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Hunting for the alpha: $B \to \rho\rho$, $B \to \pi\pi$, $B \to \pi\rho$

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The hypothesis of the smallness of penguin contribution to charmless strangeless $B_d(\bar{B}_d)$ decays allows to determine with high accuracy the value of angle α from the currently available $B \to \rho\rho$, $B \to \pi\pi$ and $B \to \rho\pi$ decay data.

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1. Introduction. Measurement of CP asymmetries in $B_d(\bar{B}_d) \to J/\psi K^0$ decays by BaBar and Belle collaborations determines angle β of CKM unitarity triangle with high accuracy [1]:

$$\sin 2\beta = 0.724 \pm 0.040, \quad \beta = 23^\circ \pm 2^\circ. \tag{1}$$

The next task is to measure the angles α and γ with comparable accuracy in order to determine if New Physics contribute to CP-violation in B decays. For precise determination of the value of angle γ one should study B_s decays and this should wait until LHC(b) era. The purpose of this Letter is to stress that (may be) angle α is already known with the accuracy comparable to that achieved in β.

2. $B \to \rho\rho$. Let us start from $B_d(\bar{B}_d) \to \rho\rho$ decays, where the smallness of QCD penguin contribution directly follows from experimental data on relative smallness of the branching ratio of $B_d(\bar{B}_d) \to \rho^0\rho^0$ decays [2]. Here are the experimental data; all the branchings are in units of 10^{-6}:

$$\begin{aligned} Br(\rho^+\rho^-) &\equiv B_{+-} = 30 \pm 5 \pm 4 \quad [3], \\ Br(\rho^\pm\rho^0) &\equiv B_{\pm 0} = 22.5 \pm 5 \pm 6 \quad [4], \\ Br(\rho^\pm\rho^0) &\equiv B_{\pm 0} = 31.7 \pm 7 \pm 5 \quad [5], \\ Br(\rho^0\rho^0) &\equiv B_{00} < 1.1 (90\%\ \text{C.L.}) \quad [6]. \end{aligned} \tag{2}$$

In order to prove the smallness of the penguin contribution let us write the amplitudes of $B_d(\bar{B}_d) \to \rho^0\rho^0$ decays as the sum of tree and penguin contributions:

$$\begin{aligned} A_{\rho^0\rho^0} &= T_{\rho^0\rho^0} e^{i\gamma} + P_{\rho^0\rho^0} e^{i\delta_{00}}, \\ \bar{A}_{\rho^0\rho^0} &= T_{\rho^0\rho^0} e^{-i\gamma} + P_{\rho^0\rho^0} e^{i\delta_{00}}, \end{aligned} \tag{3}$$

where γ is the angle of a unitarity triangle and δ_{00} is the difference of phases of the final state strong interaction amplitudes induced by the penguin and tree quark diagrams. (We use the so-called c-convention in defining penguin amplitude: penguin with an intermediate t-quark is subtracted while penguin with an intermediate u-quark is included into the tree amplitude.)

For widths we get:

$$\begin{aligned} \Gamma_{\rho^0\rho^0} &= T^2_{\rho^0\rho^0} + P^2_{\rho^0\rho^0} + 2T_{\rho^0\rho^0} P_{\rho^0\rho^0} \cos(\delta_{00} - \gamma) \ , \\ \bar{\Gamma}_{\rho^0\rho^0} &= T^2_{\rho^0\rho^0} + P^2_{\rho^0\rho^0} + 2T_{\rho^0\rho^0} P_{\rho^0\rho^0} \cos(\delta_{00} + \gamma) \ , \end{aligned} \tag{4}$$

and

$$\begin{aligned} &\frac{1}{2}(\Gamma_{\rho^0\rho^0} + \bar{\Gamma}_{\rho^0\rho^0}) = T^2_{\rho^0\rho^0} + P^2_{\rho^0\rho^0} + \\ &+ 2T_{\rho^0\rho^0} P_{\rho^0\rho^0} \cos\gamma \cos\delta_{00} \geq P^2_{\rho^0\rho^0}(1 - \cos^2\gamma). \end{aligned} \tag{5}$$

Since from the global fit of CKM matrix parameters we know that $\gamma \geq 45^\circ$ [7 – 9], one observes that the compensation of $P_{\rho^0\rho^0}$ by $T_{\rho^0\rho^0}$ is not possible and both of them are small in comparison with the amplitudes of B decays into $\rho^\pm\rho^0$, $\rho^+\rho^-$-states.

Two ρ-mesons produced in B decays should be in $I = 0$ or $I = 2$ states, and since QCD penguin amplitude has $\Delta I = 1/2$, it contributes only to $I = 0$ state. That is why $P_{\rho^\pm\rho^0} = 0$, while $P_{\rho^+\rho^-} = \sqrt{2} P_{\rho^0\rho^0} \ll T_{\rho^+\rho^-}$. Tree level $b \to u\bar{u}d$ amplitude having both $\Delta I = 1/2$ and $\Delta I = 3/2$ parts produce both $I = 0$ and $I = 2$ states of two ρ-mesons, and one can easily organize compensation of these two amplitudes in $B_d(\bar{B}_d) \to \rho^0\rho^0$ decays still satisfactorily describing $B_d(\bar{B}_d) \to \rho^+\rho^-$ and $B_u(\bar{B}_u) \to \rho^\pm\rho^0$ branching ratios.

Let us show how it works:

$$T_{\rho^0\rho^0} = \frac{1}{\sqrt{6}} A_0 - \frac{1}{\sqrt{3}} A_2 e^{i\delta}, \tag{6}$$

where δ is the difference of the phases of final state interaction (FSI) amplitudes of ρ-mesons in $I = 2$ and $I = 0$ states and in order for these two terms to compensate

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each other δ should be small. Let us suppose that $\delta = 0$, so that we can write:

$$\frac{1}{\sqrt{6}}A_0 = \frac{1}{\sqrt{3}}A_2 \pm \sqrt{B_{00}}. \quad (7)$$

We should extract the value of A_2 from $B_u(\bar{B}_u) \to \rho^\pm\rho^0$ decay branching ratio:

$$T_{\rho^\pm\rho^0} = \frac{\sqrt{3}}{2}A_2, \quad A_2 = \frac{2}{\sqrt{3}}\sqrt{B_{\pm 0}k}, \quad (8)$$

where $k = \tau_{B^0}/\tau_{B^+} = 0.92$.

Finally we get:

$$T_{\rho^+\rho^-} = \frac{1}{\sqrt{3}}A_0 + \frac{1}{\sqrt{6}}A_2 =$$
$$= \sqrt{\frac{2}{3}}A_2 \pm \sqrt{2B_{00}} + \frac{1}{\sqrt{6}}A_2 = \sqrt{2B_{\pm 0}k} \pm \sqrt{2B_{00}}, \quad (9)$$

and choosing a negative sign and the upper experimental bound on B_{00} as well as an average experimental result for $B_{\pm 0}$ we obtain for $Br(\rho^+\rho^-)$ the result which coincides with the central value from (2).

Turning to our main subject – determination of the value of the angle alpha – we should look at CP-asymmetries, measured in $B_d(\bar{B}_d) \to \rho^+\rho^-$ decays[2)]:

$$\frac{\frac{dN(\bar{B}_d^0 \to \rho^+\rho^-)}{dt} - \frac{dN(B_d^0 \to \rho^+\rho^-)}{dt}}{\frac{dN(\bar{B}_d^0 \to \rho^+\rho^-)}{dt} + \frac{dN(B_d^0 \to \rho^+\rho^-)}{dt}} =$$
$$= -C_{\rho\rho}\cos(\Delta m \Delta t) + S_{\rho\rho}\sin(\Delta m \Delta t), \quad (10)$$

where

$$C = \frac{1-|\lambda|^2}{1+|\lambda|^2}, \quad S = \frac{2Im\lambda}{1+|\lambda|^2}, \quad \lambda = \frac{q}{p}\frac{\bar{A}_{\rho^+\rho^-}}{A_{\rho^+\rho^-}}, \quad (11)$$

factor $q/p = e^{-2i\beta}$ appears from $B_d - \bar{B}_d$ mixing. Here are the experimental data [6]:

$$C_{\rho^+\rho^-} = -0.23 \pm 0.24 \pm 0.14,$$
$$S_{\rho^+\rho^-} = -0.19 \pm 0.33 \pm 0.11. \quad (12)$$

The smallness of the penguin contribution is manifested in the smallness of the $C_{\rho^+\rho^-}$ value in comparison with 1 and we see that even the value $C_{\rho^+\rho^-} = 0$ (and $P/T = 0$) does not contradict the data. Neglecting penguin amplitude

$$S_{\rho^+\rho^-} = \sin 2\alpha = -0.19 \pm 0.35\ , \quad \alpha = 95^\circ \pm 10^\circ, \quad (13)$$

where theoretical systematic uncertainty due to nonzero P/T ratio is omitted.

3. $\boldsymbol{B \to \pi\pi}$**.** As it was demonstrated in paper [10] from the experimental data on averaged branching ratios and asymmetries of $B_d(\bar{B}_d) \to \pi^+\pi^-$, $\pi^0\pi^0$ and $B_u \to \pi^+\pi^0$ decays one can extract angle α relying only on isospin relations for decay amplitudes. However, as it was noticed in the same paper, one should expect large experimental uncertainties in the parameters describing the decays to the pair of neutral pions which will prevent direct determination of α with good accuracy. And this really happens. Unfortunately unlike the case of $\rho\rho$ decays, the branching ratio of $B_d(\bar{B}_d) \to \pi^0\pi^0$ is comparable to that to charged modes preventing bounding the penguin contributions to $B \to \pi\pi$ decays. Let us note that the data of Belle and BaBar on $\pi^+\pi^-$ and $\pi^\pm\pi^0$ branching ratios well agree, while their difference in $\pi^0\pi^0$ branching ratio is within two standard deviations. However, considerable $B_d(\bar{B}_d) \to \pi^0\pi^0$ branching ratio did not necessary mean that the penguin contribution is comparable to a tree one. In order to investigate how large it is let us look at experimental data on $C_{\pi^+\pi^-}$ and $S_{\pi^+\pi^-}$. Here the data of Belle and BaBar are in disagreement [11]:

	BaBar	Belle
$C_{\pi^+\pi^-}$	-0.09 ± 0.16	-0.56 ± 0.14
$S_{\pi^+\pi^-}$	-0.30 ± 0.17	-0.67 ± 0.17

(14)

According to BaBar data the tree amplitude dominates in the decay to $\pi^+\pi^-$ ($C_{\pi^+\pi^-} \approx 0$) while according to Belle this is not so ($C_{\pi^+\pi^-}$ differs from zero by four sigmas). That is why analyzing data two strategies were implied: the Belle and BaBar data were either averaged (see for example [12]) or disregarded.

We do not want to average data which contradict each other, neither we want to disregard them. Instead we suppose that BaBar data are correct, not Belle. As an argument in favor of this statement we can suggest the results of paper [13], where the contributions of QCD penguin diagrams to $B \to \rho\rho, \rho\pi, \pi\pi$ decays were found to be small. Neglecting them, from BaBar measurement of $S_{\pi^+\pi^-}$ we obtain:

$$\sin 2\alpha = S_{\pi^+\pi^-} = -0.30 \pm 0.17, \quad \alpha = 99 \pm 5^\circ. \quad (15)$$

5. $\boldsymbol{B \to \rho\pi}$**.** The time dependence of these decays is given by the following formula [14]:

$$\frac{dN(B_d(\bar{B}_d) \to \rho^\pm\pi^\mp)}{d\Delta t} = (1 \pm A_{CP}^{\rho\pi})e^{-\Delta t/\tau} \times$$
$$\times [1 - q(C_{\rho\pi} \pm \Delta C_{\rho\pi})\cos(\Delta m \Delta t) +$$
$$+ q(S_{\rho\pi} \pm \Delta S_{\rho\pi})\sin(\Delta m \Delta t)], \quad (16)$$

2) This simple formula is valid only for the decays to longitudinally polarized ρ-mesons; fortunately $f_L = 0.99 \pm 0.03 \pm 0.03$.

where $q = -1$ describes the B_d decay probability dependence on Δt (at $\Delta t = 0$ $\bar{B}_d$ decays) and $q = 1$ corresponds to $\bar{B}_d$ decay probability dependence on Δt (at $\Delta t = 0$ B_d decays);

$$A_{CP}^{\rho\pi} = \frac{|A^{+-}|^2 - |\bar{A}^{-+}|^2 + |\bar{A}^{+-}|^2 - |A^{-+}|^2}{|A^{+-}|^2 + |\bar{A}^{-+}|^2 + |\bar{A}^{+-}|^2 + |A^{-+}|^2},$$

$$C_{\rho\pi} \pm \Delta C_{\rho\pi} = \frac{|A^{\pm\mp}|^2 - |\bar{A}^{\pm\mp}|^2}{|A^{\pm\mp}|^2 + |\bar{A}^{\pm\mp}|^2}, \tag{17}$$

$$S_{\rho\pi} \pm \Delta S_{\rho\pi} = \frac{2\mathrm{Im}(\frac{q}{p}\frac{\bar{A}^{\pm\mp}}{A^{\pm\mp}})}{1 + \left|\frac{\bar{A}^{\pm\mp}}{A^{\pm\mp}}\right|^2}.$$

The amplitudes $A^{\pm\mp}$ describe B_d decays, $\bar{A}^{\pm\mp} - \bar{B}_d$ decays and the first sign is that of produced ρ-meson (for example A^{+-} is the amplitude of $B_d \to \rho^+\pi^-$ decay). It is convenient to write the amplitudes of $B \to \rho\pi$ decays as sums of tree and penguin contributions:

$$\begin{aligned} A^{+-} &= A_1 e^{i\delta_1} e^{i\gamma} + P_1 e^{i\delta_{P_1}}, \\ A^{-+} &= A_2 e^{i\delta_2} e^{i\gamma} + P_2 e^{i\delta_{P_2}}, \\ \bar{A}^{-+} &= A_1 e^{i\delta_1} e^{-i\gamma} + P_1 e^{i\delta_{P_1}}, \\ \bar{A}^{+-} &= A_2 e^{i\delta_2} e^{-i\gamma} + P_2 e^{i\delta_{P_2}}, \end{aligned} \tag{18}$$

where amplitude A_1 corresponds to ρ-meson produced from W-boson ($b \to u\rho^-$, $\bar{b} \to \bar{u}\rho^+$) and amplitude A_2 describes π-meson produced from W-boson ($b \to u\pi^-$, $\bar{b} \to \bar{u}\pi^+$). Amplitude P_1 corresponds to a penguin diagram in which a spectator quark is involved in π-meson production, while P_2 – to participation of a spectator quark in ρ-meson production.

If one can neglect penguin amplitudes, then formulas for physical observables are as follows:

$$A_{CP}^{\rho\pi} = C_{\rho\pi} = 0, \;\; \Delta C_{\rho\pi} = \frac{A_1^2 - A_2^2}{A_1^2 + A_2^2},$$

$$\Delta S_{\rho\pi} = \frac{2A_1 A_2}{A_1^2 + A_2^2}\sin(\delta_2 - \delta_1)\cos 2\alpha, \tag{19}$$

$$S_{\rho\pi} = \frac{2A_1 A_2}{A_1^2 + A_2^2}\cos(\delta_2 - \delta_1)\sin 2\alpha.$$

We use the results of fits of the Δt distributions obtained by Belle [15] and BaBar [16].

Let us start from the experimental measurements of parameter $C_{\rho\pi}$:

$$C_{\rho\pi} = \begin{array}{ll} 0.25 \pm 0.17, & \text{Belle}\;, \\ 0.34 \pm 0.12, & \text{BaBar}, \end{array} \tag{20}$$

and we see that while Belle result is compatible with the hypothesis that $P/T \ll 1$, BaBar result almost contradicts it. Waiting for more precise data let us go on supposing that penguin contribution is negligible. By the way, the data on A_{CP} confirms the smallness of penguin amplitude:

$$A_{CP}^{\rho\pi} = \begin{array}{ll} -0.16 \pm 0.10, & \text{Belle}, \\ -0.088 \pm 0.051, & \text{BaBar}. \end{array} \tag{21}$$

For $\Delta C_{\rho\pi}$ the result is:

$$\begin{gathered} \Delta C_{\rho\pi} = \begin{array}{ll} 0.38 \pm 0.18, & \text{Belle}, \\ 0.15 \pm 0.12, & \text{BaBar}, \end{array} \\ (\Delta C_{\rho\pi})_{\text{average}} = 0.22 \pm 0.10 \end{gathered} \tag{22}$$

and it means that $A_1 \approx 1.3 A_2$.

For $\Delta S_{\rho\pi}$ we have:

$$\begin{gathered} \Delta S_{\rho\pi} = \begin{array}{ll} -0.30 \pm 0.25, & \text{Belle}, \\ 0.22 \pm 0.15, & \text{BaBar}, \end{array} \\ (\Delta S_{\rho\pi})_{\text{average}} = 0.08 \pm 0.13 \end{gathered} \tag{23}$$

and its smallness means that $\sin(\delta_1 - \delta_2) \approx 0$ (another solution, $\cos 2\alpha \approx 0$, is unacceptable). It means that both δ_1 and δ_2 are small, or that they are close to each other. Substituting $\cos(\delta_1 - \delta_2) = 1$ into expression for $S_{\rho\pi}$ we obtain:

$$S_{\rho\pi} = \sqrt{1 - (\Delta C_{\rho\pi})^2}\sin 2\alpha, \tag{24}$$

while the experimental results are:

$$\begin{gathered} S_{\rho\pi} = \begin{array}{ll} -0.28 \pm 0.25, & \text{Belle}, \\ -0.10 \pm 0.15, & \text{BaBar}, \end{array} \\ (S_{\rho\pi})_{\text{average}} = -0.15 \pm 0.13 \end{gathered} \tag{25}$$

From (22), (24) and (25) we obtain:

$$\alpha = 94^\circ \pm 4^\circ. \tag{26}$$

5. Conclusion. Averaging the results for α presented in eqs.(13), (15) and (26) we obtain:

$$\alpha = 96^\circ \pm 3^\circ, \tag{27}$$

where only the experimental error is taken into account, while the theoretical uncertainty coming from the penguin diagrams is neglected. Let us note that result (27) is in good agreement with global CKM fit results:

$$\begin{gathered} \alpha_{\text{UTfit}}^{[7]} = 94^\circ \pm 8^\circ, \alpha_{\text{CKMfitter}}^{[8]} = 94^\circ \pm 10^\circ, \\ \alpha_{\text{AOV}}^{[9]} = 100^\circ \pm 5^\circ. \end{gathered} \tag{28}$$

How large can penguin contributions be in comparison with tree ones?

Strong interaction renormalization for beauty hadron weak decays is much smaller than for strange particles, because the masses of beauty hadrons are much closer to M_W in the logarithmic scale. Here are the results of NLO calculations from Table 1 of paper [17], where we take numbers which correspond to the modern value of $\alpha_s(M_Z) = 0.12$ ($\Lambda_4 = 280\,\mathrm{MeV}$):

$$c_2 = 1.14, \quad c_1 = -0.31, \quad c_3 = 0.016,$$
$$c_5 = 0.010, \quad c_4 = -0.036,$$
$$c_6 = -0.045 \qquad (29)$$

and we observe that the renormalization coefficients of penguin operators $(O_3 - O_6)$ do not exceed 4% of that for tree-level operator (O_2). Concerning the matrix elements one can definitely state that a large enhancement factor $m_\pi^2/(m_u + m_d)m_s \approx 10$ which makes penguins so important in explaining $\Delta I = 1/2$ rule in nonleptonic weak decays of strange particles is absent in beauty hadron decays, being substituted by $m_\pi^2/(m_u+m_d)m_b \approx \approx 1/2$.

A grain of salt comes from the CKM matrix elements which enhance the penguin amplitude with respect to the tree one by factor $(\rho^2 + \eta^2)^{-0.5} \approx 2$[3].

It follows that the theoretical uncertainty in (27) coming from the penguin diagrams can be close to the experimental one.

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