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TUNNELLING SPECTROSCOPY OF LOCALIZED STATES NEAR THE QUANTUM HALL EDGE

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In the paper we discuss experimental results of M.Grayson et al. on tunneling I-V characteristics of the quantum Hall edge. We suggest a two step tunneling mechanism involving localized electron states near the edge, which might account for discrepancy between the experimental data and the predictions of the chiral Luttinger liquid theory of the quantum Hall edge.

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Measuring the tunnelling current from a normal metal to a quantum Hall (QH) edge is an attractive way to observe the Luttinger liquid behaviour of QH edge states. Scaling invariance of the Luttinger liquid should leave a clear signature in the I-V characteristic of the tunnelling current in the form of a power law dependance of the current on the applied voltage [1]

$$I \sim V^{\alpha}. \tag{1}$$

Moreover, the value of the tunnelling exponent α would give the knowledge of the decay law of the electron Green's function at the edge which would provide a direct check of the chiral Luttinger liquid theories of the edge states [1 – 3].

The predictions of the chiral Luttinger liquid theory of the QH edge states can be summarized as follows.

(1) If a QH state belongs to the principal (Laughlin) sequence of filling factors $\nu = 1/(2p+1)$ (p integer) it supports one gapless chiral edge mode. In this case the tunnelling exponent is given by

$$\alpha = 1/\nu. \tag{2}$$

(2) In a more general case of incompressible QH states corresponding to Jain filling factors $\nu = N/(2Np+1)$ there are $|N|$ edge modes and the calculations [1, 2] based on the Luttinger liquid picture predict that

$$\alpha = \min(2/\nu - 1, 3). \tag{3}$$

In [4] an alternative theory was developed capable of treating compressible states near the filling 1/2. The expression for $\alpha(\nu)$ was obtained which in the limit of vanishing compressibility coincides with (3) for Jain filling factors. This theory predicts that the slope of the tunnelling exponent $d\alpha/d\nu$ behaves discontinuously as a function of ν at points $\nu = 1$, $\nu = 1/2$.

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Recent tunnelling experiments [5] gave the following results:

(1) At low temperatures the I-V characteristic exhibits the power-law behaviour (1) up to several decades in current. The power-law I-V characteristics are observed independently of whether the 2DEG is in a compressible or in an incompressible state.

(2) The tunnelling exponent α varies continuously with the filling factor ν. The dependance of the tunnelling exponent on the filling factor is well approximated by the linear law $\alpha = 1/\nu$.

One can see that the second item is in an obvious contradiction to the predictions of the chiral Luttinger liquid theory.

This disagreement puts in doubt the generally accepted theories of the fractional QH edge and needs to be explained either within these theories or by developing a new theoretical approach. That is why a lot of attention has been paid to the problem of late [6-11].

In the discussion below we restrict ourselves to the incompressible case only. In this case the QH edge is believed to be described by a chiral Luttinger liquid theory. In it's standard form this theory [1, 2] claims that there are several edge chiral Luttinger modes which can be separated into one charged mode and a number of neutral ones. The charged mode is described by a chiral bosonic field φ_0, while the neutral modes are described by bosons $\varphi_1, \ldots, \varphi_N$ and can be both chiral (propagating along the same direction as the charged mode) or anti-chiral (counter-propagating). The dispersion of charged and neutral modes determines the velocities $s_j(q)$,

$$\omega_j(q) = s_j(q)q. \tag{4}$$

The anomalous exponent α in the I-V characteristic (1) is simply related to the asymptotic behaviour of the single particle Green's function of an electron in the QH system [12]:

$$G(t) = -i\langle T\psi(x,t)\psi^{\dagger}(x,0)\rangle \sim t^{-\alpha}. \tag{5}$$

An attempt to solve the contradiction between the theory and the experiment was made in [8, 9]. The main idea of these works is that the experimentally observed tunnelling exponent is consistent with the chiral Luttinger liquid picture under the assumption that the neutral modes are non-propagating (or their propagation velocities are negligible). In our opinion the weakness of this approach is that it's central assumption has no sufficient physical justification. In the experiment, the power law I-V characteristic is observed in a broad (about two decades) voltage range. For the picture [8, 9] it implies that the velocities of the neutral edge modes should differ from the velocity of the charged mode by minimum two orders of magnitude. This difference is attributed to the Coulomb interaction, whose contribution to the velocity of the charged mode is evidently larger than to the neutral ones. However, simple analysis shows that while the neutral modes have linear dispersion (4) with s_j constant, the velocity of the charged mode s_0 has a logarithmic q-dependence (see [10] and considerations below),

$$\frac{s_0}{s_j} \sim \ln\left(\frac{1}{qa}\right) \tag{6}$$

where a is the quantum well width. The logarithmic factor on the r.h.s of (6) evidently cannot account for the two decade difference needed for the picture of [8, 9]. In the real experimental setup [5] the tunneling occurs from a 3D metal contact separated by a $b \sim 100\,\text{Å}$ thick barier from the edge. Therefore, the Coulomb interaction must be screened at this distance, and the ratio (6) saturates at $q < b^{-1}$.

Eq. (2) corresponds to the shakeup of the charge relaxation mode at the edge. If one neglects the contribution of other bosonic modes the experimentally observed tunnelling exponent will be regained. This is exactly the way the problem is treated in the framework of the independent boson model (IBM) [6, 11, 12], where a single localized electron electrostatically interacts with the hydrodynamic charged edge mode in the incompressible case [6] or with the bulk charge relaxation modes in the compressible case [11]. Although the IBM gives a correct tunnelling exponent it says nothing about the physical origin of the localized states and it's relevance to the experiments is not clear unless the nature of these states is specified. In particular, understanding what these states are is important since the results which can be obtained in the framework of the IBM are very sensitive to the choice of the energy position of the localized state.

In our opinion, a good agreement between the observed universality of the I-V characteristics and the IBM description indicates that near the edge of the QH liquid there exist some low energy electronic states other than the excitations of the chiral Luttinger liquid. Electrons tunnelling from the metal into these states electrostatically interact with the charged mode of the edge collective excitations. Below we suggest a model of the edge where the tunnelling current is transmitted in a two-step process which involves localized states in the bulk at the intermediate stage. We show that this processes gives an experimentally observed I-V exponent providing that the intermediate localized states are spatially separated from the edge and their energy distribution function decays exponentially in the gap of the incompressible states like in the integer quantum Hall regime [13].

The series of well established plateaus observed in the experiment [5] indicates that in the gaps of incompressible QH states there exists a finite density of bulk localized states $g(\epsilon)$ created by the random impurity potential. An electron may tunnel into the QH edge in a two step process: first it tunnels into a localized state, where it may stay for some time t^*, and then decays into the the edge mode due to a finite hybridization between the edge and bulk states. If the voltage V satisfies the condition

$$\hbar/eV \ll t^*$$

then the second step of the tunnelling process does not affect the I-V characteristics.

On the time scale t^* the tunnelling process is described by the IBM model. If the QH system is incompressible, the only contribution to the polarization of the QH medium comes from gapless edge modes. An electron can polarize both charged and neutral modes (because they carry multipole moments).

First we consider the contribution of the charged mode. The corresponding Hamiltonian reads

$$H = \sum_n (\epsilon_n - w_n) a_n^\dagger a_n + \frac{1}{2} \int dx dx' \rho(x) v(x - x') \rho(x'). \qquad (7)$$

Here a_n is the annihilation operator of an electron in the localized state with the energy ϵ_n and the wave function $\psi_n(\mathbf{r})$, $e\rho(x)$ is the charge density operator of the edge plasmon, $v(r) = e^2/\kappa r$ is the Coulomb potential with the dielectric constant κ. The term

$$w_n = \int dx \rho(x) U_n(x) \qquad (8)$$

stands for the electrostatic interaction between the edge plasmon and the electron in the localized state. Here

$$U_n(x) = \int d^2\mathbf{r}' v(|\mathbf{r} - \mathbf{r}'|)|\psi_n(\mathbf{r}')|^2 \tag{9}$$

is the potential induced by the localized state at the edge $\mathbf{r} = (x, 0)$. The charge density operator $e\rho$ of the edge plasmon is given by

$$\rho = \sum_{q>0} i\sqrt{\frac{\nu q}{2\pi L}}(b_q e^{iqx} - b_q^\dagger e^{-iqx}),$$

where L is the length of the edge and $b_q, b_q^\dagger$ satisfy canonical commutation relations.

The Hamiltonian (7) is diagonalized [12] by the canonical transformation to the new fermionic operators $\bar{a}_n$

$$a_n = \bar{a}_n e^{-i\Phi_n} = \bar{a}_n T \exp\left(-i\int_{-\infty}^{t} dt' w_n(t')\right) \tag{10}$$

where $\rho(x,t)$ is the charge density operator in the interaction representation, T is the time ordering operator. Introducing the field ϕ, such that $\rho = \nu/2\pi\partial_x\phi$ and taking into account that in the interaction representation it's dynamics is given by (4) we find that the operators Φ_n in (10) are given by

$$\Phi_n(t) = \frac{\nu}{2\pi s_0}\int dx U_n(x)\phi(x,t), \tag{11}$$

where the velocity of the charged mode reads

$$s_0 = \frac{e^2}{\kappa\pi\hbar}\nu\ln\left(\frac{1}{qa}\right) \tag{12}$$

Green's function (5) of an electron in the n-th localized state is given by

$$G_n(t) = -ie^{i\tilde{\epsilon}_n t}(1 - n_F(\tilde{\epsilon}))\langle Te^{-i\Phi_n(t)}e^{i\Phi_n(0)}\rangle, \tag{13}$$

where $\tilde{\epsilon}_n$ is the exact energy of the localized eigenstate dressed by the charge mode relaxation.

The factor

$$\langle Te^{-i\Phi_n(t)}e^{i\Phi_n(0)}\rangle = \langle Te^{-i\int_0^{t'} w(t')}\rangle \tag{14}$$

is responsible for the suppression of the tunnelling density of states due to interaction of the electron with the charged mode. At large times, the main contribution to the this factor comes from the long-wavelength limit. In our case this limit is defined by $qd_n \ll 1$, where d_n is the distance from the edge to the localized state (below it will be argued that $d_n > l$). The asymptotic form of this factor does not depend on n and is given by

$$\langle Te^{-i\Phi_n(t)}e^{i\Phi_n(0)}\rangle \sim t^{-1/\nu}.$$

In the standard approximation [12] where the dependence of the tunnelling matrix element on the energy is neglected we obtain

$$I(V) \sim \int_0^{eV} d\epsilon g(\mu + \epsilon)(eV - \epsilon)^{1/\nu},$$

here μ is the chemical potential of the QH system and $g(\epsilon)$ is the density of the localized states.

The tunnelling exponent α is determined by the behaviour of the density of localized states in the vicinity of the Fermi energy. As far as we consider the incompressible QH liquid, the Fermi level must lie in the gap of the volume excitations. It looks very natural to assume that the density of localized states in the gap decays rapidly with the energy (just as in the case of the integer QH effect). If the energy scale Γ of the decay is smaller than eV the tunnelling current is given by

$$I(V) \sim \Gamma V^{1/\nu}$$

and $\alpha = 1/\nu$. In the opposite limiting case ($\Gamma > eV$) the tunnelling current is given by $I \sim V^{1/\nu+1}$.

Next we discuss the interaction of a tunnelling electron with the multipole moments of the neutral modes. This has an analogy with the model of smooth edge considered in [14], where the tunnelling exponent is much larger than $1/\nu$ due to the contribution of the modes carrying multipole moments.

Most important is the interaction with the mode responsible for the dipole moment. In contrast to the case of the charged mode this interaction depends on the concrete model of the edge, esp. on the transversal structure of the neutral modes. We take this interaction into account phenomenologically. The model Hamiltonian describing the interaction of localized states with the neutral modes should have the same form as (7) with two differences. First, one should replace $v(|\mathbf{r}-\mathbf{r}'|)$ in Eq. (9) by the dipole interaction $\delta y \partial_x v(|\mathbf{r}-\mathbf{r}'|)$, where the quantity δy is of the order of the width of the edge strip and depends of the concrete model of the edge. In what follows we assume $\delta y \approx l$. Second difference is that in contrast with (12) the velocity of the dipole mode does not contain the Coulomb logarithm $s_1 \sim e^2\nu/\kappa\pi\hbar$. Taking into account the dipole interaction in the framework of IBM shows that the factor (14) falls off more rapidly at large t:

$$\langle T \exp(-i \int_0^{t'} w(t')) \rangle \sim t^{-[(1/\nu)+(l^2/d_n^2)(1/\nu)]}.$$

For the tunnelling exponent at $\Gamma < eV$ we get

$$\alpha(d_n) = \frac{1}{\nu} + \frac{l^2}{d_n^2}\frac{1}{\nu}. \tag{15}$$

The tunnelling probability falls off as $\exp(-d_n^2/l^2)$ with the tunnelling distance leading to a decrease of the tunnelling current. On the other hand, the larger is d_n the smaller is $\alpha(d_n)$. This leads to I increasing with d_n at small values of the applied voltage. As a result, there exist optimal values of the tunnelling distance $d_{opt} > l$ and of the tunnelling exponent α_{opt}. The values of d_{opt} and of α_{opt} can be determined using the following estimate of the tunnelling current related to the n-th localized state

$$I_n(V) \sim e^{-d_n^2/l^2} \left(\frac{eV}{E_0}\right)^{\alpha(d_n)}.$$

Here E_0 is large energy ($E_0 \gg eV$) , which is of the order of the Fermi energy. From this equation we easily get

$$\alpha_{opt} = \nu^{-1}\left(1 + \left(\frac{\nu}{\ln(E_0/eV)}\right)^{1/2}\right).$$

It can be seen, that at small voltages the optimal tunnelling exponent is close to the experimentally observed value $\alpha_{opt} \approx \nu^{-1}$.

We would like to emphasise that this result is related to the fact that the electron tunnels at the distance larger than the width of the edge strip l. On the contrary, in the model used in Ref. [14] it was implicitly assumed that $d_n \sim \delta y$ and the tunnelling exponent was found to be much larger than $1/\nu$.

We conclude that the tunnelling process as a whole looks as follows. First the electron tunnels in the localized state and during the time d_{opt}/s_0 polarizes the charged mode. As a result the positive screening charge e is attracted to the edge in the region of length d_{opt} and the compensating negative charge $-e$ is carried away by the the charged mode with the velocity s_0. On time scale t such that $d_{opt}/s_0 < t < t^*$ there exists a dipole formed by the localized electron and the screening positive charge. After the time t^* this dipole vanishes due to the tunnelling of the electron from the localized state into the edge.

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