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Semiclassical theory of electron drag in strong magnetic fields

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We present a semiclassical theory for electron drag between two parallel two-dimensional electron systems in a strong magnetic field, which provides a transparent picture of the most salient qualitative features of anomalous drag phenomena observed in recent experiments, especially the striking sign reversal of drag at mismatched densities. The sign of the drag is determined by the curvature of the effective dispersion relation obeyed by the drift motion of the electrons in a smooth disorder potential. Localization plays a role in explaining activated low temperature behavior, but is not crucial for anomalous drag *per se*.

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Electron drag in double-layer two-dimensional electron systems has been established as a valuable probe of the electronic state within each layer, and also of interlayer interactions [1]. In a drag experiment a current is driven through one of the layers, which, via interlayer scattering, produces a drag voltage in the other layer. Usually the drag is positive in the sense that the drag and drive currents have the same direction, which leads to a compensating voltage opposite to the drive current. In the absence of a magnetic field, drag phenomena in 2D electron systems are well understood [2]. In that case the drag signal can be expressed in terms of the density response functions of each single layer, as long as the interlayer coupling is weak [3].

Drag experiments have also been performed in the presence of strong magnetic fields, where the formation of Landau levels plays a crucial role. Pronounced minima in the drag signal were observed at odd integer filling factors for magnetic fields far below the regime where spin splitting of Landau levels is seen in the intralayer resistivity [4, 5]. Completely unexpected was the discovery of a *negative* drag signal in case of a suitably chosen density mismatch between the two layers [6, 7]. Previous theoretical descriptions [2, 8] would predict positive drag at any filling factor. In more recent work a mechanism for a sign change of drag in strong magnetic fields was found [9], yielding however negative drag for equal densities in the two layers, unlike the experimental situation.

New hints and constraints for a theory result from a recent detailed experimental analysis of the temperature dependence of the drag resistivity by Muraki et al. [10]. At high temperatures, $k_BT \gg \hbar\omega_c$, where ω_c is the cyclotron frequency, the drag resistivity ρ_D follows the conventional T^2 behavior. It decreases with decreasing T down to scales well below $\hbar\omega_c$, but then, in the data shown for $\nu = 2n + 3/2$ with $n = 2, 3, 4, 5$, rises again to form a pronounced peak at temperatures one order of magnitude below $\hbar\omega_c$. At still lower T the drag resistivity decreases rapidly, consistent with exponentially activated behavior, $\rho_D \propto e^{-\Delta/k_BT}$. The ν-dependence of the activation energy Δ oscillates with minima near half-integer filling factors. The longitudinal intralayer resistivity ρ_{xx} also exhibits activated behavior at the lowest temperature, with an activation energy matching with Δ for those ν where both gaps could be extracted from the data. For a density mismatch $\delta\nu = 1$ between the two layers a low temperature peak with the same shape as for equal densities, but with opposite (that is negative) sign was observed.

Muraki et al. [10] interpreted their data in terms of particle-hole asymmetry and disorder induced localization. Landau levels are broadened by disorder, leading thus to a band of energies for each level. Anomalous drag is observed for parameters where the Landau level broadening is much smaller than $\hbar\omega_c$. The longitudinal resistivity ρ_{xx} of each layer as well as the drag resistivity ρ_D is due to electrons in extended states of the highest occupied Landau level. If the states with energies near the Fermi level ϵ_F are localized, charge carriers in extended states can be created only by thermal activation or scattering across the energy gap between ϵ_F and the nearest extended state energy level. This explains the observation of activated behavior of ρ_{xx} and ρ_D at low T. Muraki et al. [10] also realized that odd integer filling factors lead to a particle-hole symmetric occupation of the partially filled Landau level. Since particle-hole asymmetry is known to be crucial for drag in the absence of a magnetic field [11], it is indeed natural to relate the minima in ρ_D observed at odd integer ν to this symmetry. Judging from the experiments a necessary requirement for negative drag seems to be that the partially occupied Landau level in one layer be more, the other less than half-filled. This was noticed already by Feng et al. [6] and led them to speculate that electrons in a more than half-filled Landau level should acquire a hole-like dispersion relation due to disorder.

A comprehensive transport theory for the intralayer and drag resistivity in a strong magnetic field, which captures localization and extended states, is not available. In this work we present a simple semiclassical picture which explains why Landau quantized electrons moving in a smooth disorder potential behave effectively like band electrons with an electron-like dispersion in the lower and a hole-like dispersion in the upper half of a disorder broadened Landau level, and how this leads to the observed anomalous drag.

Let us first consider the relevant length and energy scales in the double-layer system. The distance between the layers varies from 30 to 120 nm. The disorder is due to remote donors, which leads to a smooth disorder potential with a correlation length ξ of order 50–100 nm. The Landau level broadening, which is related to the amplitude of the disorder potential, has been estimated from low-field Shubnikov-de-Haas oscillations to lie in the range 0.5–2 K [6, 12]. The magnetic fields at which drag minima at odd integer filling and/or negative drag at mismatched densities are observed vary over a relatively wide range from 0.1 T to 1 T for the cleanest samples, and up to 5 T for samples with a slightly reduced mobility. This corresponds to magnetic lengths l_B between 15 and 80 nm. Hence, ξ and l_B are generally of the same order of magnitude, which makes a quantitative theoretical analysis rather difficult. Since the anomalous drag phenomena are observed over a wide range of fields, one may hope that qualitative insight can be gained also by analysing limiting cases. For $\xi \ll l_B$ a treatment of disorder within the self-consistent Born approximation is possible [13]. This route was taken most recently by Gornyi et al. [14], who obtained negative contributions to the drag resistivity at mismatched densities in agreement with experiment. Localization is not captured by the Born approximation, and consequently the resistivities obey power-laws rather than activated behavior for $T \to 0$. For stronger magnetic fields, on the other hand, localization will become important, and a semiclassical approximation is a better starting point, which is the route we take here. Applying a criterion derived by Fogler et al. [15] we estimate that (classical) localization may set in already at 0.1 T.

We now discuss the semiclassical picture of electron states and drag on a qualitative level, ignoring spin splitting for simplicity. If the disorder potential varies smoothly, electron states lie essentially on contours of equal energy. These contours form closed loops, corresponding to localized states, except at a single energy ϵ_0 in the center of each Landau level, for which there is a percolating contour through the whole system [16]. If the Landau level broadening induced by the disorder potential and also $k_B T$ is much smaller than $\hbar\omega_c$, as is the case in the anomalous drag regime, all Landau levels except one are either fully occupied in the bulk of the whole sample or completely empty. At zero temperature and for $\epsilon_F < \epsilon_0$ the sample then consists of islands where the highest (partially) occupied Landau level is locally full, while it is locally empty in the rest of the sample; for $\epsilon_F > \epsilon_0$ the empty regions form islands. In reality, the percolating contour at the center of the Landau level is broadened to a percolation region for two reasons. First, electrons near the percolating contour can hop across saddlepoints from one closed loop to another, by using for a moment some of their cyclotron energy [15]. Second, electrons near the center of the Landau level can screen the disorder potential, such that the percolating equipotential line at ϵ_0 broadens to form equipotential terraces [17]. Hence, there is a region around the percolating path, where states are extended.

The percolating region forms a two-dimensional random network consisting of links and crossing points [18]. For simplicity we assume that the links are straight lines. The region near the link (except near the end points) can be parametrized by a cartesian coordinate system with a variable x following the equipotential lines parallel to the link and y for the transverse direction. The disorder potential varies essentially only in transverse direction, and can thus be represented by a function $U(y)$. In the Landau gauge $\mathbf{A} = (-By, 0)$ the Hamiltonian for electrons on the link is then translation invariant in x-direction and the Landau states ($\hbar = 1$ from now on)

$$\psi_{nk}(x,y) = C_n \, e^{-(y-l_B^2 k)^2/2l_B^2} \, H_n[(y - l_B^2 k)/l_B] \, e^{ikx} \tag{1}$$

are accurate solutions of the stationary Schrödinger equation. Here H_n is the n-th Hermite polynomial and C_n a normalization constant. The corresponding energy is simply

$$\epsilon_{nk} = (n + 1/2)\,\omega_c + U(l_B^2 k). \tag{2}$$

In the following we drop the Landau level index n, because only the highest occupied level is relevant. The quantum number k is the momentum associated with the translation invariance in x-direction. It is proportional to a transverse shift $y_0 = l_B^2 k$ of the wave function. The potential U lifts the degeneracy of the Landau levels and makes energies depend on the momentum along the link. The group velocity

$$v_k = \frac{d\epsilon_k}{dk} = l_B^2\, U'(y_0) \tag{3}$$

corresponds to the classical drift velocity of an electron in crossed electric and magnetic fields. In our simplified straight link approximation $U'(y_0)$ does not depend on x. In general it will depend slowly on the longitudinal coordinate, but near percolating paths away from crossing points it has a fixed sign, and hence the group velocity a fixed direction, that is the motion on the links is chiral.

The drag between two parallel layers is dominated by electrons in the percolating region, corresponding to states near the center of the Landau level, since electrons in deeper localized states are not easily dragged along. In the network picture, macroscopic drag arises as a sum of contributions from interlayer scattering processes between electrons in different links. If no current is imposed in the drive layer, both layers are in thermal equilibrium and the currents on the various links cancel each other on average. Now assume that a small finite current is switched on in the drive layer such that the electrons move predominantly in the direction of the positive x-axis (the electric current moving in the opposite direction). This means that the current on links oriented in the positive x-direction is typically larger than the current on links oriented in the negative x-direction. Interlayer scattering processes lead to momentum transfers between electrons in the drive and drag layer. The preferred direction of momentum transfers is such that the scattering processes tend to reduce the current in the drive layer, that is the interlayer interaction leads to friction. Now the crucial point is that electrons moving in the disorder potential are not necessarily accelerated by gaining momentum in the direction of their motion, or slowed down by loosing momentum (as for free electrons). The fastest electrons are those near the center of the Landau level: they have the highest group velocity on the links of the percolation network and they get most easily across the saddlepoints. Electrons in states below the Landau level center are thus accelerated by gaining some extra momentum in the direction of their motion, but electrons with an energy above ϵ_0 are pushed to still higher energy by adding momentum and are thus slowed down. To understand how negative drag arises, consider the situation with the highest occupied Landau level of the drive layer less than, and the one in the drag layer more than half filled. In that case the electrons in the drag layer receive momentum transfers with a predominantly positive x-component. As a consequence, electrons near the Fermi level of the drag layer are mostly slowed down if they move on links toward positive x-direction, and accelerated if they move on links in the negative x-direction, which is the opposite of what free electrons would do. If no net current is allowed to flow in the drag layer, an electric field is built up in negative x-direction which compensates the effective force generated by the scattering processes with the drive layer electrons. The drag signal is thus negative.

To substantiate the above qualitative picture, let us analyze the drag between two links using semiclassical transport theory. We first discuss parallel links in some detail, and then briefly the more general case of a finite angle between the links. For a fixed Landau level index, the electronic states in each link are fully labelled by their momentum k. The (non-equilibrium) occupation of states in the links is described by a distribution function $f_\alpha(k)$, where $\alpha = 1, 2$ labels the two layers. We consider the experimental standard setup where a small current flows through the "drive-layer" ($\alpha = 2$), generating a compensating drag voltage in the "drag-layer" ($\alpha = 1$). No net current is allowed to flow in the drag-layer. Under these conditions, the linear response of the drag layer to the drive current is determined by the linearized Boltzmann equation

$$\dot{k}_1 \cdot \frac{\partial f_1^0}{\partial k_1} = \left[\frac{\partial f_1}{\partial t}\right]_{\text{coll}} \tag{4}$$

with $\dot{k}_1 = -eE_1$, where E_1 is the electric field leading to the drag voltage. The interlayer collision term is given by

$$\begin{aligned}\left[\frac{\partial f_1}{\partial t}\right]^{12}_{\text{coll}} = -\int \frac{dk_2}{2\pi} \int \frac{dk_1'}{2\pi}\, W^{12}_{k_1,k_2;k_1',k_2'} \times \\ \times [\psi_1(k_1) + \psi_2(k_2) - \psi_1(k_1') - \psi_2(k_2')] \times \\ \times f_1^0(k_1) f_2^0(k_2)[1 - f_1^0(k_1')][1 - f_2^0(k_2')] \times \\ \times\, \delta(\epsilon_{k_1} + \epsilon_{k_2} - \epsilon_{k_1'} - \epsilon_{k_2'}),\end{aligned} \tag{5}$$

where $W^{12}_{k_1,k_2;k_1',k_2'}$ is the rate for a single interlayer scattering event $k_1 \to k_1'$, $k_2 \to k_2'$ and the deviation (from equilibrium) functions ψ_α are defined as usually by $f - f^0 = f^0(1 - f^0)\,\psi$. There is also an intralayer scattering term with a similar structure. The distribution function in the drive layer is obtained from the single layer Boltzmann equation as $\psi_2(k) = -\frac{\pi}{ev_{F2}T}\, j_2\, v_k$, where j_2 is the drive current and v_{F2} the equilibrium Fermi velocity. For weak interlayer coupling, the deviation function ψ_1 can be neglected in the interlayer collision term, because it would yield a contribution of order $(W^{12})^2$. To determine the relation between the drive current j_2 and the drag field E_1, we multiply the

Boltzmann equation by v_{k_1} and integrate over k_1. The left hand side yields $-e\int \frac{dk_1}{2\pi} f_1^{0\prime}(\epsilon_{k_1})\, v_{k_1}^2 E_1$ which tends to $\frac{e}{2\pi} v_{F1} E_1$ at low temperatures, where v_{F1} is the velocity at the Fermi level of the drive layer. Using the antisymmetry of the integrand of Eq. (5) under exchange of k_1 and k_1', the right hand side can be written as

$$-\frac{\pi j_2}{e v_{F2} T}\int \frac{dk_1}{2\pi}\int \frac{dk_2}{2\pi}\int \frac{dq}{2\pi} W^{12}_{k_1,k_2;k_1+q,k_2-q} \times \\ \times (v_{k_2}-v_{k_2-q})(v_{k_1}-v_{k_1+q}) \times \\ \times \frac{[f_1^0(\epsilon_{k_1})-f_1^0(\epsilon_{k_1+q})]\,[f_2^0(\epsilon_{k_2})-f_2^0(\epsilon_{k_2-q})]}{4\sinh^2[(\epsilon_{k_1+q}-\epsilon_{k_1})/2T]} \times \\ \times \delta(\epsilon_{k_1}+\epsilon_{k_2}-\epsilon_{k_1+q}-\epsilon_{k_2-q}). \quad (6)$$

The integrated intralayer scattering contributions cancel due to the condition of vanishing drag current. Note that the above integral shares several features with the general expression for the drag response function, as obtained from the Kubo formula [11, 19]. In particular it is symmetric in the layer indices, a property that depends crucially on the correct form of $\psi_2(k)$. For a simplified discussion of the most important points we assume that the interlayer scattering rate W^{12} depends only on momentum transfers q and energy transfers $\omega = \epsilon_{k_1+q}-\epsilon_{k_1}$, and that momentum transfers are so small that one can approximate $v_{k+q}-v_k$ by $q(dv_k/dk)$. The drag resistivity $\rho_D = -E_1/j_2$ can then be written as

$$\rho_D = \frac{1}{2\pi e^2}\frac{1}{v_{F1}v_{F2}}\frac{1}{m_1 m_2}\int_0^\infty dq\, q^2 \int_{-\infty}^{\infty}\frac{d\omega/T}{\sinh^2(\omega/2T)} \times \\ \times W^{12}(q,\omega)\,\mathrm{Im}\,\chi_1(q,\omega)\,\mathrm{Im}\,\chi_2(q,\omega), \quad (7)$$

where $\chi_\alpha(q,\omega)$ is the dynamical density correlation function in layer α, and the effective masses are given by the *curvature* of the dispersion relations at the Fermi level

$$\frac{1}{m_\alpha} = \left.\frac{dv_{k\alpha}}{dk}\right|_{k_{F\alpha}}. \quad (8)$$

The integral in Eq. (7) is always positive. The sign of ρ_D is thus given by the sign of the effective masses, that is by the curvature of the dispersion at the Fermi level. Negative drag is obtained when the dispersion in one layer is electron-like, and hole-like in the other. The drag vanishes if the Fermi level in one of the layers is at an inflection point of the dispersion. For a quadratic dispersion $\epsilon_k = k^2/2m$ one has $v_F = k_F/m$ and Eq. (7) reduces to a one-dimensional version of the well-known semiclassical result for drag between free electrons in two dimensions [2]. Returning to Eq. (6), it is not hard to generalize the above results on the sign of ρ_D allowing for larger momentum transfers q and general momentum dependences of W^{12}. Note that in our case of chiral electrons no backscattering is possible, unlike the situation in quantum wires [20].

For parallel links, energy and momentum conservation restrict the allowed scattering processes very strongly. At low temperatures, this leads to an exponential suppression of the drag between parallel links. This has nothing to do with the exponential suppression of drag observed in the experiments, since the links are generically not parallel. For non-parallel links the sum of momenta on the two links is no longer conserved in the scattering process. Hence scattering processes are suppressed much less at low temperatures. Computing the drag between non-parallel links from the linearized Boltzmann equation (a straightforward generalization of the above steps for parallel links) yields a quadratic temperature dependence at low T. The momentum transfers in the drag and drive links are however still correlated for non-parallel links, especially when the angle between the links is not very large, and the relative sign of the curvature of the dispersion in drive and drag layer, respectively, determines the sign of the drag. The average curvature vanishes for states in the center of the Landau level, while it is positive for energies below and negative for energies above ϵ_0. We thus understand the observation of negative drag when the Landau level in one layer is less than half-filled, and more than half-filled in the other.

Spin can be easily included in the above picture. Since the interlayer interaction is spin independent, one simply has to sum over the two spin species (up and down) in both drive and drag layer, taking the (exchange enhanced) Zeeman spin splitting of the Landau levels into account. If the Fermi level of one layer lies between the centers of the highest occupied Landau levels for up and down spins, respectively, positive and negative contributions to the drag partially cancel each other. The cancellation is complete due to particle-hole symmetry in the case of odd integer filling, as observed in experiment.

Within our semiclassical picture anomalous drag, especially negative drag, is suppressed at temperatures above the Landau level width, because then electron- and hole-like states within the highest occupied level are almost equally populated. This agrees with the results from the Born approximation [14], and also with experiments. For the low temperature asymptotics of the drag, the semiclassical theory yields two different types of behavior, depending on the filling. If the Fermi level does not hit any extended states (for either spin species), the drag should vanish exponentially for $T \to 0$, since thermal activation or scattering of electrons into extended states is then suppressed by an energy gap. By contrast, for a Fermi level within the extended states band (for at least one spin species) the gap vanishes and the drag obeys generally quadratic low temperature behavior, as obtained for the drag between non-parallel links. Within the Born approximation no localization occurs and the drag resistance always vanishes quadratically in the low temperature limit [14]. In high mobility samples localization is negligible at low magnetic fields, while an increasing amount of states gets localized at higher fields [15].

In summary, we have presented a semiclassical theory for electron drag between two parallel two-dimensional electron systems in a strong magnetic field, which provides a transparent picture of the most salient qualitative features of anomalous drag phenomena observed in recent experiments [6, 7, 10]. Localization plays a role in explaining activated low temperature behavior, but is not crucial for anomalous (especially negative) drag per se. A quantitative theory of drag which covers the whole range from low magnetic fields, where the Born approximation is valid [13, 14] to high fields, where localization becomes important, remains an important challenge for work to be done in the future.

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