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Infrared renormalons and the relations between the Gross–Llewellyn Smith and the Bjorken polarized and unpolarized sum rules

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It is demonstrated that the infrared renormalon calculus indicates that the QCD theoretical expressions for the Gross–Llewellyn Smith sum rule and for the Bjorken polarized and unpolarized ones contain an identical negative twist-4 $1/Q^2$ correction. This observation is supported by the consideration of the results of calculations of the corresponding twist-4 matrix elements. Together with the indication of the similarity of perturbative QCD contributions to these three sum rules, this observation leads to simple new theoretical relations between the Gross–Llewellyn Smith and Bjorken polarized and unpolarized sum rules in the energy region $Q^2 \geq 1\,\mathrm{GeV}^2$. The validity of this relation is checked using concrete experimental data for the Gross–Llewellyn Smith and Bjorken polarized sum rules.

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It is known that, in the traditionally used $\overline{\mathrm{MS}}$ scheme, the Borel image of the physical quantities in QCD contain infrared renormalons (IRR), namely the singularities on the positive axis of integration of this image in the complex plane of the Borel variable δ (for reviews see Refs. [1 – 3]). This related Borel integral can be defined as

$$D(a_s) = \int_0^\infty \exp(-\delta/\beta_0 a_s) B[D](\delta) d\delta, \qquad (1)$$

where $a_s = \alpha_s/4\pi$; α_s is the QCD coupling constant in the $\overline{\mathrm{MS}}$ scheme; $\beta_0 = (11/3)C_A - (4/3)T_f N_f$ is the first coefficient of the QCD β-function with $C_A = 3$, $T_f = 1/2$; and $B[D](\delta)$ is the Borel image of the physical quantity $D(a_s)$ under consideration.

From our point of view, the most important theoretical works, which pushed ahead the study of the applicability of the IRR calculus to the analysis of non-perturbative contributions to the characteristics of different processes are those of Ref. [4] and [5]. In particular, it was shown in Ref. [5] that since there exist a $1/Q^2$ non-perturbative correction of twist-4 in the characteristics of deep-inelastic scattering, the related Borel images should have the IRR pole at $\delta = 1$; this does not manifest itself in the expression for the Borel image of the Adler D-function of the e^+e^--annihilation process [4]. This crucial remark later generalized to the discussion of the Bjorken polarized sum rule in Ref. [6]. It should also be mentioned in passing that ultraviolet renormalons (UVR), associated with sign-alternating asymptotic perturbative contributions to the QCD perturbative series, manifest themselves in the Borel images as the poles at $\delta = -k$, where k are integer numbers.

The next problem, which arises in the process of applying the renormalon calculus to the analysis of the structure of both asymptotic perturbative contributions and non-perturbative corrections to physical quantities is the calculation of the corresponding Borel images $B[D](\delta)$. These calculations are usually performed using large-N_f expansion (where N_f is the number of quarks flavours). What is really calculated is the so-called one-renormalon-chain approximation to the Born expression for the physical quantity under consideration. Note that the renormalon chain is associated with the gluon propagator, dressed by a large number by quark bubbles insertions labelled by N_f. The contributions of this chain into the theoretical expression for physical quantities are gauge-invariant, but they do not reflect the whole picture of renormalon effects in QCD. The latter begin to manifest themselves after application of the naive non-abelianization ansatz [7] only, namely after the replacement $N_f \to -(3/2)\beta_0 = N_f - 33/2$ in the leading terms of the large-N_f expansion. This procedure transforms a large-N_f expansion into a large-β_0 expansion, also considered in some recent works [8], where it was associated with a BLM-type expansion [9].

In this Letter, definite new consequences of the relations between the Borel images, calculated in Ref. [10] and [11] for the Gross–Llewellyn Smith (GLS) sum rule of νN deep-inelastic scattering (DIS) [12], the Bjorken polarized (Bjp) sum rule [13] of polarized charged-lepton–nucleon DIS and the Bjorken unpolarized (Bjunp) sum rule [14] of νN DIS are discussed. In

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particular, it is argued that the values of the matrix elements of the twist-4 $1/Q^2$ corrections to the Bjp, Bjunp and GLS sum rule should have the same value. Together with the similarity in the behaviour of the perturbative corrections of all three sum rules, discussed in Ref. [11], this new observation allows us to write theoretical expressions to relate relate these a priori different physical quantities.

To be more precise, consider first the definitions of the sum rules we are interested in, taking into account twist-4 operators, evaluated in Ref. [15] in the case of GLS and Bjunp sum rules and in Ref. [16] for the Bjp sum rule:

$$\mathrm{GLS}(Q^2) = \frac{1}{2}\int_0^1 dx\left[F_3^{\nu n}(x,Q^2) + F_3^{\nu p}(x,Q^2)\right] = = 3C_{\mathrm{GLS}}(Q^2) - \frac{\langle\langle O_1\rangle\rangle}{Q^2}, \quad (2)$$

$$\mathrm{Bjp}(Q^2) = \int_0^1 dx\left[g_1^{lp}(x,Q^2) - g_1^{ln}(x,Q^2)\right] = = \frac{g_A}{6}C_{\mathrm{Bjp}}(Q^2) - \frac{\langle\langle O_2\rangle\rangle}{Q^2}, \quad (3)$$

$$\mathrm{Bjunp}(Q^2) = \int_0^1 dx\left[F_1^{\nu p}(x,Q^2) - F_1^{\nu n}(x,Q^2)\right] = = C_{\mathrm{Bjunp}}(Q^2) - \frac{\langle\langle O_3\rangle\rangle}{Q^2}. \quad (4)$$

where

$$C_{\mathrm{GLS}} = 1 - 4a_s - O(a_s^2), \quad (5)$$

$$C_{\mathrm{Bjp}} = 1 - 4a_s - O(a_s^2), \quad (6)$$

$$C_{\mathrm{Bjunp}} = 1 - \frac{8}{3}a_s - O(a_s^2). \quad (7)$$

The explicit expressions of the numerators of twist-4 contributions are defined as in the review [17], namely

$$\langle\langle O_1\rangle\rangle = \frac{8}{27}\langle\langle O^{\mathrm{s}}\rangle\rangle, \quad (8)$$

$$\langle\langle O_2\rangle\rangle = \langle\langle O^{\mathrm{NS}}_{p-n}\rangle\rangle, \quad (9)$$

$$\langle\langle O_3\rangle\rangle = \frac{8}{9}\langle\langle O^{\mathrm{NS}}\rangle\rangle, \quad (10)$$

where matrix elements on the r.h.s. of Eqs. (8)–(10) are known explicitly. Indeed, the matrix elements $\langle\langle O^{s}\rangle\rangle$ and $\langle\langle O^{NS}\rangle\rangle$ were calculated in Ref. [15], while the matrix element $\langle\langle O^{NS}_{p-n}\rangle\rangle$ is calculated in Ref. [16].

Let us return to renormalon calculus. It is known from Ref. [10] that the Borel images for the GLS and Bjp sum rules coincide and have the following form:

$$B[C_{\mathrm{GLS}}](\delta) = B[C_{\mathrm{Bjp}}](\delta) = -\frac{(3+\delta)\exp(5\delta/3)}{(1-\delta^2)(1-\delta^2/4)}. \quad (11)$$

It was shown in Ref. [11], that the Borel image $B[C_{\mathrm{Bjunp}}](\delta)$ of the Bjunp sum rule is closely related to Eq. (11), namely

$$B[C_{\mathrm{GLS}}](\delta) = \left(\frac{3+\delta}{2(1+\delta)}\right)B[C_{\mathrm{Bjunp}}(\delta)], \quad (12)$$

where

$$B[C_{\mathrm{Bjunp}}](\delta) = -\frac{2\exp(5\delta/3)}{(1-\delta)(1-\delta^2/4)}. \quad (13)$$

The consideration of Eqs. (11)–(13) allow the following conclusions to be made [11]: in the $\overline{\mathrm{MS}}$-scheme the asymptote of perturbative series for the GLS, Bjp and Bjunp sum rules is dominated by the first $\delta = 1$ IRR. Indeed, in the case of GLS and Bjp sum rules, the first UVR at $\delta = -1$, responsible for the sign-alternating perturbative QCD contribution to the asymptotic behaviour of the perturbative QCD series (for a more detailed discussion see Refs. [1, 2], is suppressed by a factor $(1/2)\exp(-10/3) = 0.018$, with respect to the dominant IRR at $\delta = 1$, responsible for sign-constant $n!$ growth of the perurbative coefficients for these two sum rules. Moreover, it is obvious from the results of Ref. [11] that in the case of the Bjunp sum rule the first UVR, created by the pole at $\delta = -1$, is absent and that the residues of the first IRR in the Borel images for the GLS, Bjunp and Bjp sum rules are the same. Therefore, it is possible to make the conclusion that the asymptotic perturbative QCD contributions will have an identical structure [11]. This fact is supported by the next-to-next-to-leading order studies of Refs. [18], performed with the help of the method of effective charges [19].

Now I will make the new conclusion, which follows from the results of Eqs. (11)–(13) and is related to twist-4 $O(1/Q^2)$ corrections. Since the IRR contribution of the first $\delta = 1$ IRR pole enter into the Borel images of the GLS, Bjp and Bjunp sum rules with the same negative residue (see (11)–(13)), the normalized to unity $O(1/Q^2)$ power correction in the GLS, Bjp and Bjunp sum rules, which are related to the $O(\Lambda^2/Q^2)$ ambiguities in the Borel integrals generated by the $\delta = 1$ IRR pole, should have the same sign and a similar numerical value. Indeed, the Λ^2/Q^2 IRR ambiguities may be coordinated with the definitions of the twist-4 contributions (see e.g. the reviews [1, 2]) and if they are the same the twist-4 $1/Q^2$ corrections should be the same also.

Let us check this statement, using concrete results of calculations of the numerical values of the matrix elements by means of three-point function QCD sum rules, namely $\langle\langle O^s\rangle\rangle = 0.33$ GeV^2, $\langle\langle O^{\rm NS}\rangle\rangle = 0.15$ GeV^2 obtained in Ref. [20]. As to the error bars, we propose to use 50% conservative uncertainty. This choice is in agreement with the error bar of the following value for twist-4 matrix element for the Bjorken polarized sum rule, namely $\langle\langle O^{\rm NS}_{p-n}\rangle\rangle = 0.09\pm0.006$ GeV^2, obtained in Ref. [21]. Taking now $g_A = 1.26$, we find the following numerical expressions for the sum rules of Eqs. (2)–(4):

$$\mathrm{GLS}(Q^2) = 3\left[1 - 4a_s - O(a_s^2) - \frac{0.098\ \mathrm{GeV}^2}{Q^2}\right], \quad (14)$$

$$\mathrm{Bjp}(Q^2) = \frac{g_A}{6}\left[1 - 4a_s - O(a_s^2) - \frac{0.071\ \mathrm{GeV}^2}{Q^2}\right], \quad (15)$$

$$\mathrm{Bjunp}(Q^2) = \left[1 - \frac{8}{3}a_s - O(a_s^2) - \frac{0.133\ \mathrm{GeV}^2}{Q^2}\right]. \quad (16)$$

One can see that, within theoretical uncertainties, typical of the application of three-point function QCD sum rules, the prediction of the IRR calculus is confirmed. So, indeed, the $O(1/Q^2)$ corrections normalized to unity, have the same negative sign and very close values.

In view of the fact that IRR calculus also indicates that known and still unknown perturbative QCD corrections to all three sum rules have comparable value as well [11], I now write the following relation between the three sum rules we are interested in, namely

$$\mathrm{Bjp}(Q^2) \approx (g_A/18)\mathrm{GLS}(Q^2) \approx (g_A/6)\mathrm{Bjunp}(Q^2). \quad (17)$$

These relations are valid in the energy region where it is possible to separate the twist-4 contribution from the twist-2 effects and $1/Q^4$ contributions are smaller than $1/Q^2$ effects. The above-mentioned features should hold at $Q^2 \geq 1$ GeV^2. Therefore, theoretical comparisons [22] of the expressions for the Bjp sum rule [23] and the GLS sum rule [24] within analytic approch [25] should possess the same feature.

Now I will consider whether the l.h.s. of the basic equation (17) is respected by experiment. I will use the values for the GLS sum rule, extracted in Ref. [26], at the energy points Q^2=1.26, 2, 3.16, 5.01, 7.94, 12.59 GeV^2. The results presented in Ref. [26] for these six energy points are GLS$(Q^2) \approx$ 2.39, 2.49, 2.55, 2.78, 2.82 and 2.80, where for simplicity we neglected both statistical and systematical uncertainties. The application of the l.h.s. of Eq. (17) gives one the following experimentally motivated values for the Bjp sum rule, namely Bjp(Q^2) $\approx$ 0.167, 0.168, 0.178, 0.195, 0.197, 0.196 for the same energy points, where again the contribution of statistical and systematical uncertainties are not taken into account.

It is interesting that the value of the Bjp sum rule extracted in Ref. [27] from the SLAC and SMC data is Bjp$(Q^2 = 3$ GeV$^2) = 0.177 \pm 0.018$ and, within existing error bars, do not contradict the value Bjp$(Q^2 = 3$ GeV$^2) = 0.164 \pm 0.011$ extracted in Ref. [28] on the basis of measurements at CERN and SLAC before 1997. It is rather inspiring that these results agree with the GLS sum rule value at $Q^2 = 3.16$ GeV^2.

At relatively high energies the SMC collaboration gives Bjp$(Q^2 = 10$ GeV$^2) = 0.195 \pm 0.029$ [29] which is consistent with high-energy results for the GLS sum rule GLS$(Q^2 = 12.59$ GeV$^2) \approx 0.196$ [26]. However, at low Q^2 the result Bjp$(Q^2 = 1.10$ GeV$^2) \approx 0.136$, extracted from CEBAF data in Ref. [30], is not consistent with the estimate Bjp$(Q^2 = 1.26$ GeV$^2) \approx 0.167$ extracted low-energy results GLS$(Q^2 = 1.26$ GeV$^2) \approx 2.39$ [26] with the help of Eq. (17). It may be interesting to clarify the origin of this disagreement, taking into account experimental uncertainties of the two independent analyses of νN DIS data and lN polarized DIS data. As the next step one could check the consistency of other experimental results for the GLS sum rule and Bjp sum rule with the IRR motivated expression of Eq. (17) for the energy points in the region 1 GeV$^2 \leq Q^2 \leq 5$ GeV^2 using NuTeV data for xF_3 structure function of νN DIS and rely on the appearance of the future Neutrino Factory, which may provide data for the Bjunp sum rule as well (for a discussion of this possibility, see Refs. [31, 32]).

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