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HOW TO CREATE ALICE STRING (HALF-QUANTUM VORTEX) IN A VECTOR BOSE – EINSTEIN CONDENSATE

U.Leonhardt[+*], *G.E.Volovik*[△□]

[+]*School of Physics and Astronomy, University of St. Andrews, KY16 9SS, Scotland*

[*]*Department of Physics, Royal Institute of Technology 100 44 Stockholm, Sweden*

[△]*Low Temperature Laboratory, Helsinki University of Technology 02015 Espoo, Finland*

[□]*Landau Institute for Theoretical Physics RAS, 117334 Moscow, Russia*

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We suggest a procedure how to prepare the vortex with $N = 1/2$ winding number – the counterpart of the Alice string – in Bose – Einstein condensates.

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Vortices with fractional winding number can exist in different condensed matter systems, see review paper [1]. Observation of atomic Bose-condensates with multi-component order parameter in laser manipulated traps opens the possibility to create half-quantum vortices there. We discuss the $N = 1/2$ vortices in the Bose-condensate with the hyperfine spin $F = 1$, and also in the mixture of two Bose-condensates.

The order parameter of $F = 1$ Bose-condensate consists of 3 complex components according to the number of the projections $M = (+1, 0, -1)$. These components can be organized to form the complex vector $\mathbf{a}$:

$$\Psi_\nu = \begin{pmatrix} \Psi_{+1} \\ \Psi_0 \\ \Psi_{-1} \end{pmatrix} = \begin{pmatrix} (a_x + ia_y)/\sqrt{2} \\ a_z \\ (a_x - ia_y)/\sqrt{2} \end{pmatrix} . \qquad (1)$$

There are two symmetrically distinct phases of the $F = 1$ Bose-condensates:

(i) The chiral or ferromagnetic state occurs when the scattering length a_2 in the scattering channel of two atoms with the total spin 2 is less than that with the total spin zero, $a_2 < a_0$ [2, 3]. It is described by the complex vector

$$\mathbf{a} = f(\hat{\mathbf{m}} + i\hat{\mathbf{n}}), \qquad (2)$$

where $\hat{\mathbf{m}}$ and $\hat{\mathbf{n}}$ are mutually orthogonal unit vector with $\hat{\mathbf{l}} = \hat{\mathbf{m}} \times \hat{\mathbf{n}}$ being the direction of the spontaneous momentum $\mathbf{F}$ of the Bose condensate, which violates the parity and time reversal symmetry; f is the amplitude of the order parameter.

(ii) The polar or superfluid nematic state, which occurs for $a_2 > a_0$, is described by the real vector up to the phase factor

$$\mathbf{a} = f\hat{\mathbf{d}}e^{i\Phi}, \qquad (3)$$

where $\hat{\mathbf{d}}$ is a real unit vector. The direction of the vector $\hat{\mathbf{d}}$ can be inverted by the change of the phase $\Phi \to \Phi + \pi$. That is why phase-insensitive properties of the polar state are also insensitive to the reversal of the direction of $\hat{\mathbf{d}}$. In this respect $\hat{\mathbf{d}}$ is similar to the director in nematic liquid crystals.

The chiral state (i) corresponds to the orbital part of the matrix order parameter in superfluid ^{3}He-A, while the nematic state (ii) corresponds to the spin part of the same ^{3}He-A order parameter. The order parameter matrix of ^{3}He-A is the product of two vector order parameters: $A_{\alpha k} \propto a_\alpha^{nematic} a_k^{chiral}$. That is why each of the two states shares some definite properties of superfluid ^{3}He-A.

In particular the chiral state (i) displays continuous vorticity [2, 3], which was heavily investigated in superfluid ^{3}He-A (see [4] and reviews [5, 6]). An isolated continuous vortex is the so called Anderson-Toulouse-Chechetkin vortex. The smooth core of the vortex represents the skyrmion, in which the $\hat{\mathbf{l}}$-vector sweeps the whole unit sphere. Outside the soft core the $\hat{\mathbf{l}}$-vector is uniform, while the order parameter phase has finite winding. In ^{3}He-A and thus in the $F = 1$ Bose-condensate too, it is the 4π winding around the soft core, i.e. the continuous vortex has winding number $N = 2$. This continuous vortex can be also represented [6, 5] as a pair of the so called continuous Mermin-Ho vortices [7], each having the winding number $N = 1$. The $\hat{\mathbf{l}}$-vector in the Mermin-Ho vortex covers only half of a unit sphere and thus is not uniform outside the soft core. Such a half-skyrmion is also called the meron. An optical method to create the meron – the Mermin-Ho vortex – in the $F = 1$ Bose-condensate has been recently discussed in Ref. [8].

For the spin-1/2 Bose-condensates the order parameter is a spinor, which represents the "half of the vector". That is why the continuous Anderson-Toulouse-Chechetkin vortex (in which the $\hat{\mathbf{l}}$ sweeps the whole unit sphere) has twice less winding number in such condensates, i.e. the skyrmion is the $N = 1$ continuous vortex [9]. The spinorial order parameter is the counterpart of the order parameter in the Standard Model of the electroweak interactions, which is the spinor Higgs field transforming under $SU(2)$ symmetry group. That is why the $N = 1$ continuous vortex in the spin-1/2 Bose-condensates simulates the continuous electroweak string in the Standard Model. The Higgs field in the continuous electroweak string (and thus the $N = 1$ continuous vortex in the spin-1/2 Bose-condensate) has the following distribution of the order parameter [10, 11]:

$$\begin{pmatrix} \Psi_\uparrow \\ \Psi_\downarrow \end{pmatrix} = f \begin{pmatrix} e^{i\phi} \cos \frac{\theta(r)}{2} \\ \sin \frac{\theta(r)}{2} \end{pmatrix}, \quad \hat{\mathbf{l}} = \hat{\mathbf{z}} \cos\theta(r) + \hat{\mathbf{r}} \sin\theta(r). \tag{4}$$

Here (z, r, ϕ) are coordinates of cylindrical system; $\theta(0) = \pi$; and $\theta(\infty) = 0$. Note that the meron configuration in such system, with $\theta(0) = \pi$ and $\theta(\infty) = \pi/2$, would have $N = 1/2$ winding number.

The $N = 1$ vortices with the order parameter described by Eq.(4), have been recently generated in the Bose-condensate with two internal levels [12], following the proposal elaborated in Ref.[13]. Though these two internal levels are not related by an exact $SU(2)$ symmetry, under some conditions there is an approximate $SU(2)$ symmetry, and the $N = 1$ vortex does represent a skyrmion. This vortex has a smooth (soft) core, which size is essentially larger than that of the conventional vortex core which has the dimension of order of the coherence length. Such enhancement of the core size allowed for the observation of the $N = 1$ vortex-skyrmion by optical methods [12]. From the Eq.(4)

it follows that this continuous $N = 1$ vortex can be also represented as the vortex in the $|\uparrow\rangle$ component whose core is filled by the $|\downarrow\rangle$ component.

The nematic state (ii) may contain a no less exotic topological object – the topologically stable $N = 1/2$ vortex [14] – which still has avoided experimental identification in superfluid ^{3}He-A. The $N = 1/2$ vortex is a combination of the π-vortex in the phase Φ and π-disclination in the nematic order parameter vector $\hat{\mathbf{d}}$:

$$\mathbf{a} = f(r)\left(\hat{\mathbf{x}}\cos\frac{\phi}{2} + \hat{\mathbf{y}}\sin\frac{\phi}{2}\right)e^{i\phi/2}. \tag{5}$$

The change of the sign of the vector $\hat{\mathbf{d}}$ when circumscribing around the core is compensated by the change of sign of the exponent $e^{i\Phi} = e^{i\phi/2}$, so that the whole order parameter is smoothly connected after circumnavigating.

This $N = 1/2$ vortex is the counterpart of the so called Alice string considered in particle physics[15]: a particle which moves around an Alice string flips its charge or parity. In a similar manner a quasiparticle adiabatically moving around the vortex in ^{3}He-A or in the Bose-condensate with $F = 1$ in nematic state (ii) finds its spin or its momentum projection M reversed with respect to the fixed environment. This is because the $\hat{\mathbf{d}}$-vector, which plays the role of the quantization axis for the spin of a quasiparticle, rotates by π around the vortex. As a consequence, several phenomena (e.g. global Aharonov – Bohm effect) discussed in the particle physics literature [16, 17] correspond to effects in ^{3}He-A physics [18, 6], which can be extended to the atomic Bose condensates.

In high-temperature superconductors with a nontrivial order parameter, the half-quantum vortex was identified as being attached to the intersection line of three grain boundaries [19], as suggested in [20]. This $N = 1/2$ vortex has been observed via the fractional magnetic flux it generates.

In the spin projection representation the order parameter asymptote in the $N = 1/2$ vortex in the nematic phase is

$$\Psi_\nu \propto e^{i\phi/2}\begin{pmatrix} e^{i\phi/2} \\ 0 \\ e^{-i\phi/2} \end{pmatrix} = \begin{pmatrix} e^{i\phi} \\ 0 \\ 1 \end{pmatrix}. \tag{6}$$

This means that the $N = 1/2$ vortex can be represented as a vortex in the spin-up component $|\uparrow\rangle$, while the spin-down component $|\downarrow\rangle$ is vortex-free. Such representation of the half-quantum vortex in terms of the regular $N = 1$ vortex in one of the components of the order parameter occurs also in the ^{3}He-A. The general form of the order parameter in the half-quantum vortex, which includes also the core structure, is

$$\Psi_\nu = \begin{pmatrix} f_1(r)e^{i\phi} \\ 0 \\ f_2(r) \end{pmatrix}, \quad f_1(0) = 0, \quad |f_1(\infty)| = |f_2(\infty)|. \tag{7}$$

Note that, since the $M = 0$ component in Eq.(6) is zero, the half-quantum vortex can be generated also in the Bose-condensate with two internal degrees of freedom, explored in Ref. [12]. The necessary condition for that is that in the equilibrium state of such condensate both components must be equally populated. This is required by the asymptote of Eq.(7), where both components have the same amplitude. If the amplitudes are not exactly equal, the half-quantum vortex acquires the tail in the form of the domain wall terminating on the vortex. The same happens in ^{3}He-A where the half-quantum vortex is the termination line of the topological soliton.

The Eq.(6) may suggest a way to generate a half-quantum vortex in an alkali Bose – Einstein condensate, simply be combining the successful idea [12, 13] for producing skyrmions with the proposal [21] for making scalar vortices by the effect of light forces. Let us start from the homogeneous state

$$\Psi_\nu(\text{initial}) = f \begin{pmatrix} e^{i\alpha} \\ 0 \\ e^{i\beta} \end{pmatrix} , \qquad (8)$$

which corresponds to the phase $\Phi = (\alpha + \beta)/2$ and the nematic vector $\hat{\mathbf{d}} = \hat{\mathbf{x}} \cos(\alpha - \beta)/2 + \hat{\mathbf{y}} \sin(\alpha - \beta)/2$. A light spot shall illuminate the condensate with an intensity distribution I that draws a half-quantized vortex,

$$I = I_0 e^{i\phi/2} \ . \qquad (9)$$

The light should be a short pulse and it should be non-resonant with respect to atomic transition frequencies. Simultaneously, uniform microwave radiation shall penetrate the condensate. The radiation should be far-detuned from the transition frequency between the spin components $|\downarrow\rangle$ and $|\uparrow\rangle$ of the condensate such that it only causes shifts in the relative phases between $|\downarrow\rangle$ and $|\uparrow\rangle$ and no population transfer. The light spot will imprint an optical mask onto the homogeneous microwave field, due to the optical Stark effect. Therefore, the generated relative phase shift will follow the half-quantum vortex drawn by the light spot. Simultaneously, the condensate gains an overall scalar phase factor, caused by the intensity kick of the light. This factor should exactly compensate the phase mismatch between the components that is left from the optically assisted microwave effect. Of course, for this the intensities of light and microwave radiation should be properly adjusted, but this could be arranged. In this simple way, an Alice string can be created in a multicomponent Bose – Einstein condensate of alkali atoms.

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