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PHONON LASER AND INDIRECT EXCITON DISPERSION ENGENEERING

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Engineering of dispersion of indirect exciton by normal electric and in-plane magnetic fields is proposed to be used for controlling of state of many-exciton system (e.g. coherent state) and its photoluminescence and producing an inverse population in excitonic system. The possibility of phonon laser creation on the basis of last effect is discussed. Phonon number distribution appears to be a fingerprint of that of exciton system. Numerical estimations for the proposed scheme are made for GaAs/AlGaAs quantum wells.

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Essential success is achieved now in experiments on coherent states of indirect excitons [1, 2], that makes search for different equilibrium phases [3] in the system of indirect excitons and study of their unusual properties [4, 5] be promising. Recently, changing of dispersion of indirect excitons by in-plane magnetic field has been experimentally demonstrated [6].

In the present work we propose a new method of controlling of condensate of indirect excitons in coupled quantum wells by normal electric and parallel magnetic fields. It occurs that by external fields one can essentially control angular dependence and intensity of photoluminescence (PL) of condensate, and, that is of main interest in this work, create inverse population in the system of excitons (spatially direct and indirect ones). The latter possibility can be used for creation of phonon laser, which frequency can be controlled by in-plane magnetic field. It occurs that statistics of number of quanta in phonon mode is a "fingerprint" of that for excitons.

In direct gap semiconductor in coupled quantum wells (CQW) *spatially* direct and indirect excitons (SDE and SIE) are direct in *momentum* space. In case of identical quantum wells (QW) both levels have two-fold degeneracy. For SIE this is due to two different relative positions of electron and hole (e ahd h) in QW's, and for SDE to two different locations of exciton in one of QW's. SDE is the ground excitonic level because of greater Coulomb interaction in the exciton.

Due to spatial separation of e and h SIE has electric dipole momentum eD normal to CQW, where D is interwell distance. Thus *moving* SIE has in-plane *magnetic* momentum normal to its velocity $eDk\hbar/cM$, where k is wave vector and M is effective mass of SIE. Interaction of SIE electric dipole with normal electric field lowers one of SIE sublevel as $eDE/\hbar$ and in sufficiently strong electric fields it becomes the ground excitonic level. Velocity selective interaction of magnetic dipole of SIE with in-plane field $eDk\hbar H/cM$ being added to quadratic dispersion law of exciton $\hbar k^2/2M$ shifts the minimum of dispersion law away from radiative zone[2] as $K_m = eDH/c\hbar$ (e.g. for $H = 10\,$T and $D = 10\,$nm the

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[2] In another approach this is equivalent to diamagnetic effect, i.e. to opposite electric currents in CQW in parallel magnetic field [4], see also [7].

shift is $K_m = 2 \cdot 10^6\,\mathrm{cm}^{-1}$); this was demonstrated experimentally in [6]. Thus, normal to CQW electric and in-plane magnetic fields controll SIE dispersion while to the first order of interaction with the fields dispersion of SDE is unaffected (Fig.1).

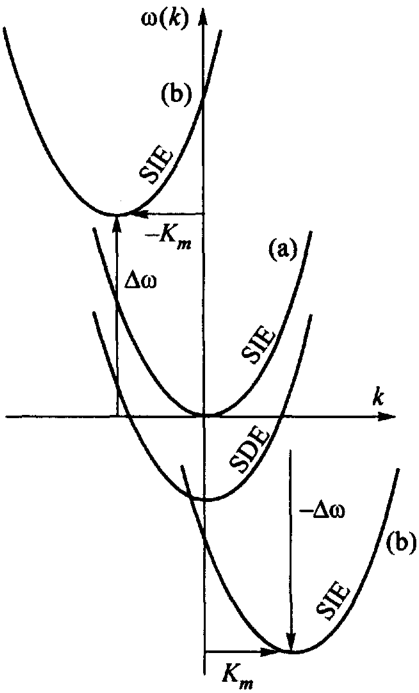

Fig.1. Dispersions of spatially direct and indirect excitons; (a) without external fields; (b) in presence of in-plane magnetic and normal electric fields, $K_m = eDH/c\hbar$, $\Delta\omega = eDE/\hbar$

Let normal electric and in-plane magnetic fields initially be such that SIE level is a ground excitonic state with its minimum of dispersion out of the radiative zone (Fig.2a). After pumping 2D SIEs gather at sufficiently low temperatures near minimum of their dispersion and form "dark" quasi-condensate with slightly fluctuating modulus of order parameter $\sqrt{n_s}$, (n_s is "local superfluid density") and short phase order; at lower temperatures in result of Kosterlitz – Thouless transition quasi-long order of phase and global exciton superfluidity must appear. For "dark" state the process of SIE's recombination is a process of the *second* order for interaction with phonon and photon fields through intermediate production of virtual $e - h$ pair or exciton in radiative zone (PL frequency after pumping can be controlled by normal electric field and can be greater than pumping frequency).

After that by swift reducing electric field value one can tune the system to the resonance of SIE's transformation into *real* SDE with production of acoustic phonon. This process has the *first* order of interaction with phonon field and its rate is much greater than the rate of SIE's recombination before tuning to the resonance. The process of SDE recombination with production of a photon is the second stage of system evolution. For our purposes it is sufficient to treat this process simply as decay of SDE level. There are several resonances which are connected with different phonon bands. The first resonance which is met by reducing electric field is one with production of transverse acoustic phonon with the lowest energy among phonon bands.

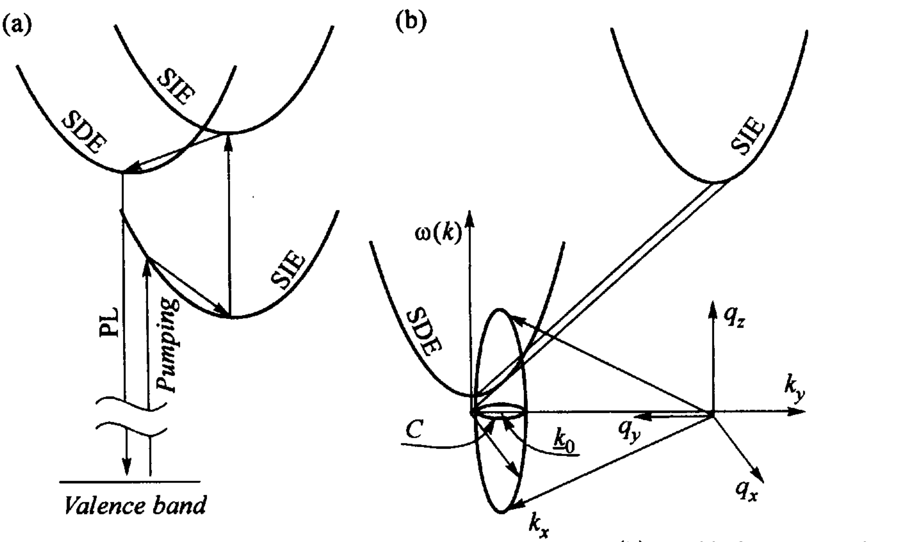

Fig.2. (a) Time evolution of exciton population in the proposed scheme (b) graphical representation of 2D momentum and energy conservation laws after tuning the system to resonance of SIE→SDE+phonon

Excitons in CQW are quasi-2D unlike 3D phonons and photons and entire system has 2D in-plane momentum conservation law. Normal component of phonon momentum, which is not fixed by 2D momentum conservation law, is an extra degree of freedom of the system which gives an additional possibility to conserve energy in the process SIE→SDE+phonon. Thus 2D momenta of resonant phonons fill continuous area. If initial SIE has $\omega / \mid k \mid < c_v$, it can not create a resonant acoustic phonon because this process is forbidden by conservation laws[3] (c_v is a speed of sound in the medium). In opposite case, SIE with momentum K_m can produce a phonon with 2D momentum laying in the circle C: $q_x^2 + (K_m - k_0 - q_y)^2 = T(E)$ (axis oriented as on Fig.2b). 3D wave vectors of phonons form a prolate ellipsoid of revolution with base C and with ratio of radii $\sqrt{K_m/k_0}$.

Analogously, one can easily obtain that at first resonance point photons are emitted as two rays declined to CQW plane and PL intensity abruptly increases. With further reducing of electric field the rays widen into cones that are gradually cover entire spatial angle and PL tends to become isotropic[4].

To analyze statistical properties of phonon radiation we will treat quantum fields as single quantum modes. In interaction representation the Hamiltonian of the system describing SIE→SDE+phonon is:

$$\hat{V} = i\frac{g\hbar}{2}\left(\hat{I}^\dagger \hat{D}\hat{P} - \hat{I}\hat{D}^\dagger \hat{P}^\dagger\right).$$

Here $\hat{I}, \hat{D}, \hat{P}$ are annihilation operators of SIE, SDE and phonon modes, g is proportional to matrix element of the transition.

[3] Dispersion laws of SDE and SIE are taken in the form: $\omega_D(\mathbf{k}) = \hbar|\mathbf{k}|^2/2M + 1/2k_0c_v$, $k_0 = Mc_v/\hbar$, $\omega_I(\mathbf{k}) = \hbar|\mathbf{k} - \mathbf{K}_m|^2/2M + c_v(K_m + T(E)/2k_0)$, where $T(E)$ is a linear function of electric field value.

[4] Details will be published elsewhere.

Initial condition (before tuning the system to the resonance) is populated SIE mode and non-populated SDE and phonon modes[5]. An equation for density operator of the system is

$$\frac{\partial \hat{\rho}}{\partial t} = -\frac{i}{g}\left[\hat{V},\hat{\rho}\right] + \frac{\gamma_P}{2}\left(2\hat{P}\hat{\rho}\hat{P}^\dagger - \hat{P}^\dagger\hat{P}\hat{\rho} - \hat{\rho}\hat{P}^\dagger\hat{P}\right) +$$

$$+\frac{\gamma_D}{2}\left(2\hat{D}\hat{\rho}\hat{D}^\dagger - \hat{D}^\dagger\hat{D}\hat{\rho} - \hat{\rho}\hat{D}^\dagger\hat{D}\right). \quad (1)$$

Here γ are decay rates of corresponding partilces. We neglected decay processes of SIE because SIE recombination has tunneling character due to spatially separated e and h and thus $\gamma_D \gg \gamma_I$. We use positive P-representation [9] of density operator:

$$\rho = \int d^2\mathbf{x}\frac{\Theta(\mathbf{x})}{\langle\alpha_1,\beta_1,\xi_1|\alpha_2^*,\beta_2^*,\xi_2^*\rangle}|\alpha_1,\beta_1,\xi_1\rangle\langle\alpha_2^*,\beta_2^*,\xi_2^*|,$$

$$|\alpha,\beta,\xi\rangle = |\alpha\rangle_P|\beta\rangle_I|\xi\rangle_D,$$

$$\mathbf{x} = (\alpha_1,\alpha_2,\beta_1,\beta_2,\xi_1,\xi_2), \quad d^2\mathbf{x} = \prod_i d(\mathrm{Re}(\mathbf{x}_i))d(\mathrm{Im}(\mathbf{x}_i)),$$

where $|x\rangle_{P(I,D)}$ is a coherent state of phonon (SIE,SDE) field with eigenvalue x of operator $\hat{P}(\hat{I},\hat{D})$. Positive P-representation is convenient for evaluating normally ordered products of operators:

$$< (\hat{P}^\dagger)^i(\hat{I}^\dagger)^j(\hat{D}^\dagger)^k\hat{P}^l\hat{I}^m\hat{D}^n >= \int d^2\mathbf{x}\Theta(\mathbf{x})\alpha_2^i\beta_2^j\xi_2^k\alpha_1^l\beta_1^m\xi_1^n = \overline{\alpha_2^i\beta_2^j\xi_2^k\alpha_1^l\beta_1^m\xi_1^n}.$$

Equivalent to (1) way to describe the time evolution of the system is analysis of infinite number of equations for average values:

$$\frac{\partial}{\partial t}\overline{\prod_{k=1}^n x_{j_n}} = \sum_{k=1}^n \overline{A_{j_k}(x)\prod_{l=1,l\neq k}^n x_{j_l}} + \frac{1}{2}\sum_{k,l=1,k>l}^n \overline{(D_{j_k,j_l}+D_{j_l,j_k})\prod_{m=1,m\neq k,l}^n x_{j_m}}, \quad (2)$$

$$A_{\alpha_1} = \xi_2\beta_1 - \lambda_P\alpha_1,\ A_{\alpha_2} = \xi_1\beta_2 - \lambda_P\alpha_2,\ A_{\xi_1} = \alpha_2\beta_1 - \lambda_D\xi_1,\ A_{\xi_2} = \alpha_1\beta_2 - \lambda_D\xi_2,$$

$$A_{\beta_1} = -\alpha_1\xi_1,\ A_{\beta_2} = -\alpha_2\xi_2,\ D_{\alpha_1,\xi_1} = D_{\xi_1,\alpha_1} = \beta_1,\ D_{\alpha_2,\xi_2} = D_{\xi_2,\alpha_2} = \beta_2.$$

Here we used the substitution $(g/2)t \to t$ and $\lambda_{P,D} = \gamma_{P,D}/g$. In real semiconductors (e.g. GaAs) the rate of SIE→SDE+phonon transition, as well as the rate of recombination of SDE with photon production, is much greater than the rate of phonon attenuation in the medium. Thus we have $\lambda_P \ll 1, \lambda_D$.

One can show using (2), that averages with different power of α_1 and α_2 or ξ_1 and ξ_2, in other words phonon and SDE phase dependent averages, depend only on the averages of the same type. All of them initially equal zero and thus they remain zero for all the time with no respect to whether emitter has or has not phase properties. This is due to that phonons and SDEs share phase of SIE system (it exists only at $T = 0$), and independently they have no definite phases.

Using (2) one can get the equations describing time evolution of populations of the modes:

$$\frac{\partial}{\partial t}N_P(t) = C(t) - 2\lambda_P N_P(t),$$

[5] We analyze pulse phonon laser. Stationary generating phonon laser will be described elsewhere. See also [8].

$$\frac{\partial}{\partial t}N_I(t) = -C(t),$$
$$\frac{\partial}{\partial t}N_D(t) = C(t) - 2\lambda_D N_D,$$
$$\frac{\partial}{\partial t}C(t) = 2(N_P(t)N_I(t) + N_D(t)N_I(t) - N_P(t)N_D(t) + N_I(t)) - (\lambda_P + \lambda_D)C(t),$$
$$N_P(t) = \overline{\alpha_1\alpha_2},\ N_I(t) = \overline{\beta_1\beta_2},\ N_D(t) = \overline{\xi_1\xi_2},\ C(t) = \overline{(\alpha_1\beta_2\xi_1 + \alpha_2\beta_1\xi_2)}.$$

We factorized four variable averages to obtain a closed system of equations. The solution to this equations (Fig.3) gives that populations of SDE and SIE modes fade out on times $\approx \lambda_D^{-1}$ and population of phonon mode does on times $\approx \lambda_P^{-1}$. The populations have or have not oscillating character (i.e. the effect of SIE$\rightarrow$SDE+phonon quantum beats) with respect to whether parameter λ_2 is greater or smaller than $\sqrt{N_I(0)}$ (for $N_I(0) \gg 1$).

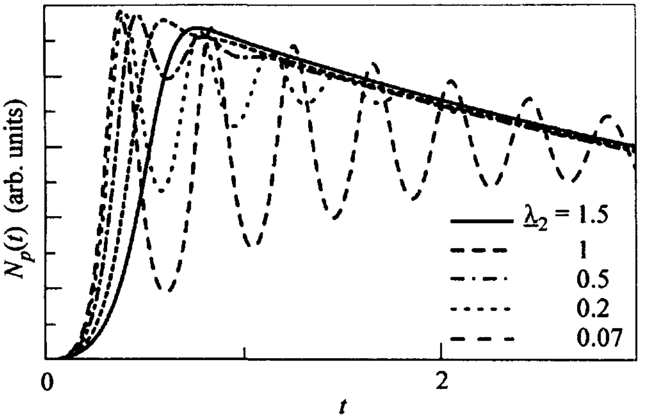

Fig.3. Time evolution of phonon mode population for different values of $\underline{\lambda}_D = \lambda_D/\sqrt{N_I(0)}$, $\lambda_P = 0.1$

When $\lambda_P^{-1} \gg t \gg \lambda_D^{-1}$ all quanta initially in SIE mode are transmitted into phonon mode, but the effect of phonon mode decay is negligible. For such times one can omit in (2) terms proportional to λ_P. In such a case the averages $\overline{(\alpha_1\alpha_2 + \beta_1\beta_2)^n}$ are the integrals of time evolution. All averages including β_1, β_2 are zero so that:

$$< (\hat{P}^\dagger(t))^n(\hat{P}(t))^n >\Big|_{\lambda_P^{-1}\gg t\gg\lambda_D^{-1}} =< (\hat{I}^\dagger(0))^n(\hat{I}(0))^n >, \quad n = 1, 2... \tag{3}$$

Phonon radiation with laser properties, i.e. low intensity fluctuations, can be viewed as quantum mode with populated Goldstone mode that is connected with phase fluctuations. Plotted as a function of variables $\hat{X}_1 = 2^{-1/2}(\hat{P}^\dagger + \hat{P})$, $\hat{X}_2 = -i2^{-1/2}(\hat{P}^\dagger - \hat{P})$, such a population is a annular with center (0,0) and with radius $\sim \sqrt{N_p}$. This can be characterized by relative dispersion of number of quanta in the mode:

$$v\left(\hat{P}^\dagger(t)\hat{P}(t)\right) = \frac{g^{(2)}(t) - N_p(t) - N_p(t)^2}{N_p(t)^2}.$$

Here $g^{(2)}(t) =< \hat{P}^\dagger(t)\hat{P}^\dagger(t)\hat{P}(t)\hat{P}(t) >$ is second order correlation function; N_p and $g^{(2)}$ can be experimentally observed by measurements with correspondingly one (intensity measurements) and two (Hunburry-Brown-Twiss–like measurements) detectors.

At sufficiently low temperatures SIEs have low n_s fluctuations and $v(\hat{I}^\dagger(0)\hat{I}(0)) \ll 1$. The lower the temperature the smaller parameter $v(\hat{I}^\dagger(0)\hat{I}(0))$ is and, therefore, the lower intensity fluctuations phonon radiation has.

Initial number of SIE is ρS, where S is the area of CQW, and they are shared between $V_q S L_z/(2\pi)^3$ phonon quantum states, where L_z is the width of the sample in z-direction and V_q is momentum space area that phonons occupy; V_q characterizes one-phonon space coherence that determines possible interference pattern between two points of phonon radiation. This is determined by the spread of SIE (k_I), and widths of SIE and SDE levels. The smallest value of V_q correspond to $T(E) = = k_0^2 + M(\gamma_I + \gamma_D)/\hbar$ and can be easily estimated from simple geometrical analysis (Fig.2) as $\sqrt{K_m/k_0}\left((k_I + k_0)^2 + 2M(\gamma_I + \gamma_D)/\hbar\right)^{3/2} \approx 10^{15}\,\mathrm{cm}^{-1}$, where we took $\gamma_D + \gamma_I \approx 10^{10}\,\mathrm{c}^{-1}$ and $k_I \approx 10^5\,\mathrm{cm}^{-1}$, k_I is mainly determined by inter-SIE dipole-dipole repulsion and can be given for estimation as $\sqrt{U(0)\rho M/\hbar}$, where $U(0)$ is zero-Fourier component of the potential. Although, Kosterlitz – Thouless transition does not directly affect phonon radiation statistics, it increases the number of quanta in each phonon quantum mode by reducing initial SIE momentum spread.

An additional condition for phonon radiation to have laser properties is a great number of quanta in each quantum state of phonon field and thus the following inequality must be true:

$$L_z \ll \rho(2\pi)^3/V_q \approx 10^{-4\div5}\,\mathrm{cm}. \qquad (4)$$

This restriction on L_z value appeared due to that phonons and SIE have different dimensionality. For convenience we propose to use a heretostructure consisting of many CQW. In such a case the distance between CQW must satisfy condition (4) for L_z.

At high temperatures source of phonon radiation – SIEs – have not low n_s fluctuations and phonon mode is just a chaotic state that can be called a phonon avalanche.

In conclusion, we proposed to use SIE dispersion engeneering for controlling of many-exciton state and for phonon laser creation. Analyzing phonon radiation statistics in the proposed scheme we obtained that phonon radiation can viewed a laser one in case of low temperatures and when condition (4) is satisfied.

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