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**В ЖУРНАЛ ЭКСПЕРИМЕНТАЛЬНОЙ**  
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**MONOPOLE CLASSICAL SOLUTIONS AND THE VACUUM  
 STRUCTURE IN LATTICE GAUGE THEORIES**

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Classical solutions corresponding to monopole-antimonopole pairs are found in  $3d$  and  $4d$   $SU(2)$  and  $U(1)$  lattice gauge theories. The stability of these solutions in various theories is studied.

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One of the most interesting and still unresolved problems is the structure of a (non-perturbative) vacuum in QCD and a confinement mechanism.

At the time being we have a number of different competing scenarios of confinement. One of the most promising approaches is a quasiclassical approach promoted by Polyakov [1] which assumes that in the treatment of infrared problems certain classical field configurations are of paramount importance. These classical field configurations ("pseudoparticles") are supposed to be stable, i.e. they correspond to local minima of the action and the interaction of these pseudoparticles creates a correlation length which corresponds to a new scale-confinement scale. This approach gives a clear field-theoretical prescription how to calculate analytically nonperturbative observables in the weak coupling region. In principle, this approach can be extended to the case of "quasistable" solutions.

Another very attractive approach is a topological (or monopole) mechanism of confinement [2]. This mechanism suggests that the QCD vacuum state behaves like a magnetic (dual) superconductor, abelian magnetic monopoles playing the role of Cooper pairs, at least, for the specially chosen ("maximally abelian") gauge [3]. At the time being this approach remains the most popular one in numerical studies in lattice QCD.

It is rather tempting to try to interpret lattice (abelian) monopoles as pseudoparticles (stable or quasistable). Recently the classical solutions have been found which correspond to Dirac sheet (i.e. flux tube) configurations [4]. It is the aim of this note to study the

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monopolelike ( $M\bar{M}$ ) abelian solutions of classical equations of motion in  $SU(2)$  and  $U(1)$  lattice gauge theories in  $d = 3$  and  $d = 4$  dimensions.

In what follows periodic boundary conditions are presumed. Lattice derivatives are:  $\partial_\mu f_x = f_{x+\mu} - f_x$ ;  $\bar{\partial}_\mu f_x = f_x - f_{x-\mu}$  and  $\bar{\nabla}_\mu(U) f_x = f_x - U_{x-\mu}^\dagger f_{x-\mu} U_{x-\mu}$ .  $\Delta = -\sum_\mu \partial_\mu \bar{\partial}_\mu$  and the lattice spacing is chosen to be unity.

### Abelian classical solutions

**Iterative procedure.** Classical equations of motion are

$$\sum_\mu \text{Im Tr} \left\{ \sigma^a \bar{\nabla}_\mu U_{x\mu\nu} \right\} = 0, \quad (1)$$

where  $U_{x\mu\nu} \in SU(2)$ . For abelian solutions  $U_{x\mu\nu} = \exp(i\sigma_3 \theta_{x\mu\nu})$  eq. (1) becomes

$$\sum_\nu \bar{\partial}_\nu \sin \theta_{x\mu\nu} = 0. \quad (2)$$

Let us represent the plaquette angle  $\theta_{x\mu\nu}$  in the form

$$\theta_{x\mu\nu} = \bar{\theta}_{x\mu\nu} + 2\pi \cdot n_{x\mu\nu}; \quad -\pi < \bar{\theta}_{x\mu\nu} \leq \pi, \quad (3)$$

and  $n_{x\mu\nu} = -n_{x\nu\mu}$  are integer numbers. The classical equations of motion (2) can be represented in the form

$$\sum_\nu \bar{\partial}_\nu \theta_{x\mu\nu} = F_{x\mu}(\theta), \quad (4)$$

where

$$F_{x\mu}(\theta) \equiv \sum_\nu \bar{\partial}_\nu (\theta_{x\mu\nu} - \sin \theta_{x\mu\nu}) \quad (5)$$

and  $\sum_\mu \bar{\partial}_\mu F_{x\mu} = 0$ . For any given configuration  $\{n_{x\mu\nu}\}$  these equations can be solved iteratively

$$\theta_{x\mu}^{(1)} \rightarrow \theta_{x\mu}^{(2)} \rightarrow \dots \rightarrow \theta_{x\mu}^{(k)} \rightarrow \dots, \quad (6)$$

where

$$\sum_\nu \bar{\partial}_\nu \theta_{x\mu\nu}^{(k+1)} = F_{x\mu}(\theta^{(k)}); \quad k = 1; 2; \dots \quad (7)$$

and

$$\sum_\nu \bar{\partial}_\nu \theta_{x\mu\nu}^{(1)} = 2\pi \sum_\nu \bar{\partial}_\nu n_{x\mu\nu}. \quad (8)$$

In the Lorentz gauge  $\sum_\mu \bar{\partial}_\mu \theta_{x\mu} = 0$  eq.'s (7), (8) are equivalent to

$$\Delta \theta_{x\mu}^{(k+1)} = J_{x\mu}^{(k)}; \quad k = 0; 1; \dots, \quad (9)$$

where  $J_{x\mu}^{(0)} = 2\pi \sum_\nu \bar{\partial}_\nu n_{x\mu\nu}$  and  $J_{x\mu}^{(k)} = F_{x\mu}(\theta^{(k)})$  at  $k \geq 1$ . Evidently,

$$\sum_\mu \bar{\partial}_\mu J_{x\mu}^{(k)} = 0; \quad \sum_x J_{x\mu}^{(k)} = 0. \quad (10)$$

Defining the propagator

$$G_{x;y} = \frac{1}{V} \sum_{q \neq 0} \frac{e^{iq(x-y)}}{\mathcal{K}^2}; \quad \mathcal{K}^2 = \sum_\mu 4 \sin^2 \frac{q_\mu}{2}, \quad (11)$$

one can easily find solutions of eq.'s (9):

$$\theta_{x\mu}^{(1)} = 2\pi \sum_{y\nu} G_{x;y} \cdot \bar{\partial}_\nu n_{x\mu\nu}; \quad (12)$$

$$\theta_{x\mu}^{(k+1)} = \sum_y G_{x;y} \cdot J_{y\mu}^{(k)}; \quad \sum_\mu \bar{\delta}_\mu \theta_{x\mu}^{(k+1)} = 0, \quad (13)$$

and  $\sum_x \theta_{x\mu}^{(k+1)} = 0$ .

The results of iterative solution can be summarized as follows.

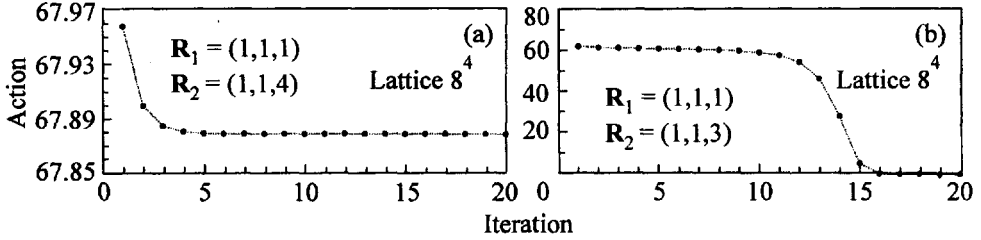


Fig.1. Iterative solution of the classical equations

1. The convergence of this iterative procedure is very fast and becomes even faster with increasing the distance between monopole and antimonopole. As an example in Fig.1a it is shown the dependence of the action on the number of iterations on the  $8^4$  lattice where  $\mathbf{R}_1$  and  $\mathbf{R}_2$  are positions of the static monopole and antimonopole, respectively. In fact, the first approximation  $\theta_{x\mu}^{(1)}$  as given in eq.(12) is a very good approximation to the exact solution.

2. There are *no* solutions when monopole and antimonopole are too close to each other. As an example, in Fig.1b one can see the dependence of the action on the number of iteration steps when  $\mathbf{R}_2 - \mathbf{R}_1 = (0, 0, 2)$ .

3. In four dimensions only *static* (i.e. three-dimensional) solutions have been found.

### Stability.

The question of stability of the classical solution  $U_{x\mu}^{cl}$  is the question of the eigenvalues  $\lambda_j$  of the matrix  $L_{xy;\mu\nu}^{ab}$  where

$$S(e^{i\delta\theta} U^{cl}) = S_{cl} + \sum_{abxy\mu\nu} \delta\theta_{x\mu}^a L_{xy;\mu\nu}^{ab} \delta\theta_{y\nu}^b + \dots, \quad (14)$$

where  $S_{cl} = S(U^{cl})$  and  $\delta\theta_{x\mu}^a$  are infinitesimal variations of the gauge field. A solution  $U_{x\mu}^{cl}$  is stable if all  $\lambda_j \geq 0$ . However, a solution can be unstable but “quasistable” if, say, only one eigenvalue is negative:  $\lambda_1 < 0$ ,  $\lambda_j \geq 0$ ,  $j \neq 1$ . A cooling history of such configuration could have demonstrated an approximate plateau. If it could have been a case, one could extend, in principle, Polyakov’s approach to the case of quasistable solutions.

It is rather easy to show that in the case of  $U(1)$  theory  $M\bar{M}$  solutions are *stable*, i.e. correspond to local minima of the action. Therefore, Polyakov’s approach based on the  $M\bar{M}$  classical solutions is expected to describe confinement and pseudoparticles are (anti)monopoles.

Stability of  $M\bar{M}$ -classical solutions in  $SU(2)$  theory has been studied numerically. To this purpose every classical  $M\bar{M}$ -configurations has been (slightly) heated and then a (soft) cooling procedure has been used. In Fig.2 one can see a typical cooling history of such configuration. The classical action  $S_{cl}$  corresponding to the  $M\bar{M}$ -configuration is  $\sim 130$ . Therefore,  $M\bar{M}$ -classical solution looks absolutely *unstable*.

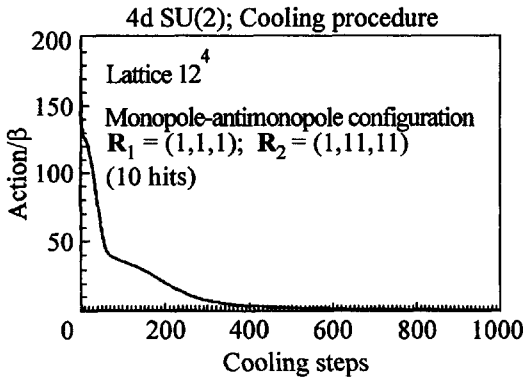


Fig.2

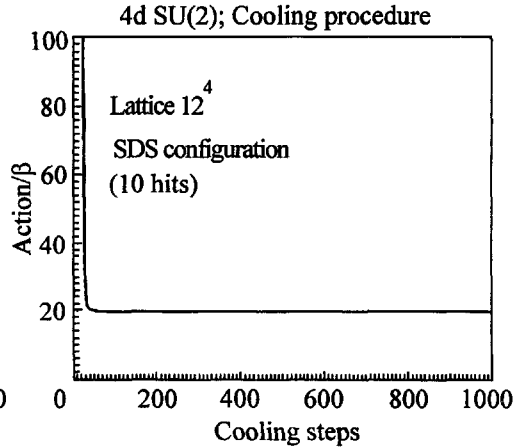


Fig.3

It is interesting to compare the stability of monopole classical solutions with that of Dirac sheet (flux tube) solutions. In Fig.3 one can see a typical cooling of the heated single Dirac sheet (SDS) [4] in  $SU(2)$  theory. Parameters of the cooling have been chosen the same for all configurations. In fact, it is also unstable. However, a strong plateau permits to define this configuration as a *quasistable*.

### Summary and discussions.

Classical solutions corresponding to monopole-antimonopole pairs in  $3d$  and  $4d$   $SU(2)$  and (compact)  $U(1)$  lattice gauge theories have been found.

In the case of  $3d$  and  $4d$   $U(1)$  theories these monopole-antimonopole classical solutions ( $M\bar{M}$ -pseudoparticles) are *stable*, i.e. correspond to local minima of the action. Therefore, the quasiclassical approach has chance to be successful.

In contrast, in  $SU(2)$  theory ( $d = 3$  and  $d = 4$ )  $M\bar{M}$ -classical solutions are completely unstable. At the moment it is not clear if Polyakov's (quasiclassical) approach can be applied to nonabelian theories (at least, with monopoles as pseudoparticles). It is very probable that the vacuum in the (compact)  $U(1)$  theory is a rather poor model of the vacuum in  $SU(2)$  theory.

It is interesting to note that the Dirac sheet (i.e. flux tube) solutions are quasistable in  $SU(2)$  theories (for  $d = 3$  and  $d = 4$ ). This observation could be interesting in view of the famous spaghetti vacuum picture where the color magnetic quantum liquid state is a superposition of flux-tubes states (Copenhagen vacuum) [5]. However, the relevance of this scenario still needs a further confirmation.

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