Pis'ma v ZhETF, vol.72, iss.6, pp.420 - 425

ATOMIC RECOIL EFFECTS IN SLOW LIGHT PROPAGATION

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Submitted 17 July 2000

Resubmitted 21 August 2000

We theoretically investigate the effect of atomic recoil on the propagation of ultra-slow light pulses through a coherently driven Bose – Einstein condensed gas. For a sample at rest, the group velocity of the light pulse is the sum of the group velocity that one would observe in the absence of mechanical effects (infinite mass limit) plus the velocity of the recoiling atoms (light dragging effect). We predict that atomic recoil may give rise to a lower bound for the observable group velocities as well as to pulse propagation at negative group velocities without appreciable absorption.

PACS: 03.75.Fi, 42.50.-p

Recent experiments [1, 2] have demonstrated a reduction in the group velocity of light down to values as low as 17 m/s in coherently driven atomic samples. This has been achieved by tuning the pulse frequency in the electromagnetically induced transparency (EIT) window of an optically dressed three-level atomic gas, where quantum coherence between two lower levels gives rise to a vanishing absorption along with a very steep dispersion [3]. Further improvements of the experimental set-up are expected [1] to enable one to reach group velocities as small as the atomic–recoil velocity. In this regime, recoil is expected to play an important role in the propagation of the pulse.

In the present letter we provide a detailed derivation of the group velocity of light pulses in a coherently driven Bose-Einstein condensed (BEC) atomic sample [4] when the effect of the atomic recoil is taken into account. Apart from the well-known light-dragging effect in uniformly moving dielectrics [5], we show that the group velocity of slow light in a sample at rest under appropriate EIT conditions is given by the group velocity in the infinite mass approximation plus the velocity of the atoms which recoil following the optical process itself. Such a dragging effect imposes a lower bound to the group velocity that can be observed in typical configurations of experimental interest. For a specific level scheme and a geometry in which atoms do recoil in directions opposite to the probe wavevector, light propagation at negative group velocities without appreciable absorption is also possible. Finally, we show that the group velocity of a light pulse is not affected by atom-atom interactions at the mean-field level.

We consider a cloud of BEC atoms [4] in a three-level Λ-type configuration as shown in Figure. All atoms are initially in the ground state $|g\rangle$ and the optical transition between the metastable $|m\rangle$ and excited state $|e\rangle$ is dressed by a nearly resonant *coupling* c.w. laser beam of amplitude $E_c(\mathbf{x})$ and frequency $\omega_c \simeq \omega_e - \omega_m$. A weak probe pulse at

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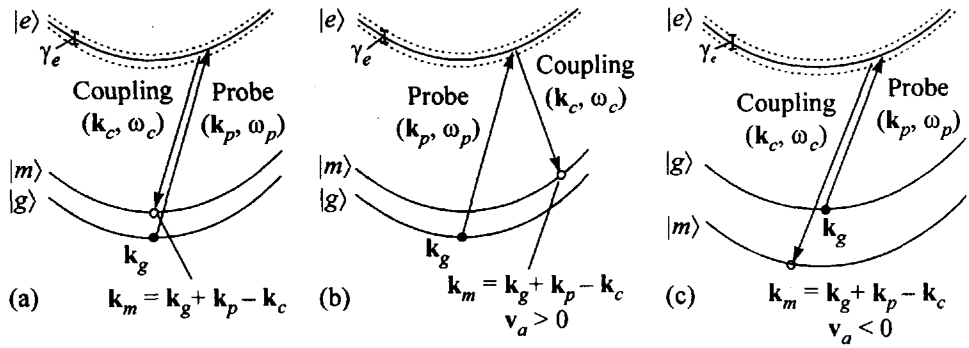

Level scheme and optical processes for co–propagating (a) and counter–propagating (b) probe and coupling beams. Proposed arrangement for obtaining negative group velocities (c)

frequency ω_p nearly resonant with the other optical transition between the ground $|g\rangle$ and the excited state $|e\rangle$ also propagates through the system. When the decay rate of the metastable level m is much smaller than the decay rate of the level e, the probe field experiences EIT with a narrow absorption dip and a very steep dispersion at frequencies around $\omega_p = \omega_c + \omega_m - \omega_g$ [3]. In a second-quantized formalism, the Hamiltonian of the system can be written as

$$\mathcal{H} = \sum_{i=\{g,e,b\}} \int d^3\mathbf{x}\, \hat{\psi}_i^\dagger(\mathbf{x}) \left(\hbar\omega_i + V_i(\mathbf{x}) - \frac{\hbar^2}{2m}\nabla^2 \right) \hat{\psi}_i(\mathbf{x}) + \\ - \left(d_p E_p(\mathbf{x},t) \hat{\psi}_e^\dagger(\mathbf{x}) \hat{\psi}_g(\mathbf{x}) + d_c E_c(\mathbf{x},t) \hat{\psi}_e^\dagger(\mathbf{x}) \hat{\psi}_m(\mathbf{x}) + \text{h.c.} \right). \tag{1}$$

The first two terms describe the internal structure of the atoms, their kinetic and potential energy while the last terms describe the coupling of the two laser beams to the atoms. The effects of the atom-atom interactions will be discussed later. Both the spontaneous emission from the excited state $|e\rangle$ and the decoherence of the two lower $|m\rangle$ and $|g\rangle$ states are responsible for a loss of atoms from the condensate and can therefore be modelled by loss terms in the equations of motion for the three-component macroscopic wavefunction ψ_i of the Bose condensate:

$$i\hbar\frac{\partial}{\partial t}\psi_g(\mathbf{x},t) = \left[-\frac{\hbar^2\nabla^2}{2m} + V_g(\mathbf{x}) + \hbar\omega_g \right] \psi_g(\mathbf{x},t) + \\ - d_p^* E_p^*(\mathbf{x},t)\psi_e(\mathbf{x},t) \tag{2}$$

$$i\hbar\frac{\partial}{\partial t}\psi_e(\mathbf{x},t) = \left[-\frac{\hbar^2\nabla^2}{2m} + V_e(\mathbf{x}) + \hbar(\omega_e - i\gamma_e) \right] \psi_e(\mathbf{x},t) + \\ - d_p E_p(\mathbf{x},t)\psi_g(\mathbf{x},t) - d_c E_c(\mathbf{x},t)\psi_m(\mathbf{x},t) \tag{3}$$

$$i\hbar\frac{\partial}{\partial t}\psi_m(\mathbf{x},t) = \left[-\frac{\hbar^2\nabla^2}{2m} + V_m(\mathbf{x}) + \hbar(\omega_m - i\gamma_m) \right] \psi_m(\mathbf{x},t) + \\ - d_c^* E_c^*(\mathbf{x},t)\psi_e(\mathbf{x},t). \tag{4}$$

In the following we shall assume that all atoms are initially condensed in the ground state and that the probe pulse is very weak; in this case, the probe will not essentially affect the (macroscopic) condensate and the optical polarization due to the non-condensed atoms generated by incoherent processes can be safely neglected. The effect of the coupling beam alone on the condensed atoms is in fact negligible for any value of its intensity since its frequency is off-resonance from any optical transition starting from the ground level. For small atomic densities N_o ($N_o/|\mathbf{k}_p|^3 \ll 1$), we can also assume that the photonic mode structure inside the condensed cloud is not strongly modified compared to the free-space one so that the excited state spontaneous emission rate γ_e can be taken to be the same as in free-space [6]. In the spirit of a semiclassical local density approximation [4, 7], we shall also neglect the effect of the external trapping potential and we consider the probe and coupling beams as monochromatic plane waves of the form $E_{p,c}(\mathbf{x},t) = \bar{E}_{p,c}e^{i[\mathbf{k}_{p,c}\mathbf{x}-\omega_{p,c}t]}$ illuminating a locally homogeneous condensate described by the field $\psi_g(\mathbf{x},t) = \bar{\psi}_g e^{i[\mathbf{k}_g\mathbf{x}-(\omega_g+\mathbf{k}_g^2/2m)t]}$ where $|\bar{\psi}_g|^2 = N_o$. For a cloud at rest, $\mathbf{k}_g = 0$ and $\psi_g(\mathbf{x},t) = \bar{\psi}_g e^{-i\omega_g t}$, while for a cloud that is uniformly and homogeneously moving with a velocity $\mathbf{v}$, $\mathbf{k}_g = m\mathbf{v}/\hbar$. Due to energy and momentum conservation, the amplitudes of the excited and metastable components of the atomic field have the same plane–wave structure as for the ground state, i.e.,

$$\psi_e(\mathbf{x},t) = \bar{\psi}_e \exp\left\{i\left[(\mathbf{k}_p+\mathbf{k}_g)\,\mathbf{x} - (\omega_p + \omega_g^{(\mathrm{eff})})t\right]\right\}, \tag{5}$$

$$\psi_m(\mathbf{x},t) = \bar{\psi}_m \exp\left\{i\left[(\mathbf{k}_p-\mathbf{k}_c+\mathbf{k}_g)\mathbf{x} - (\omega_p - \omega_c + \omega_g^{(\mathrm{eff})})t\right]\right\}. \tag{6}$$

Inserting these forms into (4) and then (3) yields

$$\bar{\psi}_m = \frac{-d_c^*\bar{E}_c^*}{\hbar\,(\Delta_m(\mathbf{k}_p,\omega_p)+i\gamma_m)}\bar{\psi}_e \tag{7}$$

and

$$\bar{\psi}_e = \frac{-d_p\bar{E}_p}{\hbar\left(\Delta_e(\mathbf{k}_p,\omega_p)+i\gamma_e - |d_cE_c|^2/(\Delta_m(\mathbf{k}_p,\omega_p)+i\gamma_m)\right)}\bar{\psi}_g \tag{8}$$

which generalize the expression used for describing EIT in Λ-type three-level atomic configuration to include for kinetic energy corrections associated with the atomic recoil. These appear in the detuning from the excited level,

$$\Delta_e(\mathbf{k}_p,\omega_p) = \omega_g^{(eff)} + \omega_p - \omega_e^{(eff)}(\mathbf{k}_p) \tag{9}$$

and from the metastable level,

$$\Delta_m(\mathbf{k}_p,\omega_p) = \omega_g^{(eff)} + \omega_p - \omega_c - \omega_m^{(eff)}(\mathbf{k}_p), \tag{10}$$

where $\omega_g^{(eff)} = \omega_g^{(eff)}(\mathbf{k}_g) = \omega_g + \hbar\mathbf{k}_g^2/2m$, $\omega_e^{(eff)}(\mathbf{k}_p) = \omega_e + \hbar(\mathbf{k}_p+\mathbf{k}_g)^2/2m$ and $\omega_m^{(eff)}(\mathbf{k}_p) = \omega_m + \hbar(\mathbf{k}_p - \mathbf{k}_c + \mathbf{k}_g)^2/2m$. Only the dependence on $\mathbf{k}_p$ and ω_p, which will be needed in the following, has been explicitely indicated whereas the dependence on the other set–up parameters ω_c, $\mathbf{k}_c$ and $\mathbf{k}_g$ has been left implicit. Since the dipole moment per unit volume at the probe frequency is given by $d_p^*\bar{\psi}_g^*\bar{\psi}_e$, (8) leads to a simple expression for the dielectric function $\epsilon(\omega_p,\mathbf{k}_p)$ of the dressed atomic cloud

$$\epsilon(\omega_p,\mathbf{k}_p) = 1 + \frac{4\pi N_o\,|d_p|^2}{\hbar\left(|\Omega_c|^2/(\Delta_m+i\gamma_m) - \Delta_e - i\gamma_e\right)}, \tag{11}$$

where $\Omega_c = |d_cE_c|/\hbar$ is the Rabi frequency of the coupling beam. If the spontanoeus decay rate γ_e is much larger than all other frequency scales and, in particular, if $\gamma_e \gg \Delta_e$, then the detuning Δ_e of the excited state can be neglected in (11). If we further assume the decoherence rate γ_m to be much smaller than $\Gamma = \Omega_c^2/\gamma_e$, then (11) simplifies to

$$\epsilon(\omega_p, \mathbf{k}_p) = 1 + \frac{4\pi N_o |d_p|^2}{\hbar\gamma_e} \left\{ i + \frac{\Gamma}{\Delta_m(\mathbf{k}_p, \omega_p) + i\Gamma} \right\}. \tag{12}$$

Provided the Rabi frequency Ω_c of the coupling beam is smaller than the excited state linewidth γ_e, nearly total transmission occurs within a small bandwidth Γ of frequencies around $\omega_p^{(o)} = \omega_m^{(eff)}(\mathbf{k}_p^{(o)}) + \omega_c - \omega_g^{(eff)}$, for which $\Delta_m(\mathbf{k}_p^{(o)}, \omega_p^{(o)}) = 0$; in this same frequency window, the refractive index, which is unit ($\omega_p^{(o)} = c|\mathbf{k}_p^{(o)}|$) at line-center, has a very steep dispersion. This implies that a narrowband pulse would propagate with a very small group velocity without being appreciably absorbed [1, 2, 8]. Approximating the atomic dispersion of the metastable $|m\rangle$ state after the absorption of a photon from the probe beam and its immediate re-emission into the coupling beam as a linear one with the group velocity

$$\mathbf{v}_a = \mathbf{v} + \frac{\hbar}{m}\left(\mathbf{k}_p^{(o)} - \mathbf{k}_c\right), \tag{13}$$

the detuning in the denominator of (12) can be approximated by $\Delta_m(\mathbf{k}_p, \omega_p) \simeq (\omega_p - \omega_p^{(o)}) - (\mathbf{k}_p - \mathbf{k}_p^{(o)})\mathbf{v}_a$ so that $\epsilon(\omega_p, \mathbf{k}_p)$ acquires the new form

$$\epsilon(\omega_p, \mathbf{k}_p) \simeq 1 + \frac{4\pi N_o |d_p|^2}{\hbar\gamma_e\Gamma} \left[(\omega_p - \omega_p^{(o)}) - (\mathbf{k}_p - \mathbf{k}_p^{(o)})\mathbf{v}_a\right]. \tag{14}$$

The dispersion law for a probe propagating in the direction of the unit vector $\hat{\mathbf{k}}_p = \mathbf{k}_p/|\mathbf{k}_p|$ with a frequency centered on the EIT transparency window can be obtained by inserting (14) into $\epsilon(\omega, \mathbf{k})\omega^2 = c^2\mathbf{k}^2$ and then linearizing around $\omega_p = \omega_p^{(o)}$ and $\mathbf{k}_p = \mathbf{k}_p^{(o)}$:

$$(\omega_p - \omega_p^{(o)}) = (\mathbf{k}_p - \mathbf{k}_p^{(o)})\frac{\eta\mathbf{v}_a + c\hat{\mathbf{k}}_p}{1+\eta}, \tag{15}$$

where $\eta = 2\pi N_o |d_p|^2 \omega_p^{(o)}/\hbar\Omega_c^2$. The relevant group velocity $\mathbf{v}_g = \nabla_{\mathbf{k}_p}\omega_p$ at two-photon resonance can finally be written as,

$$\mathbf{v}_g = \frac{c}{1+\eta}\hat{\mathbf{k}}_p + \frac{\eta}{1+\eta}\mathbf{v}_a. \tag{16}$$

For a sample at rest, in the infinite mass limit, $\mathbf{v}_a$ is negligible and the group velocity has the usual expression $\mathbf{v}_g = c\hat{\mathbf{k}}_p/(1+\eta)$ [8]. In this case, for values of η much larger than unity, light speeds much smaller than c can be observed to occur such as, e.g., in [1] where $\eta \sim 10^7$. However, we cannot neglect atomic recoil when η is much larger than unity and of the order of $c/|\mathbf{v}_a|$, since $\mathbf{v}_g$ becomes comparable magnitude to $\mathbf{v}_a$. In this case, the group velocity can then be written as

$$\mathbf{v}_g \simeq \frac{c}{\eta}\hat{\mathbf{k}}_p + \mathbf{v}_a. \tag{17}$$

While the first term $c\hat{\mathbf{k}}_p/\eta$ recovers eq.(1) in [1], the other term seems to suggest that light is dragged by the *metastable* atoms which recoil at a velocity equal to $\mathbf{v}_a$; however, we stress that in our conditions $|\bar{\psi}_e|^2 + |\bar{\psi}_m|^2 \ll |\bar{\psi}_g|^2 = N_o$ and therefore the center of mass motion of the atomic cloud is very weakly affected by light.

We now proceed to discuss novel and interesting effects associated with the result (17). For an atomic sample at rest in which $|g\rangle$ and $|m\rangle$ are hyperfine sublevels of the same ground state with energies very close to each other, $\mathbf{k}_g = 0$ and $\mathbf{v}_a$ turns out to be a negligibly small quantity for co–propagating probe and coupling beams (figure a).

Such a situation has been examined in [7], e.g., where recoil is explicitely omitted. On the other hand, for counter–propagating beams (figure b) $\mathbf{v}_a$ is nearly twice the recoil velocity of the $|g\rangle \rightarrow |e\rangle$ optical transition and it is directed as the probe wavevector; in such a geometry the group velocities are then restricted by the lower bound $|\mathbf{v}_a|$. In the case of sodium atoms (D_2-line), this quantity is approximately 6 cm/s, i.e. 300 times smaller than the lowest group velocity value of 17 m/s so far reported in sodium [1]. Since the most stringent upper bound to η is actually set by the lower bound to the coupling intensities $\Omega_c^2 > \gamma_m \gamma_e$ which have to be applied in order for the EIT to be fully developed, a substantial reduction of γ_m [1] will lead to much larger values of η so that the effect of atomic–recoil as predicted by (17) could possibly be observed.

For a sample moving with a uniform velocity $\mathbf{v}$, our theory recovers the well-known Fresnel-Fizeau light-drag [5] effect; in the slow-light case, all velocities involved are non-relativistic and the Galilean composition of velocities is obtained as in (17). Unlike for the effect of atomic recoil, the Fresnel-Fizeau drag occurs even in the infinite atomic mass limit. Recently, a related effect has been shown to lead to exotic features of light propagation in the more complex situation of non-uniformly moving media [9], but this is beyond the scope of the present paper.

With co–propagating coupling and probe beams and an appropriate choice for the atomic levels, i.e. a Λ configuration in which the m level has an energy lower than g (see figure c), the recoil velocity $\mathbf{v}_a$ is directed in the opposite direction with respect to the probe beam even for a sample initially at rest. In this case, for sufficiently small values of c/η, the probe wavevector and group velocity turn out to be oppositely directed. From a phenomenological point of view, the possibility of attaining such *negative* group velocities may be exploited to investigate rather novel effects in the domain of geometrical optics such as, e.g., negative refraction angles at the boundary with free-space [10]. Recent developments in coherently prepared atomic media have revived the interest on the issue of negative group velocities. With respect to the previous works on the subject [11, 12] our proposal is characterized by the fact that both absorption and group velocity dispersion are almost vanishing in the frequency range of interest so that the shape of the light pulse remains essentially unchanged. Negative group velocities have also been predicted to occur in an EIT configuration for coupling and probe beams co-propagating in a *hot* atomic gas [13]: because of the Doppler effect light interacts only with a narrow class of atomic velocities and the sample behaves as an effectively moving one. If the selected atoms move in the opposite direction with respect to the probe wavevector, negative group velocities may occur for sufficiently dense samples as also predicted by the present treatment when a non-zero atomic velocity is explicitely included in (13).

In actual experiments, a non-zero temperature and the finite size of the sample may cause a finite velocity spread for the ground state atoms. This can be taken into account by integrating the dielectric susceptibility (12) over the velocity distribution of ground state atoms. For a Lorentzian velocity distribution [13], a straightforward calculation leads to the same form of susceptibility where Γ in the denominator is replaced by $(\Gamma + \Gamma_D)$, being $\Gamma_D = |\mathbf{k}_p - \mathbf{k}_c|\, v_D$ the Doppler width expressed in terms of the velocity spread v_D. In physical terms, the effect of a Doppler width Γ_D comparable to the subnatural linewidth Γ would be similar to the effect of having a lower level decoherence γ_m of the order of Γ, i.e. a broadened absorption dip and a reduc ed contrast of the transparency feature, which would no more be complete. From a quantitative point of view, the broadening due to the finite size of a zero-temperature BEC is generally smaller than the recoil velocity, and thus can be safely neglected with respect to Γ. For hot samples, Γ_D is negligible only if $\mathbf{k}_p \simeq \mathbf{k}_c$, i.e. for co–propagating coupling and probe beams and small lower state energy splitting. In addition, if the Doppler broadening $|\mathbf{k}_p|\, v_D$ of the excited state is comparable to its linewidth γ_e, the detuning Δ_e can no longer be neglected in (11) and a more detailed treatment has to be carried out [13].

The theory described up to now has neglected atom-atom interactions (collisions). These are commonly modelled [4] by adding quartic terms in the Hamiltonian (1) and give rise to additional cubic terms of the form $\sum_j G_{i,j}\, |\psi_j|^2\, \psi_i$ in the mean-field wave equations (2) – (4). The coupling coefficients $G_{i,j}$ are proportional to the s-wave scattering length for collisions between atoms respectively in the i and j states ($i, j = \{e, g, m\}$). To the lowest order in the probe intensity only the $G_{i,g}\, |\psi_g|^2\, \psi_i$ terms contribute to the sum causing a mean-field shift of the e and m level–frequencies in (9) and (10). The excited level e is adiabatically eliminated in the present treatment, while the collisional frequency shift of the metastable level m gives rise to a small shift of the two-photon resonance condition in (12). This means that the photon–dragging effects of our interest originate from the independent recoil of each single atom and, thus, the Bogoliubov's sound velocity $v_s = \sqrt{G_{g,g} N_o/m}$ does not appear to be relevant to the linear propagation of light pulses in condensed media under EIT. The dispersion of Bogoliubov's phonons [4] may, on the other hand, be crucial in more complex optical processes which involve the excitation of phonons in the condensate, such as, e.g., Brillouin scattering on density fluctuations [14].

In conclusion, we have shown that even in a sample at rest, under appropriate EIT conditions, light can be dragged by the atoms which recoil after the absorption of a photon from the probe beam and the subsequent emission into the coupling beam. We hope that a feasible upgrade of the experimental set-up commonly used to study light propagation in EIT configurations [1] will soon allow for the detection of such atomic recoil effects.

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