and the metal-insulator transition in the two-dimensional electron gas

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The damping of single-particle degrees of freedom in strongly correlated two-dimensional Fermi systems is analyzed. Suppression of the scattering amplitude due to the damping effects is shown to play a key role in preserving the validity of the Landau–Migdal quasiparticle picture in a region of a phase transition, associated with the divergence of the quasiparticle effective mass. The results of the analysis are applied to elucidate the behavior of the conductivity $\sigma(T)$ of the two-dimensional dilute electron gas in the density region where it undergoes a metal-insulator transition.

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A quantitative understanding of the damping of single-particle excitations in a Fermi liquid (FL) is essential to the determination of the resistivity, thermal conductivity, and other kinetic properties of the system. When the temperature dependence of the properties of uncharged Fermi liquids is treated within Landau theory, the decay rate $\gamma(\varepsilon)$ of single-particle excitations at the relevant energies $\varepsilon \sim T$ is given by [1]

$$\gamma(T) = W(M^*)^3T^2. \tag{1}$$

Here the effective mass M^* specifies the FL single-particle spectrum $\xi(p) \equiv \epsilon(p) - \mu = p_F(p - p_F)/M^*$, where $\epsilon(p) = \delta E_0/\delta n(p)$ and μ is the chemical potential. The factor W is proportional to the square of the scattering amplitude Γ, suitably averaged over spins and momenta of incoming and outgoing particles.

Reliable experimental data on the modification of FL properties under variation of controllable variables (e.g., the density n) exist only for two-dimensional (2D) Fermi systems, notably liquid ^{3}He and the electron gas. Landau theory adequately reproduces the behavior of these data in a broad density region, *except* in the vicinity of the critical density n_∞ where the effective mass diverges, and the spectrum $\xi(p)$ becomes flat. This failure of FL theory is conventionally attributed to a strong enhancement of the dimensionless damping rate $r(T) = \gamma(T)/T$. Close to the critical point, $r(T)$ allegedly exceeds unity, invalidating the Landau-Migdal quasiparticle picture.

Here we shall demonstrate that in actuality the parameter $r(T)$ remains rather small on both sides of the phase transition associated with the divergence of the effective mass in the 2D system, and, consequently, that the quasiparticle picture does apply. We then proceed to study kinetic phenomena within the quasiparticle formalism, with particular attention to the metal-insulator transition (MIT) occurring in the 2D electron gas in the density region where the effective mass diverges [2 – 4].

Our analysis is based on the standard formula for the damping rate [1, 5],

$$\gamma(\varepsilon) \sim -\sum_{\mathbf{p}_1,\mathbf{p}'}\iint W(\mathbf{p},\mathbf{p}_1,\mathbf{p}',\mathbf{p}_1';\varepsilon,\varepsilon_1,\omega)F(\varepsilon,\varepsilon_1,\omega,T)\times \\ \times \mathrm{Im}\,G_R(\mathbf{p}_1,-\varepsilon_1)\mathrm{Im}\,G_R(\mathbf{p}',\varepsilon-\omega)\times \\ \times \mathrm{Im}\,G_R(\mathbf{p}_1',\omega-\varepsilon_1)d\varepsilon_1 d\omega, \tag{2}$$

where $\mathbf{p},\mathbf{p}_1$ and $\mathbf{p}',\mathbf{p}_1'$ are respectively the incoming and outgoing momentum pairs, and $\omega=\varepsilon-\varepsilon'$. The function W is given by the sum of absolute squares of the scalar (s) and spin-dependent (a) components Γ_s and Γ_a of the scattering amplitude $\Gamma = \Gamma_s + \Gamma_a\boldsymbol{\sigma}_1\boldsymbol{\sigma}_2$, while $F(\varepsilon,\varepsilon_1,\omega,T) = \cosh(\varepsilon/2T)[\cosh(\varepsilon_1/2T)\cosh((\varepsilon - \omega)/2T)\cosh((\omega-\varepsilon_1)/2T)]^{-1}$ and G_R is the retarded Green function. In what follows we assume that the dependence of the mass operator $\Sigma(p,\varepsilon)$ on ε is not crucial, and then

$$\mathrm{Im}\,G_R(p,\varepsilon) = -\gamma(\varepsilon)/[(\varepsilon - \xi(p))^2 + \gamma^2(\varepsilon)]. \tag{3}$$

To begin, we note that in the collision integral (2), all the quasiparticle energies must lie close to the Fermi surface, so that $|\xi(p)| \le T$, $|\xi(p_1)| \le T$, and $|\xi(|\mathbf{p}-\mathbf{q}|)| \le T$, $|\xi(|\mathbf{p}_1 + \mathbf{q}_1|)| \le T$, since, as we shall see, broadening of the single-particle states is insignificant. In 2D, these conditions are easily met, if (i) the momentum transfer $q = |\mathbf{p}-\mathbf{p}'|$ in the longitudinal particle-hole channel is

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small, i.e., $q \leq q_c(T) = T\,(dp/d\xi)_T \sim TM^*/p_F$, or equivalently, if (ii) the momentum transfer $q_1 = |\mathbf{p} - \mathbf{p}'_1|$ in the transverse particle-hole channel is small, or (iii) the total momentum $P = |\mathbf{p} + \mathbf{p}_1|$ is close to zero. Outside these regions, contributions to the collision integral appear to be minor.

In dealing with small momentum transfers, we first address the scalar component Γ_s of the scattering amplitude Γ, which obeys the standard equation [5]

$$\Gamma_s(q,\omega) = f + f\Pi_0(q,\omega)\Gamma_s(q,\omega) \equiv [f^{-1} - \Pi_0(q,\omega)]^{-1}, \tag{4}$$

where f is the scalar part of the Landau interaction function. In FL theory, the polarization loop Π_0 is an integral over the product of two quasiparticle Green functions $G(p,\varepsilon) = (\varepsilon - \xi(p))^{-1}$, given by

$$\Pi_0(q,\omega) = 2\int \frac{n(\mathbf{p}) - n(\mathbf{p}-\mathbf{q})}{\xi(\mathbf{p}) - \xi(\mathbf{p}-\mathbf{q}) - \omega}\, dv, \tag{5}$$

in which $dv = d^2p/(2\pi)^2$ is the volume element in 2D momentum space, and $n(p) = 1/[1 + \exp(\xi(p)/T)]^{-1}$ is the quasiparticle momentum distribution.

The value of $\mathrm{Re}\,\Pi_0$ is of order of $N(0)$, the density of states, proportional to M^*. In a strongly correlated FL obeying Landau theory, this quantity, whose sign depends on the ratio ω/q, is enhanced by the factor M^*/M compared to the corresponding ideal Fermi-gas value. On the other hand at small ω and $q > q_{\min} = M^*\omega/p_F$, the imaginary part of $\Pi_0(q,\omega,T=0)$, given by

$$\mathrm{Im}\,\Pi_0(q,\omega,T=0) \simeq -\frac{\omega(M^*)^2}{\pi q p_F\sqrt{1 - (M^*\omega/qp_F)^2}}\,, \tag{6}$$

has the same order as $\mathrm{Re}\,\Pi_0$. Thus in strongly correlated systems f^{-1} can be neglected, and Eq. (4) reduces to $|\Gamma_s(q \sim q_c, \omega \sim T)| \simeq N^{-1}(0)$ [6].

A similar situation applies for the spin-dependent part Γ_a of the scattering amplitude Γ, which satisfies the same equation (4) with the replacement $f \to f_a$, where f_a is the spin-dependent part of the Landau interaction function. The 2D Fermi systems in question do not exhibit ferromagnetism, in spite of the negative sign of f_a derived from experimental data on the spin susceptibility. This means that the Pomeranchuck stability condition [1] $1 + f_a N(0) > 0$ is not violated, implying that $|f_a N(0)| < 1$ holds even if the enhancement of the effective mass is large. The estimate $|\Gamma_a(q \sim q_c, \omega \sim T)| \leq N^{-1}(0)$ follows straightforwardly. In the transverse particle-hole channel, where small momentum transfer corresponds to $q \simeq 2p_F$, the situation is evidently the same, so that in the collision term (2), integration over q can be restricted to the region of small q, and the result is doubled.

In the third relevant momentum region where the total momentum P is small, the scattering amplitude $|\Gamma(P \to 0)| \simeq -1/[N(0)\ln P^2]$ contains an additional suppression factor $1/\ln P^2$ due to the BCS logarithmic divergence of the particle-particle propagator [5]. Therefore in what follows the respective contribution will be neglected. Thus we conclude that a proper treatment of damping effects in the strongly correlated system leads to substantial suppression of the interaction factor W governing the damping rate (2).

In the foregoing analysis, the momentum dependence of the Landau interaction function $\mathcal{F} = f + f_a\boldsymbol{\sigma}_1\boldsymbol{\sigma}_2$ has been neglected. Upon its inclusion, relation (4) is replaced by the integral equation

$$\Gamma(\mathbf{n}\mathbf{n}_1, q,\omega) = \mathcal{F}(\mathbf{n}\mathbf{n}_1) + \\ + \Pi_0(q,\omega)\int \mathcal{F}(\mathbf{n}\mathbf{n}')\,\Gamma(\mathbf{n}'\mathbf{n}_1, q,\omega)\, d\varphi'/2\pi, \tag{7}$$

with $\mathbf{n} = \mathbf{p}/p_F$ and $\mathbf{n}_1 = \mathbf{p}_1/p_F$. We now make use of the smallness of the quantity $1/\Pi_0 \sim M/M^*$ and rewrite the scattering amplitude as

$$\Gamma(\mathbf{n}\mathbf{n}_1, q,\omega) = X(\mathbf{n}\mathbf{n}_1)/\Pi_0(q,\omega). \tag{8}$$

Neglecting small corrections, Eq. (7) becomes

$$0 = \mathcal{F}(\mathbf{n}\mathbf{n}_1) + \int \mathcal{F}(\mathbf{n}\mathbf{n}')\, X(\mathbf{n}'\mathbf{n}_1)\, d\varphi'/2\pi. \tag{9}$$

Inserting the expression (8) into the collision formula (2), standard algebra [7] converts it to

$$\gamma(\varepsilon \sim T) \sim \int\limits_0^{\varepsilon\sim T}\int\limits_{q_{\min}}^{q_c}\int X^2(\mathbf{n}\mathbf{n}_1)\,\frac{\mathrm{Im}\Pi_0(q,\omega)}{|\Pi_0(q,\omega)|^2}\times \\ \times\, \mathrm{Im}\, G_R(\mathbf{p}-\mathbf{q}, \varepsilon - \omega)\, d\omega\, q dq\, d\varphi, \tag{10}$$

where $\mathrm{Im}G_R(p,\varepsilon)$ is given by Eq. (3). In the 2D dilute electron gas where $X = 1$, this result practically coincides with that derived in Ref. [7]. As usual [8], integration over angles in Eq. (10) is replaced by integration over energies $\xi(l)$ where $\mathbf{l} = \mathbf{p} - \mathbf{q}$, and after some algebra we arrive at [7]

$$\gamma(T) \sim \frac{T^2 M^*}{M\varepsilon_F^0}\ln\left(\frac{M^*T}{M\varepsilon_F^0}\right). \tag{11}$$

The same estimate is valid for other strongly correlated 2D Fermi systems where the spectrum $\xi(p)$ is specified only by the effective mass.

The result (10) holds in the density region where the effective mass M^* diverges, since in its derivation only relation (8) has been employed. Here at relevant q, ω, the value of $\mathrm{Re}\Pi_0(q,\omega)$ turns out to be of order $(dp/d\xi)_{\xi\simeq T}$. As for $\mathrm{Im}\,\Pi_0(q,\omega)$, its value is evaluated on the base of the general formula [5]

$$\mathrm{Im}\,\Pi_0(q,\omega) = \iint \left[\tanh\frac{\varepsilon-\omega}{2T} - \tanh\frac{\varepsilon}{2T}\right] \times$$
$$\times\,\mathrm{Im}\,G_R(\mathbf{p}-\mathbf{q},\varepsilon-\omega)\,\mathrm{Im}\,G_R(\mathbf{p},\varepsilon)\,\frac{d\varepsilon}{\pi}\,dv. \quad (12)$$

Insertion of the explicit form of $\mathrm{Im}\,G_R$ and integration over $\xi(p),\xi(l)$ along the same lines, as before, gives

$$\mathrm{Im}\,\Pi_0(q,\omega) \sim \frac{p_F}{q}\int\left[\tanh\frac{\varepsilon-\omega}{2T} - \tanh\frac{\varepsilon}{2T}\right]\left(\frac{dp}{d\xi}\right)_T^2 d\varepsilon, \quad (13)$$

where the product $(dp/d\xi)_{\xi=\varepsilon}(dp/d\xi)_{\xi=\varepsilon-\omega}$ has been replaced by $(dp/d\xi)_T^2$. As a result, one finds

$$|\mathrm{Im}\Pi_0(q,\omega{\sim}T)| \sim \frac{Tp_F}{q}\left(\frac{dp}{d\xi}\right)_T^2. \quad (14)$$

Upon inserting this result into Eq. (10) we are led to

$$\gamma(T) \sim \frac{T^2}{p_F}\left(\frac{dp}{d\xi}\right)_T \ln\left[\frac{T}{p_F}\left(\frac{dp}{d\xi}\right)_T\right]. \quad (15)$$

Thus for evaluation of the damping rate $\gamma(T)$ in the density region where M^* diverges one needs to know the spectrum $\xi(p)$ close to the Fermi surface. To date, microscopic calculations in this density region have been performed only for the electron gas in 2D and 3D and only at $T=0$ [9–11]. In Fig.1, we display results for the spectrum $\xi(p)$ of the 2D electron gas, calculated at $T=0$ within a functional approach [12]. Close to the Fermi surface, the electron spectrum $\xi(p,n_\infty)$, given in Fig.1, behaves as $(p-p_F)^3$. The leading FL term re-emerges at finite temperatures [13], so that

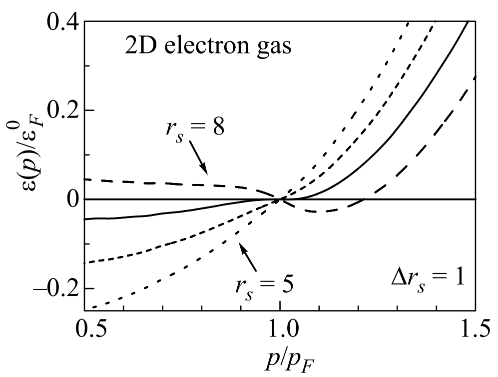

Fig.1. Single-particle spectrum $\xi(p)$ of the homogeneous two-dimensional electron gas in units of $\varepsilon_F^0 = p_F^2/2M$, evaluated at $T=0$ for different values of $r_s = (\pi n)^{-1/2}/a_B$, where a_B is Bohr radius

$$\xi(p,T,n_\infty) = p_F(p-p_F)/M^*(T,n_\infty)+\xi_3(p-p_F)^3, \quad (16)$$

with the effective mass [14] going like $M^*(T,n_\infty) \sim (dp/d\xi)_T \sim T^{-2/3}$. With this result, the damping rate evaluated with the help of Eq. (15), becomes $\gamma(T,n_\infty) \sim T^{4/3}\ln(\varepsilon_F^0/T)$.

The critical single-particle spectrum $\xi(p,T{=}0,n_\infty) \sim (p-p_F)^3$ is not universal. In a broader context, the Landau state is known (e.g. from Refs. [15, 16]) to lose its stability at a density n_b for which a bifurcation point $p=p_b$ emerges in equation

$$\xi(p,T=0,n_b)=0, \quad (17)$$

which ordinarily has only the single root $p=p_F$. The particular form $\xi(p,T=0,n_\infty) \sim (p-p_F)^3$ corresponds to the case in which the bifurcation point p_b coincides with p_F. Obviously, in the general case one has $p_b \neq p_F$, and the Landau state loses its stability before M^* becomes infinite. If the distance between p_b and p_F is small, then the single-particle spectrum has the form

$$\xi(p,T=0,n_b) \sim (p-p_b)^2(p-p_F). \quad (18)$$

Suppose the temperature T lies below the maximum value ξ_m of $|\xi(p,T=0,n_b)|$ in the momentum interval $[p_b,p_F]$. In this case, the dominating contributions to the properties of interest come from the momentum region adjacent to the bifurcation point p_b, where according to Eq. (18), $(dp/d\xi)_T \sim T^{-1/2}$. From this result and Eq. (15) one obtains $\gamma(T,n_b) \sim T^{3/2}\ln(\varepsilon_F^0/T)$.

Beyond the critical density n_b, Eq. (17) possesses two additional roots $p_1<p_b<p_2$. The single-particle spectrum $\xi_{FL}(p,T{=}0,n)$, evaluated with the Landau momentum distribution $n_{FL}(p){=}\theta(p_F{-}p)$, has the form

$$\xi_{FL}(p,T=0,n) \sim (p-p_1)(p-p_2)(p-p_F). \quad (19)$$

If $p_b{\neq}p_F$, the roots p_1,p_2 are both located either in the interior of the Fermi sphere or both outside it. If $p_b{=}p_F$, then $p_1{<}p_F{<}p_2$. In all these cases, the Landau occupation numbers $n_L(p)$ are rearranged. As a rule, the Fermi surface becomes multi-connected, but the quasiparticle occupation numbers $n(p)$ continue to take values 0 or 1. Hence the Landau-Migdal quasiparticle picture holds, with $n(\xi){=}1$ for $\xi{<}0$ and 0 otherwise. Consider first the case $p_1{<}p_2{<}p_F$. Then according to Eq. (19), the single-particle states remain filled in the intervals $p{<}p_1$ and $p_2{<}p{<}p_F$, while the states corresponding to $p_1{<}p{<}p_2$ are empty. We call this new phase the bubble phase. If the bifurcation point p_b coincides with the Fermi momentum p_F, then $p_1{<}p_F$ and $p_2{>}p_F$, and the states with $p{<}p_1$ and with $p_F{<}p{<}p_2$ are occupied, while those for $p_1{<}p{<}p_F$ are empty. Again one deals with the bubble phase.

At this point, we observe that the solution (19) is not self-consistent, since the spectrum is evaluated with $n_{FL}(p)$ while the true Fermi surface is doubly-connected. Following Ref. [15], we consider the feedback of the rearrangement of $n_{FL}(p)$ on the spectrum $\xi(p)$ in the bubble phase based on the Landau relation [5]

$$\frac{\partial\epsilon(p)}{\partial\mathbf{p}}=\frac{\mathbf{p}}{M}+\int f(\mathbf{p},\mathbf{p}_1)\frac{\partial n(p_1)}{\partial\mathbf{p}_1}dv_1, \qquad (20)$$

where, as before, f is the scalar part of the Landau interaction function and $n(p)=[1+\exp(\xi(p)/T)]^{-1}$ is the quasiparticle momentum distribution. Solutions of this nonlinear integral equation are known only in 3D Fermi systems with phenomenological functions f depending on $q=|\mathbf{p}-\mathbf{p}_1|$. Despite of the diversity of forms assumed for $f(q)$, the resulting single-particle spectra and momentum distributions bear a close family resemblance. Figs.2 and 3 display results from solution of Eq. (20) for

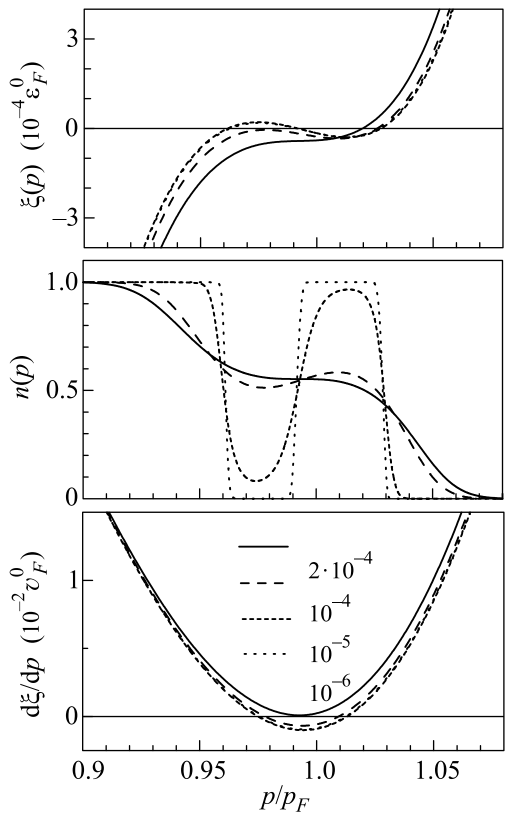

Fig.2. Single-particle spectrum $\xi(p)$ in units of $10^{-4}\,\varepsilon_F^0$ (top panel), occupation numbers $n(p)$ (middle panel), and $d\xi/dp$ in units of $10^{-2}\,v_F^0$, where $v_F^0=p_F/M$ (bottom panel), plotted versus p/p_F at four line-type-coded temperatures relevant to the bubble phase, in units of ε_F^0. The model (21) is assumed with parameters $\beta=0.48$ and $\lambda=3\,N_0$, where $N_0=p_FM/\pi^2$

the function

$$f(q)=\lambda/[((q/2p_F)^2-1)^2+\beta^2]. \qquad (21)$$

Let us briefly summarize how solutions of Eq. (20) evolve under variation of T. When the bubble range p_2-p_1 is small, then heating to $T\sim T_{FL}=(p_2-p_1)^2/M$ results in its dissolution (see Fig.2). With further increase of T, the function $\xi(p)$ becomes smoother, and in the region of a new critical temperature T_Z, a flat portion $\xi\simeq0$ appears in the spectrum over an interval $[p_i,p_f]$ surrounding the Fermi momentum p_F, as shown in the left panel of Fig.3. Since $\xi(p)=\epsilon(p)-\mu$ and $\epsilon(p)=\delta E_0/\delta n(p)$, the equality $\xi=0$ can be rewritten as a variational condition [17]

$$\frac{\delta E_0}{\delta n(p)}=\mu, \quad p_i<p<p_f, \qquad (22)$$

with $E_0=\sum_{\mathbf{p}}\epsilon^0_{\mathbf{p}}n(\mathbf{p})+\frac{1}{2}\sum_{\mathbf{p},\mathbf{p}_1}f(\mathbf{p}-\mathbf{p}_1)\,n(\mathbf{p})\,n(\mathbf{p}_1)$ and $\epsilon^0_{\mathbf{p}}=p^2/2M$. The solution $n_0(p)$ of Eq. (20), or equivalently of Eq. (22), is a continuous function of p with a nonzero derivative dn_0/dp (see Fig.3, top-right panel). The set of single-particle states with $\xi(p)=0$ is called the fermion condensate (FC), since the corresponding density of states $\rho(\varepsilon)$ contains a Bose-liquid-like term $\eta n\delta(\varepsilon)$. The dimensionless constant $\eta\simeq(p_f-p_i)/p_F$ is naturally identified as a characteristic parameter of the FC phase.

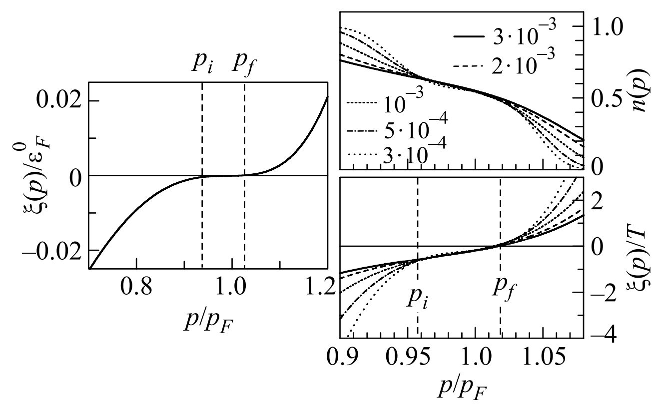

Fig.3. Single-particle spectrum $\xi(p)$ in units of ε_F^0 at the critical temperature $T_Z=3\cdot10^{-4}\,\varepsilon_F^0$ (left panel), occupation numbers $n(p)$ (right-top panel), and $\xi(p)/T$ (right-bottom panel), plotted versus p/p_F at five line-type-coded temperatures relevant to the phase with a FC, in units of ε_F^0. The model (21) is assumed

It has been demonstrated [18] that the FC "plateau" in $\xi(p)$ has a small slope, evaluated by inserting $n_0(p)$ into the above Fermi-Dirac formula for $n(\xi)$ to yield

$$\xi(p,T\geq T_Z)=T\ln\frac{1-n_0(p)}{n_0(p)}, \; p_i<p<p_f. \qquad (23)$$

As indicated in the bottom-right panel of Fig.3, at $T\geq T_Z$ the ratio $\xi(p)/T$ is indeed a T-independent function of p in the FC region. The presence of this flat portion of $\xi(p)\sim T$ is a signature of the phenomenon called fermion condensation [17–19]. The width

$\xi(p_f)-\xi(p_i)\equiv\xi_f-\xi_i$ of the FC "band" appears to be of order T, almost independently of $\eta>\eta_{\min}\sim10^{-2}$. Thus at $\eta>\eta_{\min}$ the FC group velocity is estimated as

$$\left(\frac{d\xi(p)}{dp}\right)_T\sim\frac{T}{\eta p_F}\,,\quad p_i<p<p_f. \tag{24}$$

As shown in Ref. [10], the effective mass diverges before attaining the critical point $r_{\rm CDW}$ for the charge-density-wave instability. Microscopic calculations confirm this assertion: M^* diverges at $r_s{=}r_\infty{\simeq}7$, while the condensate of the charge-density waves occurs at $r_{\rm CDW}{\simeq}10$. Thus in the interval $r_\infty{<}r_s{<}r_{\rm CDW}$ one deals with the homogeneous ground state having a FC. In this case, the damping rate of single particle excitations is evaluated on the base of Eqs. (15) and (24) that yields

$$\gamma(T)\sim\eta\,T\ln(1/\eta). \tag{25}$$

Thus, as long as the FC density remains small, the dimensionless damping rate $r(T)=\gamma(T)/T$ proves to be small as well, so the presence of the FC does not destroy the quasiparticle picture. It is worth noting that at greater energies $\varepsilon\gg T$, the damping $\gamma(\varepsilon)$ grows as $\sqrt{\varepsilon}$ with increasing ε [20]. Thus at these energies, the ratio $\gamma(\varepsilon)/\varepsilon$ exceeds unity, and the quasiparticle picture fails independently of the η value.

Let us now apply our results to the elucidation of behavior of the conductivity of the dilute 2D electron gas from data obtained in samples with silicon inversion layers [2, 4]. We focus our attention at low densities where the 2D electron gas undergoes a metal-insulator transition (MIT), as signaled by a change in sign of the derivative $d\rho(T\to0)/dT$. In high-quality samples, the sign change occurs at the density $n_{\rm MIT}\simeq0.9\cdot10^{11}{\rm cm}^{-2}$. On the metallic side of the MIT, this derivative has positive sign, while on the insulating side, it is negative, the separatrix $\rho_{\rm MIT}(T)\simeq3h/e^2\simeq75\,{\rm k}\Omega$ between the two phases being almost horizontal [2, 4].

At these densities, the electron-electron interaction, first taken into account within perturbation theory in Ref. [21], becomes a "play maker". A crucial point is that close to the critical density $n_{\rm MIT}$, the effective mass $M^*(n)$ diverges [2, 4]. In this situation, a standard method of treatment of kinetic phenomena on the base of the Boltzmann equation fails. Therefore we employ a different approach where the conductivity $\sigma(T)$ is expressed in terms of the imaginary part of the polarization operator $\Pi(\mathbf{j},\omega\to0,T)$ through [5]

$$\sigma(T)\sim-\lim_{\omega\to0}\omega^{-1}\,{\rm Im}\,\Pi(\mathbf{j},\omega,T). \tag{26}$$

It provides contributions of two different types, namely from (i) imaginary parts of the quasiparticle Green functions and (ii) imaginary parts of the scattering amplitudes. As a rule, both these contributions provide the same T-dependence of $\sigma(T)$. E.g. this is seen from homogeneous systems without impurities where the two types of contributions cancel each other to ensure vanishing of the resistivity due to momentum conservation. (In solids, the resistivity $\rho(T)$ differs from 0 due to umclapp processes). Such cancellation allows one to find out the T-dependence of the resistivity in the critical density region, where the spectrum $\xi(p)$ becomes flat by retaining in Eq. (26) only contributions coming from ${\rm Im}\,G_R$. Thereby the calculations are simplified considerably, and the expression for ${\rm Im}\,\Pi$ acquires the form

$$\begin{aligned}{\rm Im}\,\Pi(\mathbf{j},\omega{\to}0,T)\sim&\iint\left(\tanh\frac{\varepsilon-\omega}{2T}-\tanh\frac{\varepsilon}{2T}\right)\times\\ &\times|\mathcal{T}(\mathbf{j},\omega{=}0)|^2{\rm Im}\,G_R(\mathbf{p},\varepsilon{-}\omega){\rm Im}\,G_R(\mathbf{p},\varepsilon)d\varepsilon dv,\end{aligned} \tag{27}$$

Here, $\mathcal{T}$ is the vertex part, whose static limit is given by [5] $\mathcal{T}(\mathbf{j},\omega=0)=e\partial\xi(p)/\partial\mathbf{p}$. Upon inserting the explicit form for ${\rm Im}\,G_R$ into Eqs. (27) and (26), the latter becomes [22]

$$\sigma(T){=}2e^2\iint\frac{(d\xi/dp)^2\,\gamma^2(\varepsilon)\,d\varepsilon\,dv}{2T[(\varepsilon{-}\xi(p))^2{+}\gamma^2(\varepsilon)]^2\cosh^2(\varepsilon/2T)}. \tag{28}$$

Converting, as before, the momentum integration to an integration over ξ and taking into account that the overwhelming contributions to this integral come from the vicinity of the point $\xi=\varepsilon$, we arrive at

$$\sigma(T)=2\pi\frac{ne^2}{p_F}\int\frac{(d\xi/dp)}{2T\,\gamma(\xi)\cosh^2(\xi/2T)}\,d\xi, \tag{29}$$

where $n=p_F^2/2\pi$. Remembering that in the region of the critical density of 2D electron gas, where the effective mass diverges, one has $\gamma(T)\sim T^{4/3}\ln(\varepsilon_F^0/T)$, and then Eq. (29) gives us

$$\sigma(T)\sim T^{-2/3}/\ln(\varepsilon_F^0/T). \tag{30}$$

Since $d\sigma(T\to0)/dT<0$, this point is situated on the metallic side of the MIT. Beyond this density, i.e. at $r_\infty<r_s<r_{\rm CDW}$, and greater but still very low temperatures $T\sim T_Z$, we pass the point of fermion condensation. The FC contribution to $\sigma(T)$ is evaluated with the help of Eqs. (25) and (24), yielding

$$\sigma_{FC}(T)=\sigma_0e^2/\eta^2\ln(1/\eta), \tag{31}$$

where σ_0 is a T-independent constant.

At $r_s>r_{\rm CDW}$, the spontaneous generation of the condensate of the charge density waves occurs, and the ground state of 2D electron gas becomes nonhomogeneous. Consequently, a gap in the single-particle spectrum opens that results in exponential falling of the conductivity $\sigma(T)$ at $T\to0$, implying that one deals with the insulating side of the MIT. Thus the separatrix, dividing the metallic and insulating domains, is situated in the FC region, and according to Eq. (31), it is a straight line. This result is in agreement with available experimental data [2–4].

Flattening of the single-particle spectrum entails the change of the Hall coefficient $R_H = \sigma_{xyz}/\sigma_{xx}^2$ [23]. In homogeneous matter at $H \to 0$, $\sigma_{xx} = \sigma/3$, with σ given by Eq. (29), while σ_{xyz} is recast to

$$\sigma_{xyz} = \frac{e^3}{3c\gamma^2}\int\left(\frac{d\xi}{dp}\right)^2\frac{\partial n(\xi)}{\partial \xi}d\xi, \qquad (32)$$

where $n(\xi)$ is the Fermi-Dirac distribution function. Far from the critical density n_∞, these formulas lead to the standard result $R_H = 1/nec$. The critical spectrum of 2D electron gas has the form $\xi(p, n_\infty, T = 0) \sim \sim (p - p_F)^3$, and with the help of Eqs. (29), (32), one then finds $R_H = K/nec$ where

$$K(n_\infty, T\to 0, H\to 0) = \frac{\int z^{4/3}e^z[1+e^z]^{-2}dz}{\left(\int z^{2/3}e^z[1+e^z]^{-2}dz\right)^2} \simeq 1.5. \qquad (33)$$

We see that at the critical density, the effective volume of the Fermi sphere considerably shrinks. It is important that even quite close to the critical point where the effective mass still remains finite, the value $K = 1$ holds, so that at low T, the critical behavior (33) of K emerges abruptly. On the other hand, imposition of static magnetic field H on the system at the critical density n_∞ renders the effective mass finite [13, 14] and hence, one can expect the abrupt change of the Hall coefficient $R_H(n_\infty, T \to 0, H)$ as a function of H.

In conclusion, we have analyzed damping effects in the strongly correlated 2D Fermi liquid in a density region where the effective mass diverges. We have demonstrated that in spite of the enhancement of the dimensionless constants specifying the strength of the effective interaction between quasiparticles, the Landau-Migdal quasiparticle picture is applicable on both sides of the phase transition associated with the divergence of the effective mass. The results of the analysis have been applied to the interaction-driven metal-insulator transition in the 2D electron gas, demonstrating that the separatrix between the metallic and insulating regions is a straight line.

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