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THE ION CYCLOTRON RESONATOR IN THE MAGNETOSPHERE

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Our concern here is to present the idea of ion cyclotron resonator in the planetary magnetosphere, and to discuss briefly the experimental status of the corresponding theory. The resonator confines the ion cyclotron waves to a thin equatorial zone, so that it keeps the wave field from coming in contact with the ionosphere resulting in a decrease of energy losses. The properties of resonator are illustrated by adopting plausible distribution of magnetic field in the equatorial zone, which yields an expression for the discrete spectrum of the waves just above the gyrofrequency of heavy ions. We show that the resonator is remarkable for many reasons, including the frequency dependence of its size and specific structure of the spectrum.

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The important and very much discussed problems centre around the electromagnetic ion cyclotron waves in the Earth's magnetosphere. The literature on this subject is quite voluminous (see [1 – 3] for references). Considerable recent attention has been also focused on the ion cyclotron waves in the magnetospheres of other planets [4]. It is generally agreed that the study of ion cyclotron waves allows us to broaden our conceptions of the space plasma physics [5]. The observation of these waves provides the basis for useful practical applications [1].

Up to now attention was paid only to the travelling ion cyclotron waves (e.g., [6 – 8]). In this paper we would like to discuss the standing ion cyclotron waves as the discrete eigen modes of the ion cyclotron resonator (ICR). Our main concern here is to present the idea of a possible existence of such resonators "suspended" in the equatorial zones of planetary magnetospheres. We will show that the physical properties of ICR are remarkable for many reasons, including the specific structure of the spectrum and the frequency dependence of ICR size. One point generates a particular interest, namely, the resonator holds the ion cyclotron waves in the magnetosphere. This keeps the wave field from coming in contact with ionosphere resulting in a decrease of energy losses. In this regard the ICR differs from the familiar Alfvén resonator [5], and it resembles the toroidal magnetosonic waveguide which exists in the equatorial zone of the Earth's magnetosphere [9, 10].

Let us consider the wave equation

$$\frac{d^2E_\pm}{dz^2} + \left[\frac{\omega}{c}\, n_\pm(z,\omega)\right]^2 E_\pm = 0\,, \tag{1}$$

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where $n_\pm^2 = 1+\sum \Omega_i^2/\omega_{Bi}(\omega_{Bi} \mp \omega)$, descriptive the left-hand ($E_+ = E_x + iE_y$) and right-hand ($E_- = E_x - iE_y$) circularly polarized low-frequency ($\omega \ll \omega_{Be}$) electromagnetic waves in the framework of 1D slab plasma model with the external magnetic field **B** which points in z direction. Here $\omega_{Be} = eB/m_e c$ is the electron gyrofrequency, e is the elementary charge, m_e is the mass of electron, c is the velocity of light, $\omega_{Bi} = e_i B/m_i c$ is the ion gyrofrequency, e_i and m_i are the charge and mass of ion, $\Omega_i = (4\pi e_i^2 N_i/m_i)^{1/2}$ is the ion plasma frequency, N_i is the number density of ions; the summation is made over the ion species; the upper sign in Eq. (1) refers to the ion cyclotron waves, and the bottom sign refers to the helicon waves (or whistlers) [11].

The multicomponent composition of the plasma is essential to the formation of ICR. We have restricted ourselves to the simplest case of a binary mixture of light ($i = 1$) and heavy ($i = 2$) positive ions for better visualization of the idea. Let us introduce the designations

$$\omega_\times = \omega_{B2}[(1+\eta/\mu)/(1+\eta\mu)]^{1/2}, \quad \omega_0 = \omega_{B2}(1+\eta)/(1+\eta\mu), \quad \omega_\infty = \omega_{B2}, \tag{2}$$

where $\mu = m_1 e_2/m_2 e_1$, $\eta = \rho_2/\rho_1$, $\rho_i = m_i N_i$. It can be shown that the function $n_+^2(\omega)$ has the pole at the frequency ω_∞, and zero at the frequency ω_0. The opaqueness band ($n_+^2 < 0$) is situated between these two singularities. The crossover frequency $\omega_\times$ is determined by the relation $n_+^2(\omega_\times) = n_-^2(\omega_\times)$. It is easy to check that $\omega_\infty < \omega_0 < \omega_\times$ for $\mu < 1$. Note that the formulae for $\omega_\times$ and ω_0 in Eqs. (2) are appropriate only in the case of dense plasma in that $\rho \gg B^2 / 4\pi c^2$, where $\rho = \rho_1 + \rho_2$.

Suppose that the opaqueness band $\omega_0 - \omega_\infty$ is thick enough. (The physical meaning of this condition will be discussed below.) Besides, let us assume that $\omega_\infty < \omega < \omega_\times$ and furthermore the frequencies ω and ω_0 are close together, i.e. $\omega \sim \omega_0$. In such an event, the square of refractive index for the ion cyclotron waves can be approximated by the equation

$$n_+^2(\omega) = \alpha\,(\omega - \omega_0)\ , \tag{3}$$

where $\alpha = \left(\partial n_+^2/\partial\omega\right)_0$. The function $\omega_0(z)$ will be assumed as smoothly slowly varying with a minimum at the point $z = 0$. For example, this is take place in the equatorial zone of the Earth's magnetosphere, since $B(z)$ has a minimum at the equator. In that case there exist the frequencies $\omega > \omega_0(0)$ and the points $z_- < 0$, $z_+ > 0$ such that $n_+^2(\omega, z_\pm) = 0$, and $n_+^2(\omega, z) > 0$ for $z_- < z < z_+$. Then, over the interval $z_- < z < z_+$ the non-trivial solutions of Eq. (1) exist if, and only if the frequencies ω belong to the discrete spectrum ω_s with $s = 0, 1, 2, \ldots$ The wave field $E_+(s, z)\exp(-i\omega_s t)$ has the form of standing ion cyclotron wave with s nodes. In other words, we are concerned with ICR "suspended" in the magnetosphere.

To simplify the treatment let us use a parabolic model to approximate the geomagnetic field in the equatorial zone

$$B(z) = \frac{B_E}{L^3}\left[1 + \frac{9}{2}\left(\frac{z}{R_E L}\right)^2\right]. \tag{4}$$

Here B_E is the magnetic field at the Earth surface, R_E is the Earth radius, and L is the McIlwain parameter. Rewriting Eq. (1) in view of Eqs. (3), (4) we obtain

$$\frac{d^2 E_+}{d\zeta^2} + \xi(\omega)\left[\zeta_0^2(\omega) - \zeta^2\right] E_+ = 0\,, \tag{5}$$

where

$$\xi = \frac{2}{\eta}\left[\frac{R_E L\,\omega(1+\eta\mu)}{3\,c_A\,(1-\mu)}\right]^2,\ \zeta_0 = \left[\frac{\omega(1+\eta\mu)}{\omega_{B2}(1+\eta)} - 1\right]^{1/2}, \tag{6}$$

and $\varsigma = 3z\,/\sqrt{2}R_E L$. The values η, ω_{B2} , and $c_A = B\,/(4\pi\rho)^{1/2}$ in Eqs. (6) are taken at the point $\zeta = 0$. The solutions of Eq. (5) are $D_s\left[(4\xi)^{1/4}\,\zeta\right]$ with $\zeta_0^2\sqrt{\xi} = 2s+1$, where $D_s(z) = H_s(z/\sqrt{2})\exp\left(-z^2/4\right)$ are the functions of parabolic cylinder, and $H_s(z)$ are the Hermite polynomials. It is natural that $E_+(s,\zeta) \to 0$ at $\zeta \to \pm\infty$ in our parabolic model of ICR. This leads to the quantization condition $s = 0,\ 1,\ 2,\ ...$, so that the equation

$$\left[\xi\,(\omega_s)\right]^{1/2}\,\left[\zeta_0\,(\omega_s)\right]^2 = 2s+1 \tag{7}$$

describes the discrete spectrum of ion cyclotron oscillations in the equatorial zone of the magnetosphere.

If $\eta\mu \ll 1$, the roots of Eq. (7) are

$$\omega_s = (1+\eta)\,\omega_{B2} + 3\sqrt{2\eta}\,(c_A\,/\,R_E L)\,(s+1/2), \tag{8}$$

on the condition that the second term at the right-hand side of Eq. (8) is small in comparison with the first one. This condition asserts in the oxyhydrogen magnetospheric plasma at least at low values of s. We can see that the spectrum is equidistant, but the intervals $\Delta\omega = \omega_{s+1} - \omega_s$ between the adjacent spectral lines

$$\Delta\omega = 3\sqrt{2\eta}c_A\,/\,R_E L \tag{9}$$

is much smaller than the frequency $\omega_{s=0}$ of fundamental harmonic. It is well to bear in mind that this result has been obtained in the dissipation-free limit. There can be little doubt that a natural broadening of the spectral lines in the real plasma causes a flattening of the ICR spectrum. This has led us to believe that the gap

$$\delta\omega = \eta\,\omega_{B2} + 3\sqrt{\eta\,/\,2}\,c_A\,/\,R_E L \tag{10}$$

between the gyrofrequency of heavy ions ω_{B2} and fundamental frequency $\omega_{s=0}$ is of greater interest for the experimental study of ICR than $\Delta\omega$. Likewise, the ICR size

$$\Delta z = 2\,\eta^{1/4}\left[(c_A R_E L\,/\,\omega_{B2})\,(s+1/2)\right]^{1/2} \tag{11}$$

is of immediate interest to the experimentalist. Here $\Delta z = z_+ - z_-$, $z_\pm = \pm(\sqrt{2}\,/\,3)\,\zeta_0\,R_E L$, $\eta \ll 1$. We notice that the size of resonant region is minimal for the fundamental harmonic.

A small size of ICR is noteworthy. By way of illustration let us assume that $L = 7.48$, $\rho_{O^+} = 0.21\,\rho$, $c_A = 2\cdot 10^7\mathrm{cm}\cdot\mathrm{s}^{-1}$, as measured by ISEE-1 satellite [12]. Then $(\Delta z)_{\min} = 4.6\cdot 10^8$ cm which is correspond to the interval of geomagnetic latitudes between $\pm 2.3^o$ (see Eq. (11) for $s = 0$). The ISEE-1 satellite recorded the ion cyclotron waves in the frequency band from 0.1 to 0.2 Hz just above the O^+ cyclotron frequency at the distance 6.5^o from the geomagnetic equator. According to Eq. (11) this means that satellite was liable to detect the standing waves in ICR if, and only if $s \geq 3$. The corresponding gap between the gyrofrequency of oxygen ions at the equator and the lower boundary of the wave spectrum equals 28 mHz. It is reasonable to say that this estimate does not contradict with ISEE-1 observations (see the ion cyclotron wave spectrum in Ref. [12], Fig.10a).

There are number of spikes in the spectra observed at ISEE-1 and 2 satellites. However, the interval between adjacent spikes is of the order of 10 mHz which is several times greater than it follows from Eq. (9). We have reanalyzed the satellite data, and we have concluded that in any case we could not resolve the predicted line spacing with the 4 mHz spectral resolution at ISEE-1 and 2. Thus, while we feel that the idea of magnetospheric ICR is plausible, we have not yet confirm it experimentally in full measure. We urge other researches to look for the quasi-discrete structure of spectra when examining the ion cyclotron waves near magnetic equator.

In conclusion, let us take up the applicability of theory. Generally this is a widespread problem, but here we restrict the discussion to the elementary aspects, namely, to the foregoing conditions of applicability of the Eqs. (3)–(5). It has been assumed that the value $\omega_0 - \omega_\infty$ is large enough so that the poles and zeros of $n_+^2(z)$ are far apart. Physically this means that we are in position to neglect the energy leakage from ICR due to the tunnelling of the ion cyclotron waves through the opaqueness bands disposed bilaterally just beyond the turning points z_- and z_+. In the low latitudes the tunnel-effect is negligibly small when $\Omega_2 \gg c\,/\,R_E L$ (e.g., see [2] where the tunnel-effect was considered in the high latitudes). This sufficient condition asserts in the case of satellite observations cited above.

One possible mechanism of the energy leakage from ICR is associated with the linear mode conversion. The essence of this process is that the left-hand ion cyclotron waves may couple to the right-hand whistler waves due to inhomogeneity of the medium [13]. Without going into detail, we are simply note that the coupling between the modes is especially strong adjacent to the cross-over points $z_\times$ which are the roots of the equation $n_+(z_\times, \omega) = n_-(z_\times, \omega)$. Needless to say that the concept of mode conversion, as applied to ICR, needs further consideration. For our present purposes it will suffice to mention that n_+ and n_- as the functions of z have the maximum and minimum at $z = 0$ respectively, so that in ICR the cross-over points $z_\times$ are absent at all if $\omega < \omega_\times$ since $n_+(0, \omega) < n_-(0, \omega)$ at this condition. It is easy to check that the spectrum described by the Eq. (8) satisfies the condition $\omega_s < \omega_\times$ at least at low values of s. The last remark pertains equally to the parabolic model for geomagnetic field which is appropriate in the limit of small Δz (see Eqs. (4) and (11)).

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