

DYNAMICALLY STABLE ELECTRON BUNCHES IN BEAM INTERACTION WITH AN ELECTROMAGNETIC WAVE PACKET

A.S.Volokitin, C.Krafft⁺

*Institute of Terrestrial Magnetism, Ionosphere and Radiowave Propagation RAS
142092 Troitsk, Moscow Region, Russia*

*⁺Laboratoire de Physique des Gaz et des Plasmas, Université Paris Sud
91405 Orsay Cedex, France*

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Nonlinear interaction of electron beam with a whistler wave packet which effectively dissipates through collisions or wave leakage is studied. Independently of dissipation type and nature of waves, self-organization of beam structure leads to the formation of bunches continuously decelerated by waves. Strong dissipation prevents the phase mixing required for quasilinear theory and keeps wave phases in the packet correlated. Thus, dynamically stable bunches are present together with a plateau in the velocity distribution; asymptotically, wave emission by bunches is the main process.

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The study of energetic beam interaction with electromagnetic waves in magnetized plasmas is motivated by many laboratory and space experiments involving beam injection [1–9] and by observations of natural suprathermal particles fluxes traveling in planetary, solar and astrophysical plasmas [10–12]. For example, emission of very low frequency waves by modulated and pulsed electron beams injected by satellites in the Earth's ionosphere and magnetosphere have been observed by several space active experiments [4–9].

In particular, theoretical investigations of nonlinear dynamics of electron beam resonant interaction with whistler waves were mainly devoted to the single monochromatic wave case or to the quasilinear theory of boundless beam relaxation in a plasma with a continuous spectrum of waves [13–15]. It was namely shown [16–19] that, in presence of strong dissipation, the nonlinear interaction of a monoenergetic electron beam with a single wave differs considerably from the nondissipative case: the beam-wave system exhibits a strong tendency to self-organization. Indeed, dissipative effects – due to collisions in plasma or to effective wave radiation out of the bounded volume of a thin beam – make the system not conservative and, as a result of the irreversible loss of wave momentum and energy, prevent the periodic energy exchange between the beam and the wave. Meanwhile, the nonlinear evolution of resonant particles is characterized by the formation of dynamically stable electron bunches which are continuously decelerated and supply energy to the wave through resonant Cherenkov interaction owing to a self-adjusted nonlinear shift of the parallel wave number [16–18].

In this letter we evidence and explain for the first time such physical effects during the nonlinear interaction of an electron beam with a packet of whistlers; moreover, it is shown that different types of dissipative beam-waves interactions can be described by similar mechanisms. In the case of a wave packet, bunched particles exchange energy with several waves and one could expect that the beam-waves system should evolve according to quasilinear theory (diffusion to lower velocities and plateau formation). However, in

the presence of strong losses of wave energy, the phases of all waves can become strongly correlated and thus can prevent the stochastic phase mixing required for the validity of quasilinear theory. This letter presents a theoretical model and related numerical simulations explaining the nonlinear evolution of the beam-waves system in terms of dynamic energy exchange, particles trapping, slowing down of the beam, self-organization of complex bunched electron structures and quasilinear diffusion.

As practically interesting example of dissipative system, we consider here a nonlinear model developed to study whistler emission through Cherenkov resonance by a density modulated thin electron beam. The beam of small radius r_b and cylindrical symmetry is injected along the ambient magnetic field $B_0 = B_0 \mathbf{z}$ with a fixed modulation frequency. The evolution of the beam current modulation is considered self-consistently as the result of the nonlinear beam particles motion in the whistlers' fields. Wave fields outside the beam are described with the approximation of a fixed beam radius as B_0 stabilizes the total perpendicular size of the beam, although the radial profile of the beam current can be modified by the action of the whistlers. All nonlinearity is held in electrons' motion; the drift approximation is considered for the motion in the direction perpendicular to B_0 . The evolution of the emitted cylindrical whistlers is characterized by slow variations of the waves' spatial structure along the distance z from the injector or, similarly, by slow changes of the parallel wave numbers.

A packet of M sheared whistlers with frequencies ω_m below the electron gyrofrequency ω_c and well above the low hybrid frequency ω_{lh} are considered (here $\omega_c \ll \omega_p$, where ω_p is the electron plasma frequency); the whistlers' parallel wave number k_{zm} is much smaller than the perpendicular one, $k_{zm} \ll k_{\perp m}$. This allows to describe electromagnetic wave fields with the potentials A_z and φ only; the perpendicular component k_{\perp} of the vector potential is small and can be expressed with the help of A_z . At given ω_m , there are two resonant whistlers with the same k_{zm} (verifying the Cherenkov resonance condition $k_{zm} v_{bz} = \omega_m$, where v_{bz} is the parallel beam velocity) and with two different perpendicular wave numbers k_{1m} and k_{2m} . Properties of whistler dispersion allow the beam to interact simultaneously with several waves of the same phase velocity, but with different ω_m and $k_{\perp m}$. The situation considered here is realistic as the spectral analysis of the modulated current of beams injected from guns on board satellites or in vacuum chambers typically exhibits not only the modulation frequency but also higher harmonics [2, 9]; moreover, modulation at different frequencies can also be applied simultaneously to the beam density [9].

As the beam is thin (i.e., $k_{\perp m} r_b \ll 1$), all fields and potentials inside and outside the beam can be described in terms of potential amplitudes at the beam center, $\Psi_m = \Theta_{1m}(z)H_{1m} + \Theta_{2m}(z)H_{2m}$, where $\Theta_{im}(z)$ are slowly varying amplitudes of cylindrical waves and $H_{im} \equiv H_1^{(i)}(k_{im} r_b)$ are Hankel functions. Then, using Maxwell equations, matching conditions at the beam boundary as well as conditions of free wave propagation to infinity, one can find the M equations describing the self-consistent nonlinear evolution of whistlers' amplitudes along the beam, including the slow modulation of the parallel beam current j_{bz} as a source term

$$\frac{\partial \Psi_m}{\partial z} + \kappa_m \Psi_m = -i \frac{2\pi v_{bz}}{\omega_p^2} \langle j_{bz,m} \rangle = i \frac{2\pi e n_b v_{bz}^2}{\omega_p^2} \frac{1}{N} \sum_{j=1}^N e^{-i\theta_{j,m}} \quad (1)$$

where n_b is the initial beam density and e the electron charge; N is the total number of beam macroparticles; $\theta_{j,m} = k_{zm} z_j - \omega_m t \equiv m\theta_j$, where $\theta_j = k_{z1} z_j - \omega_1 t$, is the phase of

the particle j in the fields of the wave harmonic m . The complex factor κ_m , depending on r_b and $k_{\perp m}$, describes new effects of energy loss by wave emission out of the beam to infinity (linear process of effective damping). The analogy to effective dissipation through collisions is not full: whereas $\text{Re}(\kappa_m)$ represents the rate of emission, $\text{Im}(\kappa_m)$ controls the reversible exchange of wave fields energy inside the beam with the outside waves. The right hand side of (1) describes the nonlinear interaction of the harmonic m with resonant electrons and results from averaging the beam current on the beam cross section. The slow evolution of the current results from variations of particles' phases θ_j due to the parallel motion of electrons in the M wave fields

$$\frac{d^2\theta_j}{dz^2} = -\frac{ek_{z1}}{m_e v_{bz}^2} \left(E_z + \frac{B_\theta E_\theta + B_r E_r}{B_0} \right) \simeq m=1 \sum^M a_m(r) \Psi_m e^{im\theta_j} + \text{c.c.} \quad (2)$$

where m_e is the electron mass; $E_{r,\theta,z}$ and $B_{r,\theta,z}$ are respectively the total electric and magnetic fields at the particle position (r, θ, z) ; $a_m(r)$ depends on the parameters of the beam-plasma system.

A similar model can describe the simple and well studied case of electrostatic waves in a collisional plasma. In normalized form, the evolution of the complex amplitude of the wave electric field $E_k = \mathcal{E}_k e^{i v_{bz} (k - k_0) t}$ and the motion of beam electrons can be presented as

$$\left(\frac{d}{d\tau} + \delta_k \right) A_k = \frac{1}{N} \sum_{j=1}^N e^{-ik'\theta_j}, \quad \frac{d^2\theta_j}{d\tau^2} = - \sum_k A_k e^{ik'\theta_j} + \text{c.c.} \quad (3)$$

where $A_k = \eta^2 E_k / \sqrt{n_0 m_e v_{bz}^2}$ and $\eta = (n_b/n_0)^{-1/3}$; n_0 is the background plasma density; $\delta_k = \eta [\nu_c - i(\omega_k - v_{bz} k)] / \omega_{k_0}$, where ν_c is a collision frequency; k_0 is the Cherenkov resonant parallel wave number, $k' = k/k_0$; $\theta_j = k_0 z_j - \omega_{k_0} t$ is the phase of the particle j in the frame moving with velocity v_{bz} and $\tau = \omega_{k_0} \eta t$. After proper normalization, equations (1), (2) and (3) take a very similar form characteristic of the dissipative beam-wave packet system.

Results of numerical simulations (solutions of systems of equations) of both above-mentioned dissipative systems show various stages in their evolution. During the so-called initial stage, waves grow according to linear instability, saturate in amplitude and begin to trap beam particles. During the trapping process, typical vortexes appear in phase spaces; some of them evolve further on in clumps or electron bunches localized in space and velocity. Then, in contradiction with the well known case of a conservative system (no dissipation), the energy given by the beam to the waves cannot be returned to them due to the irreversible loss of wave energy and momentum: no quasi-periodic exchange of energy between the waves and the trapped particles is observed. At the same time, the phase velocity of waves is self-adjusted owing to a nonlinear shift of their parallel wave number so that Cherenkov resonance with bunched particles holds. The stable nonlinear structures thus formed, so-called bunches, are continuously decelerated (slowing down of the beam); however, as the rate of energy transfer from beam to waves decreases, those damp.

In the case of a single quasi-monochromatic whistler, continuous bunch deceleration and Cherenkov resonance tuning processes can be explained with the help of a simple model describing bunches as nonlinear resonant structures. At the asymptotic stage of the interaction, a well formed bunch can be considered as a single particle with a weight $n_{tr} = N_{tr}/N$ proportional to the number N_{tr} of particles it contains; current modulation is only due to the bunched particles. As confirmed by the numerical solution, the wave

and the bunch remain in phase, i.e., $\theta_b + \psi + \pi/2 = \varphi \approx \text{const}$. Then, the bunch interaction with the dissipating wave results from the normalized form of (1), (2)

$$|A| \simeq -\frac{n_{tr}}{v} \sin \varphi, \quad \frac{dv}{d\tau} \simeq -\frac{n_{tr}}{v} \sin 2\varphi, \quad \frac{dv^2}{d\tau} \simeq -2n_{tr} \sin 2\varphi \simeq \text{const}, \quad (4)$$

where $A = |A| e^{i\psi}$ is the normalized wave amplitude and $v \simeq -d\psi/d\tau$ the nonlinear shift of phase velocity. Even in the single wave case, not only one but several bunches with different velocities can exist; they are resonant with waves present in the Fourier spectrum: indeed, slow changes in the main wave characteristics can be considered as result of the superposition of several waves whose wave numbers slightly differ. The case of the beam-wave packet interaction is much more complex, as the different waves can trap successively electrons and as result form a more wide variety of bunches (in size, velocity spread, number of particles and deceleration); those can be accelerated or decelerated according to their phase matching with waves. As the bunches' velocity decreases continuously, the bunched electrons can start interactions with waves of smaller phase velocities than the initial trapping wave. Bunches can catch up each other and merge together to form bigger bunches. Each formed bunch gives a finite contribution to the amplitude A_m of each harmonic m according to the phenomenological estimation (derived from (1), with δ_m standing as normalized form of κ_m)

$$A_m \simeq i \sum_{l=1}^{M_b} \frac{mn_l e^{-im\theta_b}}{(i\partial\theta_b/\partial\tau + m\delta_m)} \quad (5)$$

where n_l is the relative number of particles inside the bunch l and M_b the total number of bunches; θ_b is the phase of the bunch l (all particles it contains have roughly the same phase). When the formed bunches have similar characteristics, their behavior with respect to the waves is roughly equivalent and all can be resonant with waves. On the contrary, when one bunch contains much more electrons than others, it dominates the system's dynamics as only it interacts strongly with waves. Other bunches participate weakly to the radiation process, even though they are in resonance with waves. But, as each A_m results from the sum of components resonant with each bunch (see (5)), a non vanishing (in average) resonant deceleration force $F_l = d^2\theta_b/d\tau^2$ acts on particles in the bunch l ; phases of all waves being well correlated, this force is proportional to the relative number of particles in the bunch, $F_l \propto \text{Re} [\sum_m iA_m e^{im\theta_b}] \propto n_l$.

At the asymptotic stage of the interaction, beam particles are definitively separated in two distinct groups: dynamically stable bunches continuously decelerated in resonance with waves and a stochastic population of non resonant electrons, so-called bulk, presenting more or less diffusion to lower velocities. The number of particles in each bunch as well as in the bulk is established during the trapping process by waves and the subsequent bunches' formation; it depends on beam parameters and initial conditions. This picture is different from that usually expected from quasilinear theory where the system evolves asymptotically toward a plateau distribution through velocity diffusion; in our case, additional nonlinear stable structures are present in the velocity distribution which allow the beam to radiate energy out of its volume on a significative distance from its injection point. Thus, strong effective dissipation can prevent the stochastic phase mixing required for the validity of quasilinear theory and keep the phases of all waves well correlated. Sequences of bunches (strongly modulated electron beam) propagating together

with forced electric field perturbations (modulated wave packet) can be considered as nonlinear Van-Kampen modes in a plasma including dissipation.

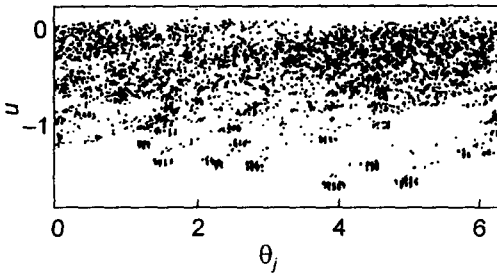


Fig.1. Thin beam interaction with a packet of $M = 20$ whistlers : Electron dynamics in phase space $\theta_j - u$, where u and θ_j are respectively the normalized parallel velocity of electrons in the frame moving with the initial beam velocity v_{bz} and their total phase. An uniform initial particle distribution has been used as well as dimensionless parameters corresponding to active space experiments; $M=20$, $N=6000$, $n_b/n_0 \simeq 0.02$

Main features of the beam-waves system evolution are illustrated on Fig.1 and 2 presenting results of numerical simulations of the interaction of a thin modulated electron beam with a packet of whistlers in presence of dissipative effects (wave radiation out of the bounded beam volume). Fig.1 shows the electron phase space at a stage where most of bunches are well formed after trapping by the wave packet and well separated from the non resonant bulk. On Fig.2 one can see the evolution with the distance to the injector of the parallel beam velocity distribution, exhibiting bunch deceleration, bulk heating, velocity diffusion and formation of a plateau with fine bunched structures, as well as the electromagnetic energy carried by the waves.

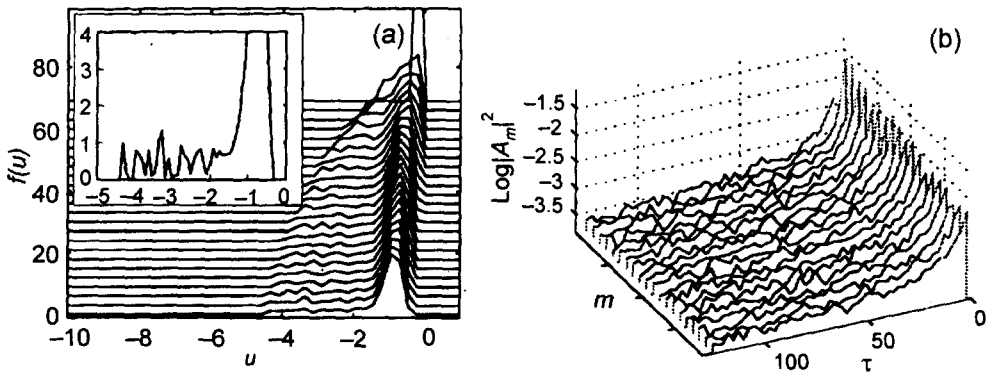


Fig.2. Thin beam interaction with a packet of $M = 20$ whistlers : Variation as a function of the normalized distance τ to the injector ($0 < \tau < 125$) of : (a) the parallel velocity distribution $f(u)$ ($\tau = 0$ for the upper curve); the small picture on the left upper corner represents a zoom of $f(u)$ in the asymptotic stage, at $\tau = 125$; a diffusion plateau together with bunched structures can be observed; (b) the normalized electromagnetic energy $|A_m|^2$ (in logarithmic scale) carried by the M waves m . Parameters are the same as in Fig.1

In conclusion, numerical simulations of beam interaction with a finite number of waves in presence of effective dissipation have shown that, independently of the dissipation type and the nature of the considered waves, the nonlinear evolution of the particles' distribution has a tendency to self-organization, leading to the formation of strongly concentrated electron structures. These bunches of resonant particles are decelerated continuously by friction on waves and their dynamics shows noticeable stability in a range of time exceeding several characteristic times of their formation. When the number of waves in the packet is big and the wave spectrum is continuous, quasilinear diffusion of particles in velocity space and plateau formation in the velocity distribution are usually

expected to occur during the beam relaxation stage. In the strongly dissipative case however, our calculations show the coexistence in the velocity distribution of a wide and very low plateau together with small peaks (at lower velocities) corresponding to stable electron bunches, which typically contain around 1-10% of the total number of particles. On the other hand, the plateau itself exhibits a fine structure consisting in a big set of small and almost indistinguishable bunches. At the asymptotic stage of the evolution, the deceleration rate of bunches and, correspondingly, their whistler emission rate are proportional to the number of particles they contain. At the same time, each wave is supported by all bunches and in average the waves in the packet have almost the same amplitudes, but not the same emission rates (which in the whistler case are proportional to the wave frequency). If the total number of particles gathered in bunches is not very small, the whistler energy emitted during the long asymptotic stage of beam relaxation (and the total loss of beam energy) can exceed the whistler emission in the initial stage of the interaction (i.e., near the injector) although the latter is much more intense. So, it is essential to find a way how to control the amount of particles organized into bunches in order to increase the emission efficiency. Our calculations show that one possibility to achieve this aim is to premodulate the beam at one or several given frequencies.

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