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INERTIAL PARAMETERS AND SUPERFLUID-TO-NORMAL PHASE TRANSITION IN SUPERDEFORMED BANDS

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The quasiclassically exact solution for the second inertial parameter $\mathcal{B}$ is found in a self-consistent way. It is shown that superdeformation and nonuniform pairing arising from the rotation induced pair density significantly reduce this parameter. The new signature for the transition from pairing to normal phase is suggested in terms of the variation $\mathcal{B}/\mathcal{A}$ versus spin. Experimental data indicate the existence of such transition in the three SD mass regions.

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One of the amazing features of superdeformed (SD) rotational bands is the extreme regularity of their rotational spectra: a SD nucleus is the best quantum rotor known in nature. In spite of the fact that numerous theoretical calculations successfully reproduce the measured intraband γ-ray energies (see e.g. [1, 2]) the underlying microscopic mechanism of this phenomenon is still not well understood. To explain this regularity we use in the present letter the parameterization of its rotational energy by the two term formula

$$E(I) = \mathcal{A}I(I+1) + \mathcal{B}I^2(I+1)^2, \tag{1}$$

which is valid for an axially symmetric deformed nucleus with $K = 0$. The inertial parameters $\mathcal{A} = \hbar^2/2\Im^{(1)}$ ($\Im^{(1)}$ is the kinematic moment of inertia) and $\mathcal{B}$ are the objects of our investigation. The coefficient $\mathcal{B}$ characterizes the nonadiabatic properties of a band and is very sensitive to its internal structure. The ratio $\mathcal{B}/\mathcal{A}$ determines the convergence radius [3] of the two parameter formula (1), which is of order 100 for the bands in the 80 and 150 mass regions and 40 in the 130 and 190 ones. Faster convergence is obtained with Harris formula

$$E(\omega) = E_0 + \frac{1}{2}\alpha\omega^2 + \frac{3}{4}\beta\omega^4, \tag{2}$$

which is based on the fourth-order cranking expansion

$$\alpha = \frac{1}{\omega}Sp(\ell_x\rho^{(1)}), \qquad \beta = \frac{1}{\omega^3}Sp(\ell_x\rho^{(3)}), \tag{3}$$

where $\rho^{(n)}$ is the nth correction to the nucleus density matrix, ℓ_x is the single-particle angular momentum projection on the rotational axis x perpendicular to the symmetry axis z, and ω is the rotational frequency. For simplicity we will deal with the parameter β.

The problem of the microscopic calculation of the parameter $\mathcal{B}$ for normal deformed (ND) nuclei was attracted considerable attention. This value is formed mainly by four

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effects: vibration-rotation interaction, centrifugal stretching, perturbation of the quasiparticle motion, and attenuation of pair correlation by the Coriolis force (Mottelson – Valatin effect). The last two are dominant for well deformed nuclei as it has already been shown in the first attempts at obtaining $\mathcal{B}$ [4, 5]. Unfortunately the results of these and many other works cannot be used for the superdeformation. The formulas of Ref. [5] have been obtained in the limit of the monopole pairing interaction (the uniform pairing), which is not adequate at SD shapes as shown in Ref. [6]. In the more sophisticated work [4] the gauge invariant pairing interaction allows to study the effect of nonuniform pairing. However this approach is also limited because it neglects the coupling between major shells (the limit of close transitions in Migdal's terminology [7]). Thus, despite a number of publications on the subject the correct cranking selfconsistent solution for the $\mathcal{B}$ coefficient has not been found.

We used the quasiclassical method of Ref. [4] to derive the following expression for the β parameter in the superfluid phase

$$\beta_s = -\frac{\hbar^4}{4\Delta^2}\sum \ell^x_{12}\ell^x_{23}\ell^x_{34}\ell^x_{41}F(x_{12}, x_{23}, x_{34}, x_{41})\delta(\varepsilon_1 - \varepsilon_F), \tag{4}$$

where the summation indices i=1,2,3,4 refer to the single-particle states i of the nonrotating mean field with the energy ε_i. The δ-function means that the summation over the states 1 is restricted by a small interval at the Fermi energy ε_F [7]. The dimensionless values $x_{ii'} = (\varepsilon_i - \varepsilon_{i'})/2\Delta$, where Δ is the state independent pairing gap at $\omega = 0$, correspond to energy differences between states permitted by the selection rules for the matrix element of ℓ_x. The function F depending on these values may be written as follows

$$F = \sum_{k=0}^{3} \hat{P}_k G_{12} + \sum_{k=0}^{1} \hat{P}_k H_{13} - 8D_2^2 x_{12}x_{23}x_{34}x_{41}h(x_{13}), \tag{5}$$

where

$$G_{12} = \frac{g(x_{12})}{x_{23}x_{41}x_{13}x_{24}}\{(1 - D_1x_{12}^2)[-1 - x_{12}^2 - x_{23}x_{41} + D_1[x_{23}^2(1 - x_{12}x_{24}) + x_{41}^2(1 + x_{12}x_{13})] +$$
$$+ D_1^2x_{23}x_{34}x_{41}(x_{23} + x_{41}) + D_1^3x_{12}x_{23}^2x_{34}x_{41}^2] + D_1(x_{34} - D_1x_{12}x_{23}x_{41})(x_{34} +$$
$$+ x_{12}x_{13}x_{24} - D_1x_{12}x_{23}x_{41})\},$$

$$H_{13} = \frac{h(x_{13})}{x_{12}x_{23}x_{34}x_{41}}[1 - D_1(x_{12}^2 + x_{23}^2 + x_{34}^2 + x_{41}^2)^2 + D_1^2(x_{12}x_{41} + x_{23}x_{34})], \tag{6}$$

and

$$h(x) = (1 + x^2)g(x), \quad g(x) = \frac{\mathrm{argsh}x}{x\sqrt{1 + x^2}}. \tag{7}$$

Here $\hat{P}_k$ are permutation operators in the space of four indices i, i mod4 $= i$: $\hat{P}_k x_{i,i'} = x_{i+k,i'+k}$.

Equation (4) multiplied by ω^3 is the third order cranking correction to the total angular momentum of the neutron or proton system. Its derivation will be described in a forthcoming paper. We want now to emphasize that Eq. (4) represents the first theoretically correct expression for the high order effect of the Coriolis-pairing interaction at fixed deformation. The result is obtained by taking into account the effect of rotation on the Cooper pairs in the gauge invariant form. This effect is described by the first, $\Delta^{(1)}(\mathbf{r}) = -i\hbar^2\omega D_1\dot{\ell}_x/2\Delta$, and the second, $\Delta^{(2)}(\mathbf{r}) = \hbar^4\omega^2 D_2\dot{\ell}_x^2/4\Delta^3$, corrections to the pairing energy. The amplitudes D_1 and D_2 of the nonuniform pairing fields are found in a self-consistent way. The coordinate dependent pairing field is crucial for conservation of a nucleon current. The theory incorporating the nonuniform pairing allows also to consider the different limiting cases for the inertial parameters, which make possible the study of an interplay between rapid rotation, pairing correlations and mean field deformation in a SD band.

In order to consider this problem quantitatively we will use the axially deformed oscillator potential with the frequencies ω_x and ω_z on the corresponding axes. In this model the matrix element ℓ_{12}^x is non-zero for the two types of transitions: (*i*) transitions inside a single oscillator shell, for which $x_{12} = \pm\nu_1$; (*ii*) transitions over a shell with $x_{12} = \pm\nu_2$. The quantities ν_1 and ν_2 are the well known parameters involved in the moment of inertia [7]:

$$\nu_{1,2} = \frac{\hbar(\omega_x \mp \omega_z)}{2\Delta} = \frac{k \mp 1}{2\xi k^{2/3}}, \qquad \xi = \frac{\Delta}{\hbar\omega_\circ}, \tag{8}$$

where $\hbar\omega_\circ = 41A^{-1/3}$ MeV. Here and later we use the axis or frequency ratio $k = c/a = \omega_x/\omega_z$ and the volume conservation condition $\omega_x^2\omega_z = \omega_\circ^3$. Both of the values ν_1 and ν_2 are large for the superdeformation. The final expression for the parameter β_s in the oscillator potential is

$$\beta_s = \beta_\circ\Phi(\xi,k) = \frac{AM^3(k+1)^4}{1875\hbar^2k^{4/3}}R^6\Phi(\xi,k), \tag{9}$$

where $R = 1.2A^{1/3}$ fm is the radius of the sphere, which volume is equal to that of the spheroid with the half-axes $a < c$, M is the nucleon mass, and A is the number of nucleons. The function Φ along with its limiting cases is shown in Fig.1. It is seen that nonuniform pairing reduces substantially the parameter β_s in agreement with the estimation of Ref. [6]. On the other hand, the contribution of distant transitions is minor for small ξ. Nevertheless the later are necessary to obtain the hydrodynamic limit (see below). Since $\Phi \sim 1$ for a reasonable pairing gap, $\Delta \sim 0.5$ MeV, the order of the value β_s is $\hbar^4(\mathcal{A}/\varepsilon_F)^3$. This, along with the estimation $\mathcal{A} \sim \varepsilon_F A^{-5/3}$, gives $\mathcal{B}/\mathcal{A} \sim A^{-2}$, which overestimates the minimal value of this ratio in all the SD mass regions. Thus a small Δ and nonuniform pairing does not solve the problem of the SD band regularity.

Let us consider first the limiting case $\Delta = 0$. The correct expression for the β parameter in the normal phase has the form

$$\beta_n = -\hbar^4 \sum \ell_{12}^x\ell_{23}^x\ell_{34}^x\ell_{41}^x \sum_{k=0}^{3}\hat{P}_k\Big\{\frac{n_1}{\varepsilon_{12}\varepsilon_{13}\varepsilon_{14}}\Big\}. \tag{10}$$

The odd function of the differences $\varepsilon_{ii'} = \varepsilon_i - \varepsilon_{i'}$ leads to the cancelation of the main terms in sum (10) that decreases substantially the value of β_n, $\beta_n \sim \hbar^4A^{7/3}/\varepsilon_F^3$. Note that the centrifugal stretching effect has the same order $\beta_{str} \sim \beta_n$. Its contribution is

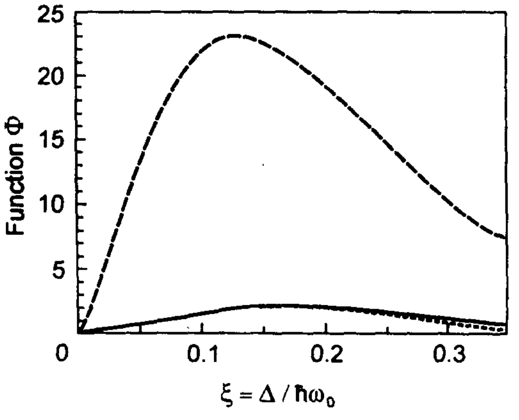

Fig.1. Plot of the function Φ from Eq.(9) against the dimensionless value ξ, for the axis ratio $c/a = 2$. The solid, dotted, and dashed lines correspond respectively to the exact value, the limit of close transitions, and the uniform pairing. The scale on the abscissa should be multiplied by a factor of approximately 7.7 for nuclei in the $A \sim 150$ mass region to obtain a gap energy in MeV

small compared to that of β_s, but it should not be overlooked for an unpaired system. For the oscillator potential, we have the following expression for a nucleus consisting of Z protons and N neutrons

$$\frac{\mathcal{B}_n}{\mathcal{A}_n} = -2.56\frac{(k^4-10k^2+1)k^{2/3}}{(k^2+1)^3 A^{8/3}}\left[\left(\frac{2Z}{A}\right)^{1/3} + \left(\frac{2N}{A}\right)^{1/3}\right]. \tag{11}$$

It is seen that this value is positive for the prolate nuclei with $c/a < 3.15$, whereas $\mathcal{B}_s$ is always negative. Thus, with an increase of the spin I, the ratio $\mathcal{B}/\mathcal{A}$ has to change sign and to approach its limiting value $\mathcal{B}_n/\mathcal{A}_n \sim A^{-8/3}$ ($\sim 10^{-6}$ for the SD bands in the 80 and 150 mass region).

One can therefore conclude that there are two distinct regions in the variation of $\mathcal{B}/\mathcal{A}$ versus I. The lower part of a SD band is characterized by a gradual decrease of the pairing gap Δ. According to Eq. (9) the ratio $\mathcal{B}/\mathcal{A}$ should exhibit a sharp increase. Then it changes sign and approaches the plateau (11) at the top of a band because the deformation c/a depends weakly on spin in both superdeformed and normal phase. Such behavior of the $\mathcal{B}/\mathcal{A}$ ratio is the signature of the pairing phase transition.

We have analyzed all the SD bands of Ref. [8] with known or suggested spins of levels. Fig.2 shows the variation of the $\mathcal{B}/\mathcal{A}$ ratio with I obtained for bands with different internal structure and different rotational frequencies. Apart from the bands ^{192}Hg(1) and ^{194}Hg(3), where frequencies are so low that $\mathcal{B}/\mathcal{A}$ rises continuously in the superfluid phase, and ^{84}Zr(1), for which pairing is quenched completely and $\mathcal{B}/\mathcal{A}$ is close to the limiting value (11), all other bands display the behavior described above. It is important to note that such behavior is observed for the ND yrast band of ^{84}Zr, where $\mathcal{B}/\mathcal{A}$ reaches the same limiting value (11) as in the SD band ^{84}Zr(1). There are other ND yrast bands of ^{168}Yb and ^{168}Hf with the pair phase transition which experimental evidence has been discussed previously in terms of the canonical variables [9] and the spectrum of single-particle states [9, 10]. These bands exhibit the same features. Thus the plots of Fig.2 demonstrate the universality of the superfluid-to-normal phase transition for SD and ND bands.

The next limit we want to consider is that of a large pairing gap Δ. In this case, the nonuniform pairing is essential and the leading terms in the function Φ are those proportional to the powers of D_1 and D_2^2. They result in the limiting expression $\Phi \sim (\hbar\omega_o/\Delta)^2$. Thus for the very strong pairing ($\Delta \gg \hbar\omega_o$), when the size of the Cooper

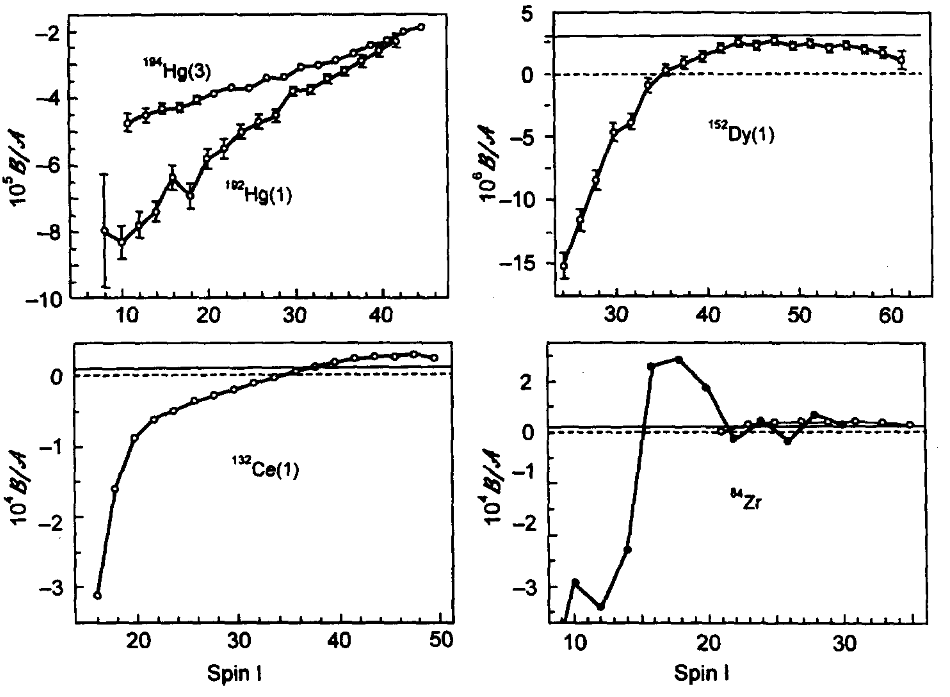

Fig.2. $\mathcal{B}/\mathcal{A}$ ratio versus spin for the SD (open circles) and ND (full circles) bands with mainly collective behavior. The solid straight line is the limiting value (11) with the deformation c/a found from the quadrupole moment. Error bars (if they are greater than symbols) include γ-ray energy uncertainties only

pair $R\hbar\omega_o/\Delta$ becomes much less than the nuclear radius, the parameter $\mathcal{B}_s$ vanishes in agreement with the hydrodynamic equations of the ideal liquid [3]. In the limit of an extremely large deformation, $c/a \to \infty$, a needle shaped nucleus with pairing correlations rotates as a rigid body, $\Im = \Im_{rig}$, $\mathcal{B}_s = 0$. For the finite but large deformation the deviations from these values are proportional to $(a/c)^{4/3}$. This means that all nucleons with the exclusion of a small sphere in the center of a nucleus are completely involved in rotational motion. Finally, for small deformations we have $\mathcal{B}_s \sim (c/a-1)^{-6}$ that is comparable to the vibration-rotation interaction [4].

Unlike the limiting value (11), it is impossible to compare with experiment the ratio $\mathcal{B}_s/\mathcal{A}_s$ because the proton (Δ_π) and neutron (Δ_ν) pairing gaps are unknown for the SD bands. In such a case we try to solve an inverse problem. The equations for the two inertial parameters

$$\alpha A/\Im_{rig} = Z\varphi(\xi_\pi, k) + N\varphi(\xi_\nu, k),$$
$$\beta A/\beta_o = Z\Phi(\xi_\pi, k) + N\Phi(\xi_\nu, k), \qquad (12)$$

allow in principle to find ξ_π and ξ_ν if we use the quadrupole moment for extraction of the axis ratio k. Unfortunately this system does not have a solution because the oscillator potential overestimates the moment of inertia and, to a greater extent, the β parameter. A more realistic potential should be used to solve this problem.

In summary, the exact solution for the inertial parameter $\mathcal{B}$ in the superfluid phase allows to show that neither superdeformation nor nonuniform pairing arising from rotation induced pair density is responsible for the extreme regularity of the SD rotational spectra. The regularity of the SD bands in the 80 and 150 mass regions is explained by the transition from the superfluid to normal phase. The new signature of this transition reveals itself in the characteristic dependence of the ratio $\mathcal{B}/\mathcal{A}$ with the spin I. Application of this criterion to experimental data indicates the existence of the phase transition in the SD bands of the three mass regions.

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