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NON-FORWARD COLOUR OCTET BFKL KERNEL

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The kernel of the non-forward BFKL equation is obtained in the case of the antisymmetric colour octet state of the two Reggeized gluons in the t-channel. The explicit form of the kernel is presented.

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The most common basis for the description of processes at small values of $x = Q^2/s$ (Q^2 is a typical virtuality and $\sqrt{s}$ is the c.m.s. energy) in the framework of the perturbative QCD is the BFKL equation [1]. Originally the equation was derived in the leading logarithmic approximation (LLA), which means resummation of all terms of the type $[\alpha_s \ln s]^n$. Recently, the kernel of the equation was obtained in the next-to-leading approximation (NLA) [2] for the case of the forward scattering, i.e. for the momentum transfer $t = 0$ and the vacuum quantum numbers in the t-channel. The next step should be calculation of the non-forward kernel. Here we present the non-forward kernel in the case of the gluon channel (antisymmetric colour octet state of the two Reggeized gluons). This channel is extremely important since the derivation of the BFKL equation is based on one of the remarkable properties of QCD – the gluon Reggeization. The colour octet kernel enters the "bootstrap" equations [3] appearing as the requirement of the compatibility of the gluon Reggeization with the s-channel unitarity.

We will consider pure gluodynamics, since the quark part of the kernel was calculated in [4]. We use the Sudakov decomposition for particle momenta, assuming, without any loss of generality, masses of the colliding particles with momenta p_A and p_B equal zero: $p_A^2 = p_B^2 = 0, \quad (p_A + p_B)^2 = 2(p_A p_B) = s$. The BFKL equation is the equation for the Mellin transform of the Green's function of two Reggeized gluons. If the initial momenta of these gluons in the s-channel are $q_1 \simeq \beta p_A + q_{1\perp}$ and $-q_2 \simeq \alpha p_B - q_{2\perp}$, the momentum transfer is $q \simeq q_\perp$, then the equation has the form [3]:

$$\omega G_\omega^{(\mathcal{R})}(\mathbf{q}_1, \mathbf{q}_2, \mathbf{q}) =$$

$$= \mathbf{q}_1^2 \mathbf{q}_1'^2 \delta^{(D-2)}(\mathbf{q}_1 - \mathbf{q}_2) + \int \frac{d^{D-2}r}{\mathbf{r}^2 \mathbf{r}'^2} \mathcal{K}^{(\mathcal{R})}(\mathbf{q}_1, \mathbf{r}; \mathbf{q})\, G_\omega^{(\mathcal{R})}(\mathbf{r}, \mathbf{q}_2; \mathbf{q}), \qquad (1)$$

where $\mathcal{R}$ denotes the representation of the colour group in the t-channel. The transverse momenta are spacelike and we use the vector sign for them. Here and below we use for brevity $v' \equiv v - q$ for any v. The space-time dimension $D = 4 + 2\epsilon$ is taken different from 4 to regularize the infrared divergences. We use the normalization adopted in [3].

The non-forward kernel, as well as the forward one, is given by the sum of the "virtual" part, defined by the gluon trajectory $j(t) = 1 + \omega(t)$, and the "real" part $\mathcal{K}_r^{(\mathcal{R})}$, related to the real particle production in the Reggeon – Reggeon collisions

$$\mathcal{K}^{(\mathcal{R})}(\mathbf{q}_1, \mathbf{q}_2; \mathbf{q}) =$$

$$= \left[\omega\left(-\mathbf{q}_1^2\right) + \omega\left(-\mathbf{q}_1'^{\,2}\right)\right] \mathbf{q}_1^2 \mathbf{q}_1'^{\,2} \delta^{(D-2)}\left(\mathbf{q}_1 - \mathbf{q}_2\right) + \mathcal{K}_r^{(\mathcal{R})}\left(\mathbf{q}_1, \mathbf{q}_2; \mathbf{q}\right). \tag{2}$$

As it is seen from (2), the gluon trajectory enters in the equation in the universal (independent from $\mathcal{R}$) way. It was calculated [5] in the NLA and we will not discuss it here. The "real" part for the gluon channel in the LLA is [1,6]:

$$\mathcal{K}_r^{(8)B}\left(\mathbf{q}_1, \mathbf{q}_2; \mathbf{q}\right) = \frac{g^2 N}{2(2\pi)^{D-1}} \left(\frac{\mathbf{q}_1^2 \mathbf{q}_2'^{\,2} + \mathbf{q}_2^2 \mathbf{q}_1'^{\,2}}{(\mathbf{q}_1 - \mathbf{q}_2)^2} - \mathbf{q}^{\,2} \right), \tag{3}$$

where the superscript B means the LLA (Born) approximation. Let us stress that in this paper we will systematically use the perturbative expansion in terms of the bare coupling g. In the NLA the "real" part of the kernel can be presented as [3]:

$$\mathcal{K}_r^{(8)}\left(\mathbf{q}_1, \mathbf{q}_2; \mathbf{q}\right) =$$

$$= \frac{f_{c_1 c_1' c} f_{c_2 c_2' c}}{2N(N^2-1)} \int_0^\infty \frac{ds_{RR}}{(2\pi)^D} \theta\left(s_\Lambda - s_{RR}\right) \sum_{\{f\}} \int \gamma_{c_1 c_2}^{\{f\}}\left(q_1, q_2\right) \left(\gamma_{c_1' c_2'}^{\{f\}}\left(q_1', q_2'\right)\right)^* d\rho_f -$$

$$-\frac{1}{2} \int \frac{d^{D-2} r}{\mathbf{r}^2 \mathbf{r}'^{\,2}} \mathcal{K}_r^{(8)B}\left(\mathbf{q}_1, \mathbf{r}; \mathbf{q}\right) \mathcal{K}_r^{(8)B}\left(\mathbf{r}, \mathbf{q}_2; \mathbf{q}\right) \ln\left(\frac{s_\Lambda^2}{(\mathbf{r} - \mathbf{q}_1)^2 (\mathbf{r} - \mathbf{q}_2)^2} \right). \tag{4}$$

Here f_{abc} are the group structure constants, N is the number of colours ($N = 3$ in QCD), $s_{RR} = \left(q_1 - q_2\right)^2 = s\alpha\beta - (\mathbf{q}_1 - \mathbf{q}_2)^2$ is the squared invariant mass of the Reggeons, the sum $\{f\}$ is performed over all states f which are produced in the Reggeon-Reggeon collisions and over all discreet quantum numbers of these states, $\gamma_{c_1 c_2}^{\{f\}}\left(q_1, q_2\right)$ is the effective vertex for the production of the state f and $d\rho_f$ is the phase space element for this state,

$$d\rho_f = \frac{1}{n!} (2\pi)^D \delta^{(D)}(q_1 - q_2 - \sum_{i \in f} k_i) \prod_{i \in f} \frac{d^{D-1} k_i}{(2\pi)^{D-1} 2\epsilon_i}, \tag{5}$$

where n is a number of identical particles in the state f. The second term in the r.h.s. of Eq. (4) serves for the subtraction of the contribution of the large s_{RR} region in the first term, in order to avoid a double counting of this region in the BFKL equation. The intermediate parameter s_Λ in Eq. (4) must be taken tending to infinity. At large s_{RR} only the contribution of the two-gluon production does survive in the first integral, so that the dependence on s_Λ disappears in Eq. (4) due to the factorization property of the two-gluon production vertex [3]. In the LLA only the one-gluon production does contribute and Eq. (4) gives for the kernel its LLA value (3); in the NLA the contributing states include also the two-gluon and the quark-antiquark states. The normalization of the corresponding vertices is defined in Ref. [3].

The gluon contribution to the Reggeon-Reggeon-gluon vertex in the limit $\epsilon \to 0$ was calculated in [7]. Putting the results of [7] in (4) we obtain the contribution to the kernel from the one-gluon production in the Reggeon-Reggeon collisions:

$$\mathcal{K}_{RRG}^{(8)}(\mathbf{q}_1, \mathbf{q}_2; \mathbf{q}) = \frac{g^2 N}{2(2\pi)^{D-1}} \left\{ \left(\frac{\mathbf{q}_1^2 \mathbf{q}_2'^{\,2} + \mathbf{q}_1'^{\,2} \mathbf{q}_2^2}{\mathbf{k}^{\,2}} - \mathbf{q}^{\,2} \right) \times \right.$$

$$\times \left(\frac{1}{2} + \frac{g^2 N \Gamma(1-\epsilon)}{2(4\pi)^{2+\epsilon}} \left[-(\mathbf{k}^{\,2})^\epsilon \left(\frac{2}{\epsilon^2} - \pi^2 + 4\epsilon\zeta(3) \right) - \ln^2\left(\frac{\mathbf{q}_1^2}{\mathbf{q}_2^2} \right) \right] \right) +$$

$$+\frac{g^2N\Gamma(1-\epsilon)}{6(4\pi)^{2+\epsilon}}\left(\left[\frac{\mathbf{q}_1'^2-\mathbf{q}_2'^2}{\mathbf{q}_1^2-\mathbf{q}_2^2}-\frac{\mathbf{k}^2}{(\mathbf{q}_1^2-\mathbf{q}_2^2)^2}\left(\mathbf{q}_1^2+\mathbf{q}_2^2+4\mathbf{q}_1'\mathbf{q}_2'-2\mathbf{q}^2\right)\right]\times\right.$$

$$\times\left[\frac{2\mathbf{q}_1^2\mathbf{q}_2^2}{\mathbf{q}_1^2-\mathbf{q}_2^2}\ln\left(\frac{\mathbf{q}_1^2}{\mathbf{q}_2^2}\right)-\mathbf{q}_1^2-\mathbf{q}_2^2\right]+11\left[\frac{2\mathbf{q}_1^2\mathbf{q}_2^2}{\mathbf{q}_1^2-\mathbf{q}_2^2}+\frac{\mathbf{q}_1^2\mathbf{q}_2'^2-\mathbf{q}_1'^2\mathbf{q}_2^2}{\mathbf{k}^2}-\frac{\mathbf{q}_1^2+\mathbf{q}_2^2}{\mathbf{q}_1^2-\mathbf{q}_2^2}\mathbf{q}^2\right]\times$$

$$\left.\left.\times\ln\left(\frac{\mathbf{q}_1^2}{\mathbf{q}_2^2}\right)-2\mathbf{q}_1'\mathbf{q}_2'\right)\right\}+\frac{g^2N}{2(2\pi)^{D-1}}\left\{\mathbf{q}_i\longleftrightarrow\mathbf{q}_i'\right\}, \tag{6}$$

where $\mathbf{k}=\mathbf{q}_1-\mathbf{q}_2$, $\Gamma(x)$ is the Euler gamma-function and $\zeta(n)$ is the Riemann zeta-function. The effective vertex for the two-gluon production in the Reggeon-Reggeon collisions was calculated in [8]. The contribution to the kernel from the two-gluon production can be obtained putting the results of [8] in (4), taking the sum over colour and spin states of the produced gluons and performing the integration. Unfortunately, the integral (4) can not be expressed in terms of elementary functions (and dilogarithms) at arbitrary ϵ. Therefore we present the result (see details of the calculation in [9]) in a "combined" form, leaving untouched the terms which can not be integrated in elementary functions:

$$\mathcal{K}_{RRGG}^{(8)}(\mathbf{q}_1,\mathbf{q}_2;\mathbf{q})=\frac{4g^4N^2\Gamma(1-\epsilon)}{(4\pi)^D\pi^{1+\epsilon}}\frac{\Gamma^2(1+\epsilon)}{\Gamma(1+2\epsilon)}\left\{\frac{(\mathbf{k}^2)^{\epsilon-1}}{4\epsilon}\left[(\mathbf{q}_1'^2\mathbf{q}_2^2+\mathbf{q}_2'^2\mathbf{q}_1^2)\times\right.\right.$$

$$\left.\times\left(\frac{1}{\epsilon}+\psi(1)+\psi(1+\epsilon)-2\psi(1+2\epsilon)-\frac{11+7\epsilon}{2(1+2\epsilon)(3+2\epsilon)}\right)-\frac{\mathbf{q}^2\mathbf{k}^2}{\epsilon}\right]+$$

$$+\frac{(\mathbf{q}^2)^{\epsilon+1}}{4\epsilon}\left(-\frac{1}{\epsilon}+\psi(1)-\psi(1-\epsilon)+2\psi(1+\epsilon)-2\psi(1+2\epsilon)-\frac{11+7\epsilon}{2(1+2\epsilon)(3+2\epsilon)}\right)-$$

$$-\frac{11+7\epsilon}{4\epsilon(1+2\epsilon)(3+2\epsilon)}\mathbf{q}_1^2\mathbf{q}_2^2\frac{(\mathbf{q}_1^2)^\epsilon-(\mathbf{q}_2^2)^\epsilon}{\mathbf{q}_1^2-\mathbf{q}_2^2}+\frac{\mathbf{q}^2}{\epsilon(1+2\epsilon)}\frac{(\mathbf{q}_1^2)^{\epsilon+1}-(\mathbf{q}_2^2)^{\epsilon+1}}{\mathbf{q}_1^2-\mathbf{q}_2^2}+$$

$$+\frac{\mathbf{q}^2}{4\epsilon}\left(\frac{1}{2\epsilon}-\psi(1+\epsilon)+\psi(1+2\epsilon)\right)\left((\mathbf{q}_1^2)^\epsilon+(\mathbf{q}_2^2)^\epsilon\right)+\frac{1}{8\epsilon(1+2\epsilon)(3+2\epsilon)}\times$$

$$\times\left[\left(2(1+\epsilon)\mathbf{q}_1^2\mathbf{q}_2^2\left((\mathbf{q}_1^2)^\epsilon-(\mathbf{q}_2^2)^\epsilon\right)-\epsilon\left(\mathbf{q}_1^2+\mathbf{q}_2^2\right)\left((\mathbf{q}_1^2)^{\epsilon+1}-(\mathbf{q}_2^2)^{\epsilon+1}\right)\right)\times\right.$$

$$\times\left(\frac{\mathbf{k}^2}{(\mathbf{q}_1^2-\mathbf{q}_2^2)^3}\left(\mathbf{q}_1^2+\mathbf{q}_2^2+2\mathbf{q}_1'^2+2\mathbf{q}_2'^2-2\mathbf{k}^2-2\mathbf{q}^2\right)-\frac{\mathbf{q}_1'^2-\mathbf{q}_2'^2}{(\mathbf{q}_1^2-\mathbf{q}_2^2)^2}\right)+$$

$$\left.\left.+\frac{(\mathbf{q}_1^2)^{\epsilon+1}-(\mathbf{q}_2^2)^{\epsilon+1}}{\mathbf{q}_1^2-\mathbf{q}_2^2}\left(\epsilon\left(\mathbf{q}_1'^2+\mathbf{q}_2'^2-\mathbf{k}^2\right)-2(1+\epsilon)\mathbf{q}^2\right)\right]+\frac{\Gamma(1+2\epsilon)}{4\Gamma^2(1+\epsilon)}I(\mathbf{q}_1,\mathbf{q}_2;\mathbf{q})\right\}+$$

$$+\frac{4g^4N^2}{(4\pi)^D\pi^{1+\epsilon}}\Gamma(1-\epsilon)\frac{\Gamma^2(1+\epsilon)}{\Gamma(1+2\epsilon)}\{q_1\leftrightarrow q_1',\ q_2\leftrightarrow q_2'\}, \tag{7}$$

where $\psi(x)=\Gamma'(x)/\Gamma(x)$ and

$$I(\mathbf{q}_1,\mathbf{q}_2;\mathbf{q})=$$

$$=4\hat{\mathcal{S}}\int_0^1\frac{dx}{x(1-x)_+}\int\frac{d^{2+2\epsilon}k_1}{\pi^{1+\epsilon}\Gamma(1-\epsilon)}\left(\frac{x\mathbf{q}_2^2}{2\mathbf{\Lambda}^2\bar{t}_1}\left[\frac{x\left(\mathbf{q}_1^2(\mathbf{k}_1\mathbf{q}_1')-\mathbf{q}_1'^2(\mathbf{k}_1\mathbf{q}_1)\right)}{\mathbf{k}_1^2}-(\mathbf{q}\mathbf{q}_1')\right]-\right.$$

$$\left.-\frac{x^2(1-x)}{4\mathbf{\Lambda}^2\bar{t}_1}\left[\frac{\mathbf{q}_1'^2\mathbf{q}_2^4}{\Sigma}+\frac{\mathbf{q}_1^4\mathbf{q}_2'^2-\mathbf{q}_1^2\mathbf{q}^2\mathbf{k}^2}{\mathbf{k}_1^2}\right]\right)-\frac{\Gamma^2(\epsilon)}{\Gamma(2\epsilon)}\frac{(\mathbf{k}^2)^{\epsilon-1}}{2\epsilon}(\mathbf{q}_1^2\mathbf{q}_2'^2+\mathbf{q}_2^2\mathbf{q}_1'^2-\mathbf{q}^2\mathbf{k}^2)-$$

$$-\frac{(\mathbf{q}^2)^2}{4}\frac{1}{\pi^{1+\epsilon}\Gamma(1-\epsilon)}\int\frac{d^{2+2\epsilon}l}{(\mathbf{q}_1-\mathbf{l})^2(\mathbf{q}_1'-\mathbf{l})^2}\ln\left(\frac{\mathbf{l}^2(\mathbf{l}-\mathbf{k})^2}{\mathbf{q}^4}\right). \qquad (8)$$

Here $\hat{\mathcal{S}}$ denotes the operator of symmetrization with respect to the substitution $\mathbf{q}_1 \leftrightarrow -\mathbf{q}_2$, $\mathbf{q}_1' \leftrightarrow -\mathbf{q}_2'$, $(1-x)_+$ means the subtraction:

$$\int_0^1 \frac{dx}{(1-x)_+}f(x) \equiv \int_0^1 \frac{dx}{(1-x)}[f(x)-f(1)], \qquad (9)$$

$$\Sigma = (1-x)\mathbf{k}_1^2 + x(\mathbf{k}-\mathbf{k}_1)^2,\ \tilde{t}_1 = -\frac{1}{x}\left((1-x)\mathbf{k}_1^2 + x(\mathbf{k}_1-\mathbf{q}_1)^2\right), \qquad (10)$$

and $\mathbf{\Lambda} = \mathbf{k}_1 - x\mathbf{k}$. In the limit $\epsilon \to 0$ we have

$$\mathcal{K}_{RRGG}^{(8)}(\mathbf{q}_1,\mathbf{q}_2;\mathbf{q}) = \frac{g^4N^2\Gamma(1-\epsilon)}{(4\pi)^D\pi^{1+\epsilon}}\left\{(\mathbf{k}^2)^{\epsilon-1}\left(\mathbf{q}_1'^2\mathbf{q}_2^2+\mathbf{q}_2'^2\mathbf{q}_1^2-\mathbf{q}^2\mathbf{k}^2\right)\left(\frac{1}{\epsilon^2}-\frac{11}{6\epsilon}+\frac{67}{18}-\right.\right.$$

$$\left.-4\zeta(2)+\epsilon\left(-\frac{202}{27}+9\zeta(3)+\frac{11}{6}\zeta(2)\right)\right)+\mathbf{q}^2\left[\frac{1}{2}\left(\frac{1}{\epsilon}+\ln\mathbf{q}^2\right)\ln\left(\frac{\mathbf{q}_1^2\mathbf{q}_2^2}{\mathbf{q}^4}\right)-2\zeta(2)+\right.$$

$$\left.+\frac{11}{6}\left(\ln\left(\frac{\mathbf{q}_1^2\mathbf{q}_2^2}{\mathbf{k}^2\mathbf{q}^2}\right)+\frac{\mathbf{q}_1^2+\mathbf{q}_2^2}{\mathbf{q}_1^2-\mathbf{q}_2^2}\ln\left(\frac{\mathbf{q}_1^2}{\mathbf{q}_2^2}\right)\right)+\frac{1}{4}\ln^2\left(\frac{\mathbf{q}_1^2}{\mathbf{q}^2}\right)+\frac{1}{4}\ln^2\left(\frac{\mathbf{q}_2^2}{\mathbf{q}^2}\right)\right]+\frac{\mathbf{q}_1\mathbf{q}_2}{3}-$$

$$-\frac{11}{6}(\mathbf{q}_1^2+\mathbf{q}_2^2)-\frac{1}{6}\left(11-\frac{\mathbf{k}^2}{(\mathbf{q}_1^2-\mathbf{q}_2^2)^2}\left(\mathbf{q}_1^2+\mathbf{q}_2^2+4(\mathbf{q}_1'\mathbf{q}_2')-2\mathbf{q}^2\right)+\frac{\mathbf{q}_1'^2-\mathbf{q}_2'^2}{\mathbf{q}_1^2-\mathbf{q}_2^2}\right)\times$$

$$\left.\times\left(\frac{2\mathbf{q}_1^2\mathbf{q}_2^2}{\mathbf{q}_1^2-\mathbf{q}_2^2}\ln\left(\frac{\mathbf{q}_1^2}{\mathbf{q}_2^2}\right)-\mathbf{q}_1^2-\mathbf{q}_2^2\right)+I(\mathbf{q}_1,\mathbf{q}_2;\mathbf{q})\right\}+\frac{g^4N^2\Gamma(1-\epsilon)}{(4\pi)^D\pi^{1+\epsilon}}\left\{\mathbf{q}_i\longleftrightarrow\mathbf{q}_i'\right\}. \qquad (11)$$

In this limit the function $I(\mathbf{q}_1,\mathbf{q}_2;\mathbf{q})$ takes the form

$$I(\mathbf{q}_1,\mathbf{q}_2;\mathbf{q}) = \frac{1}{2}\int_0^1\frac{dx}{(\mathbf{q}_1(1-x)+\mathbf{q}_2x)^2}\ln\left(\frac{\mathbf{q}_1^2(1-x)+\mathbf{q}_2^2x}{\mathbf{k}^2x(1-x)}\right)\times$$

$$\times[\mathbf{q}^2(\mathbf{k}^2-\mathbf{q}_1^2-\mathbf{q}_2^2)+2\mathbf{q}_1^2\mathbf{q}_2^2-\mathbf{q}_1^2\mathbf{q}_2'^2-\mathbf{q}_2^2\mathbf{q}_1'^2+\frac{\mathbf{q}_1^2\mathbf{q}_2'^2-\mathbf{q}_2^2\mathbf{q}_1'^2}{\mathbf{k}^2}(\mathbf{q}_1^2-\mathbf{q}_2^2)]+$$

$$+\frac{\mathbf{q}^2}{2}\left(4\zeta(2)-\frac{1}{2}\ln^2\left(\frac{\mathbf{q}_1^2}{\mathbf{q}_2^2}\right)\right)-\frac{\mathbf{q}_1^2\mathbf{q}_2'^2-\mathbf{q}_2^2\mathbf{q}_1'^2}{4\mathbf{k}^2}\ln\left(\frac{\mathbf{q}_1^2}{\mathbf{q}_2^2}\right)\ln\left(\frac{\mathbf{q}_1^2\mathbf{q}_2^2}{\mathbf{k}^4}\right)-$$

$$-\frac{\mathbf{q}^2}{4}\left[\left(\frac{1}{\epsilon}+\ln\mathbf{q}^2\right)\ln\left(\frac{\mathbf{q}_1^2\mathbf{q}_2^2\mathbf{q}_1'^2\mathbf{q}_2'^2}{\mathbf{q}^8}\right)+\frac{1}{2}\ln^2\left(\frac{\mathbf{q}_1^2}{\mathbf{q}_1'^2}\right)+\frac{1}{2}\ln^2\left(\frac{\mathbf{q}_2^2}{\mathbf{q}_2'^2}\right)\right]. \qquad (12)$$

The integral in (12) can be presented in another form:

$$\int_0^1\frac{dx}{(\mathbf{q}_1(1-x)+\mathbf{q}_2x)^2}\ln\left(\frac{\mathbf{q}_1^2(1-x)+\mathbf{q}_2^2x}{\mathbf{k}^2x(1-x)}\right)=$$

$$=\int_0^\infty\frac{dz}{z+\mathbf{k}^2}\frac{1}{\sqrt{(\mathbf{q}_1^2+\mathbf{q}_2^2+z)^2-4\mathbf{q}_1^2\mathbf{q}_2^2}}\times$$

$$\times\ln\left(\frac{\mathbf{q}_1^2+\mathbf{q}_2^2+z+\sqrt{(\mathbf{q}_1^2+\mathbf{q}_2^2+z)^2-4\mathbf{q}_1^2\mathbf{q}_2^2}}{\mathbf{q}_1^2+\mathbf{q}_2^2+z-\sqrt{(\mathbf{q}_1^2+\mathbf{q}_2^2+z)^2-4\mathbf{q}_1^2\mathbf{q}_2^2}}\right). \qquad (13)$$

It is possible also to express the integral in (12) in terms of dilogarithms, but this expression is not very convenient:

$$\int_0^1 \frac{dx}{(\mathbf{q}_1(1-x)+\mathbf{q}_2 x)^2} \ln\left(\frac{\mathbf{q}_1^2(1-x)+\mathbf{q}_2^2 x}{\mathbf{k}^2 x(1-x)}\right) = -\frac{2}{|\mathbf{q}_1||\mathbf{q}_2|\sin\phi}\times$$

$$\times\left[\ln\rho\arctan\frac{\rho\sin\phi}{1-\rho\cos\phi} + \mathrm{Im}\left(L\left(\rho\exp i\phi\right)\right)\right], \tag{14}$$

where ϕ is the angle between $\mathbf{q}_1$ and $\mathbf{q}_2$,

$$\rho = \min\left(\frac{|\mathbf{q}_1|}{|\mathbf{q}_2|}, \frac{|\mathbf{q}_2|}{|\mathbf{q}_1|}\right), \quad L(z) = \int_0^z \frac{dt}{t}\ln(1-t). \tag{15}$$

The "real" part of the kernel, related to real particle production in the Reggeon-Reggeon collisions, in the NLA is given by the one-gluon and the two-gluon contributions. Since the radiative corrections to the effective vertex of the one-gluon production are known only in the limit $\epsilon \to 0$, the total "real" part of the kernel can be obtained only in this limit. It is given by the sum of (6) and (11). After powerful cancellations (in particular, between the terms with singularities $1/\epsilon^2$ and all terms with $\mathbf{q}_1^2 - \mathbf{q}_2^2$ in denominators) we obtain

$$\mathcal{K}_r^{(8)}(\mathbf{q}_1,\mathbf{q}_2;\mathbf{q}) = \frac{g^2 N}{2(2\pi)^{D-1}}\left\{\left(\frac{\mathbf{q}_1^2\mathbf{q}_2'^2+\mathbf{q}_1'^2\mathbf{q}_2^2}{\mathbf{k}^2} - \mathbf{q}^2\right)\times\right.$$

$$\times\left(\frac{1}{2} + \frac{g^2 N\Gamma(1-\epsilon)(\mathbf{k}^2)^\epsilon}{(4\pi)^{2+\epsilon}}\left(-\frac{11}{6\epsilon}+\frac{67}{18}-\zeta(2)+\epsilon\left(-\frac{202}{27}+7\zeta(3)+\frac{11}{6}\zeta(2)\right)\right)\right)+$$

$$+\frac{g^2 N\Gamma(1-\epsilon)}{(4\pi)^{2+\epsilon}}\times$$

$$\times\left[\mathbf{q}^2\left(\frac{11}{6}\ln\left(\frac{\mathbf{q}_1^2\mathbf{q}_2^2}{\mathbf{q}^2\mathbf{k}^2}\right)+\frac{1}{4}\ln\left(\frac{\mathbf{q}_1^2}{\mathbf{q}^2}\right)\ln\left(\frac{\mathbf{q}_1'^2}{\mathbf{q}^2}\right)+\frac{1}{4}\ln\left(\frac{\mathbf{q}_2^2}{\mathbf{q}^2}\right)\ln\left(\frac{\mathbf{q}_2'^2}{\mathbf{q}^2}\right)+\frac{1}{4}\ln^2\left(\frac{\mathbf{q}_1^2}{\mathbf{q}_2^2}\right)\right)-\right.$$

$$-\frac{\mathbf{q}_1^2\mathbf{q}_2'^2+\mathbf{q}_2^2\mathbf{q}_1'^2}{2\mathbf{k}^2}\ln^2\left(\frac{\mathbf{q}_1^2}{\mathbf{q}_2^2}\right)+\frac{\mathbf{q}_1^2\mathbf{q}_2'^2-\mathbf{q}_2^2\mathbf{q}_1'^2}{\mathbf{k}^2}\ln\left(\frac{\mathbf{q}_1^2}{\mathbf{q}_2^2}\right)\left(\frac{11}{6}-\frac{1}{4}\ln\left(\frac{\mathbf{q}_1^2\mathbf{q}_2^2}{\mathbf{k}^4}\right)\right)+$$

$$+\frac{1}{2}[\mathbf{q}^2(\mathbf{k}^2-\mathbf{q}_1^2-\mathbf{q}_2^2)+2\mathbf{q}_1^2\mathbf{q}_2^2-\mathbf{q}_1^2\mathbf{q}_2'^2-\mathbf{q}_2^2\mathbf{q}_1'^2+\frac{\mathbf{q}_1^2\mathbf{q}_2'^2-\mathbf{q}_2^2\mathbf{q}_1'^2}{\mathbf{k}^2}(\mathbf{q}_1^2-\mathbf{q}_2^2)]\times$$

$$\left.\left.\times\int_0^1\frac{dx}{(\mathbf{q}_1(1-x)+\mathbf{q}_2 x)^2}\ln\left(\frac{\mathbf{q}_1^2(1-x)+\mathbf{q}_2^2 x}{\mathbf{k}^2 x(1-x)}\right)\right]\right\}+\frac{g^2 N}{2(2\pi)^{D-1}}\left\{\mathbf{q}_i\longleftrightarrow\mathbf{q}_i'\right\}. \tag{16}$$

After the cancellation of the terms $\sim 1/\epsilon^2$ the leading singularity of the kernel is $1/\epsilon$. It turns again into $\sim 1/\epsilon^2$ after subsequent integrations of the kernel because of the singular behaviour of the kernel at $\mathbf{k}^2 = 0$. The additional singularity arises from the region of small $\mathbf{k}^2$, where $\epsilon|\ln\mathbf{k}^2| \sim 1$. Therefore we have not expanded in ϵ the term $(\mathbf{k}^2)^\epsilon$. The terms $\sim\epsilon$ are taken into account in the coefficient of the divergent at $\mathbf{k}^2 = 0$ expression in order to conserve all nonvanishing in the limit $\epsilon \to 0$ contributions after the integrations.

In conclusion let us note that in [10] the octet kernel was obtained using as a basis the bootstrap relation and a specific ansatz to solve it. Our results disagree with the results obtained in [10]. To see the disagreement it is sufficient to observe that the kernel obtained in [10] is expressed in terms of elementary functions. We conclude that the ansatz used in [10] is not correct.

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