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COMPTON TENSOR WITH HEAVY PHOTON FOR LONGITUDINALLY POLARIZED ELECTRON WITH NEXT-TO-LEADING ACCURACY

G.I.Gakh, M.I.Konchatnij, N.P.Merenkov

National Science Center "Kharkov Institute of Physics and Technology", 61108 Kharkiv, Ukraine

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Compton tensor for scattering of the longitudinally polarized electron on heavy photon has been considered. Obtained result can be used for analysis of electromagnetic corrections to spin-spin correlation in quasi-elastic and deep-inelastic scattering with next-to-leading accuracy.

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1. The recent polarized experiments on deep inelastic scattering [1, 2] cover the kinematical region of Bjorken variable $y \simeq 0.9$, where the electromagnetic corrections to the cross section are extremely large. Within the framework of one-photon-exchange approximation the matrix element squared of DIS process can be written as a contraction of leptonic $L_{\mu\nu}$ and hadronic $H_{\mu\nu}$ tensors

$$|M|^2 = \frac{1}{q^4} L_{\mu\nu} H_{\mu\nu}, \tag{1}$$

where q is momentum transferred. The model-independent radiative correction implies the corresponding correction to leptonic tensor on the right side of Eq. (1). The first–order QED correction to $L_{\mu\nu}$ includes contributions due to real and virtual single photon emission. It has been computed in Ref.[3], and at large values of variable y it is of the order unity. Therefore, the calculation of the second-order QED correction to $L_{\mu\nu}$ becomes very important for interpretation of these polarized experiments in terms of hadronic structure functions.

The second-order correction to leptonic tensor includes the contributions due to e^+e^--pair production [4], double hard photon emission [5], one-loop corrected single-photon emission [6] and two-loop vertex function [7]. In the case of longitudinally polarized electron beam all these contributions are known with the next-to-leading accuracy, except one due to one-loop corrected single-photon emission (Compton tensor). The calculation performed in Ref.[6] corresponds to the case when hard photon is radiated on large angle relative to both, the initial and the scattered electron 3-momentum. Therefore, in Ref.[6] some contributions, which can become essential at small angles, are omitted. The goal of our letter is to calculate these contributions.

2. We define Compton tensor for the process

$$e^-(p_1) + \gamma^*(q) \to \gamma(k_1) + e^-(p_2) \tag{2}$$

in the following way

$$L_{\mu\nu} = -\frac{1}{4}\mathrm{Sp}(\hat{p}_2 + m)[Q^{(0)}_{\mu\rho} + Q^{(1)}_{\mu\rho}](\hat{p}_1 + m)[1 + \eta\gamma_5(1 + \frac{m\hat{g}}{p_1 g})][Q^{(0)}_{\nu\rho} + Q^{(1)}_{\nu\rho}]^+ , \tag{3}$$

where $m(\eta)$ is the electron mass (doubled helicity of the initial electron) and 4-vector g has components ε, $-\mathbf{p}$, provided 4-vector $p_1 = (\varepsilon, \mathbf{p})$. Tensor $Q^{(0)}_{\mu\rho}$ describes the Born approximation of amplitude of the process (for corresponding Born and one-loop corrected Feynman diagrams of the process (2) see Ref.[6, 8]) and reads

$$Q^{(0)}_{\mu\rho} = \gamma_\mu \frac{(\hat{p}_2 - \hat{q} + m)}{t}\gamma_\rho + \gamma_\rho \frac{(\hat{p}_1 + \hat{q} + m)}{s}\gamma_\mu, \tag{4}$$

where $t = -2p_1k_1$, $s = 2p_2k_1$.

Tensor $Q^{(1)}_{\mu\rho}$ corresponds to one-loop corrected amplitude of the process (2). After some algebra it can be written in gauge invariant form

$$Q^{(1)}_{\nu\rho} = \frac{\alpha}{4\pi}(1+\hat{T}\hat{S})\Big[\frac{2a}{m}Q_\mu\gamma_\rho + \Big(\frac{2}{(0)(q)t} - \frac{2b}{m^2}\Big)P_{2\mu}\gamma_\rho +$$

$$+\frac{\gamma_\lambda(\hat{p}_2 - \hat{k} + m)(P_{2\mu} - 2\tilde{k}_\mu)\gamma_\lambda(\hat{p}_2 - \hat{q} + m)}{(0)(2)(q)t}\gamma_\rho +$$

$$+\frac{\gamma_\lambda(\hat{p}_2 - \hat{k} + m)(P_{2\mu} - 2\tilde{k}_\mu)\gamma_\rho(\hat{p}_1 - \hat{k} + m)\gamma_\lambda}{(0)(1)(2)(q)} +$$

$$+\frac{\gamma_\lambda(\hat{p}_2 - \hat{q} - \hat{k} + m)\gamma_\rho(\hat{p}_1 - \hat{k} + m)\gamma_\lambda P_{2\mu}}{(0)(1)(q)t} - \frac{\gamma_\lambda Q_\mu \gamma_\rho(\hat{p}_1 - \hat{k} + m)\gamma_\lambda}{(0)(1)(q)}\Big], \tag{5}$$

where k is 4-momentum of virtual photon, and we use such notation

$$(0) = k^2 - \lambda^2,\ \ (1) = (p_1 - k)^2 - m^2,\ \ (2) = (p_2 - k)^2 - m^2,\ \ (q) = (p_2 - k - q)^2 - m^2 ,$$

$$Q_\mu = \gamma_\mu - \frac{q_\mu \hat{q}}{q^2},\ \ P_{2\mu} = 2\tilde{p}_{2\mu} + q_\mu - \gamma_\mu\hat{q},\ \ \tilde{k}_{/mu} = k_\mu - \frac{kq}{q^2}q_\mu,\ \ \tilde{p}_{2\mu} = p_{2\mu} - \frac{p_2 q}{q^2}q_\mu ,$$

$$a = \frac{m^2}{2(t+m^2)}\Big(1 - \frac{2m^2+3t}{t+m^2}l_t\Big),\ \ b = -\frac{m^2}{2t}\Big[\frac{2m^2+t}{t+m^2} + \frac{t^2 - 4m^2t - 4m^4}{(t+m^2)^2}l_t + 2\Big(1 + \ln\frac{\lambda^2}{m^2}\Big)\Big] ,$$

$$l_t = \ln\frac{-t}{m^2} , \tag{6}$$

where λ is the photon "mass".

The substitution operator $\hat{S}$ on the right side of Eq.(5) acts as follows

$$\hat{S}F(p_2, q) = F(p_1, -q) , \tag{7}$$

and the operator $\hat{T}$ changes the order of matrix $\gamma's$:

$$\hat{T}\{\gamma_\alpha\gamma_\beta\cdots\gamma_\sigma\gamma_\rho\} = \{\gamma_\rho\gamma_\sigma\cdots\gamma_\beta\gamma_\alpha\}. \tag{8}$$

For every term on the right side of Eq.(5), which depends on 4-momentum k of virtual photon, we bear in mind the 4-dimension loop integration, for example,

$$\frac{1}{(0)(q)} \to \int \frac{d^4k}{i\pi^2}\frac{1}{(0)(q)} \tag{9}$$

and so on.

It is easy to see that under substitution $\hat{S}$ large logarithm l_t becomes complex

$$\hat{S}l_t \to l_s = \ln\frac{-s}{m^2} = \ln\frac{s}{m^2} - i\pi \ . \tag{10}$$

Below we will neglect the imaginary part on the right side of Eq. (5) that can appear by means of operator $\hat{S}$ action in accordance with relation (10). Such kind approximation is suitable in very case when one wants to apply our result for analysis of spin-spin correlations in quasi-elastic and deep-inelastic scattering processes because imaginary part in (5) can contribute in one-spin correlation only. Moreover, in order to observe such type correlations , in principle, it needs to fix perpendicular components with respect to initial electron 3-momentum of both, the hard photon and the scattered electron [9]. As we will see below the term we want to calculate does not contributes in this case. This circumstance justifies chosen approximation.

In this approximation the spin-dependent part of Compton tensor can be expressed in term of two gauge invariant structures $\epsilon_{\mu\nu\alpha\beta}q_\alpha p_{1\beta}$ and $\epsilon_{\mu\nu\alpha\beta}q_\alpha p_{2\beta}$ which appear on the Born level (in general case, when we keep imaginary part that arise on the right side of Eq.(5), the structure of leptonic tensor is more complicated as it is shown in Ref.[6]). Neglecting terms of the order α^2 on the right side of Eq. (3) we have

$$L_{\mu\nu} = S_{\mu\nu} + i\eta P_{\mu\nu}, \quad P_{\mu\nu} = P^{(0)}_{\mu\nu} + \frac{\alpha}{2\pi}P^{(1)}_{\mu\nu} \ , \tag{11}$$

where $P^{(0)}_{\mu\nu}$ is the well known Born contribution [3, 6]:

$$P^{(0)}_{\mu\nu} = 2\{(\mu\nu q p_1)[\frac{u+t}{st} - 2m^2(\frac{1}{s^2}+\frac{1}{t^2})] + (\mu\nu q p_2)(\frac{u+s}{st} - \frac{2m^2 s}{t^2 u})\} \ , \tag{12}$$

and here we use notation $u = (p_1 - p_2)^2$ and $(\mu\nu q p_{1,2}) = \epsilon_{\mu\nu\alpha\beta}q_\alpha p_{1,2\beta}$.

As concerns tensor $P^{(1)}_{\mu\nu}$, we can write it in the following form

$$P^{(1)}_{\mu\nu} = \rho P^{(0)}_{\mu\nu} + R_{\mu\nu} \ , \quad R_{\mu\nu} = R^{(0)}_{\mu\nu} + 2m^2 R^{(m)}_{\mu\nu} . \tag{13}$$

If we consider loop correction only the Born-like structure on the right side of Eq. (13) will depend on photon "mass" λ. To eliminate it the corrections due to soft photon emission (with the energy smaller than $\Delta\varepsilon$) must be added. The corresponding procedure has been described in Ref.[6, 8] and overall effect can be absorbed by quantity ρ. We follow Ref.[8] and use

$$\rho = 2(l_u - 1)\ln\frac{\Delta^2}{Y} + 3l_q - \ln^2 Y - \frac{\pi^2}{3} - \frac{9}{2} + 2L_{i2}(\cos\theta), \tag{14}$$

where

$$Y = \varepsilon_2/\varepsilon, \quad l_u = \ln(-u/m^2), \quad l_q = \ln(-q^2/m^2),$$

and ε_2 is the energy of the scattered electron. This form of ρ slightly differs from one used in Ref.[6].

Tensor $R^{(0)}_{\mu\nu}$ includes singular and finite terms in the limit $m \to 0$ and reads:

$$R^{(0)}_{\mu\nu} = \Big\{\frac{2s(s+u)+u(u+t)}{s^2 t}G + \frac{u(t-u)}{st^2}\tilde{G} + \frac{s+u-t}{st} - \frac{u+t}{t(s+u)} - \frac{4u}{s(s+t)} -$$

$$-\frac{2u(s-t)}{st(s+u)}(l_t - l_u) + \frac{3}{t+u}(l_q - l_u) + \frac{t+u}{(s+u)^2}(l_q - l_t) - \frac{2u}{st}(l_s - l_t) +$$

$$+2\Big[\frac{u+t}{t(s+u)}+\frac{u}{s(s+t)}-\frac{uq^2(s-t)}{st(s+t)^2}\Big](l_q-l_u)\Big\}(\mu\nu qp_2)+(t\leftrightarrow s,\ p_2\to p_1)\ . \qquad (15)$$

For functions G and $\widetilde{G}$ see Refs.[6, 8]. In fact both, ρ and $R^{(0)}_{\mu\nu}$, can be reconstructed from results of Ref.[6] if we eliminate imaginary part of l_s in the last.

3. The main result of our calculations is the tensor $R^{(m)}_{\mu\nu}$. It can be written in the following form

$$R^{(m)}_{\mu\nu}=A_1(\mu\nu qp_1)+A_2(\mu\nu qp_2)\ , \qquad (16)$$

$$A_1=-\frac{4s^2}{u^2t^2}G-\frac{4}{s^2}\widetilde{G}+\frac{\pi^2}{3}\Big(-\frac{1}{s^2}-\frac{2}{t^2}-\frac{m^2s(2s+u)}{u^2t^3}\Big)+\frac{9u^2-8s^2}{u^2t^2}+\frac{9}{s^2}-\frac{m^2}{s^2(s+m^2)}+$$
$$+\frac{2(s^2+t^2)}{s^2t^2}(l_q^2-l_u^2)+\frac{-ut^2(2s+3u)-m^2t(2u^2-4s^2-us)+2m^4s(2s+u)}{t^2u^2(t+m^2)^2}l_t+$$
$$+\frac{2}{s^2}L_{i2}\Big(1+\frac{s}{m^2}\Big)+\frac{2s^2}{u^2t^2}\Big(2+\frac{m^2}{t}\Big)L_{i2}\Big(1+\frac{t}{m^2}\Big)\ , \qquad (17)$$

$$A_2=-\frac{4}{t^2}G+\frac{\pi^2}{3}\Big(\frac{u-2s}{ut^2}-\frac{m^2t}{us^3}\Big)+\frac{7(s-u)}{ut^2}-\frac{9t+7u}{us^2}-\frac{m^2}{t^2(t+m^2)}+\frac{2s}{ut^2}(l_q^2-l_u^2)+$$
$$+\frac{-s^2(2t+3u)+m^2s(t-2u)+2m^4t}{us^2(s+m^2)^2}l_s+\frac{2m^2t}{us^3}L_{i2}\Big(1+\frac{s}{m^2}\Big)+\frac{2(ut+m^2s)}{ut^3}L_{i2}\Big(1+\frac{t}{m^2}\Big)\ . \qquad (18)$$

As one can see from (17) and (18) tensor $R^{(m)}_{\mu\nu}$, in contrast with $R^{(0)}_{\mu\nu}$, lost a symmetry relative the substitution $(t\leftrightarrow s, p_2\leftrightarrow p_1)$. Besides, 4-vector g escapes the final result in the same way as in the cases of pair production [4] and double-photon emission [5]. It is obvious also that this tensor is important only in collinear regions $(\mathbf{k}_1\parallel\mathbf{p}_1)$ (or $(\mathbf{k}_1\parallel\mathbf{p}_2)$) where invariant t (or s) can be of the order of m^2. (In fact, vector g enters via combination of the type $m^2(k_1g)/(t^2(p_1g))$, and we can use approximation $k_1=xp_1$, $x=-s/u$ to eliminate it.) In these collinear regions we can perform model-independent angular integration for hard photon with 4–momentum k_1. In region $(\mathbf{k}_1\parallel\mathbf{p}_1)$ we have

$$\frac{\alpha}{4\pi^2}\int\frac{d^3k_1}{\omega_1}P_{\mu\nu}(\mathbf{k}_1\parallel\mathbf{p}_1)=i\eta\frac{\alpha}{2\pi}\Big\{\Big[\frac{1+(1-x)^2}{x}\ln z_0-\frac{2(1-x+x^2)}{x}\Big]\Big(1+\frac{\alpha}{2\pi}\rho\Big)+$$
$$+\frac{\alpha}{2\pi}\Big[\frac{1+(1-x)^2}{x}\big((\ln z_0-2l_u)\ln(1-x)-2F(1-x)\big)+\frac{2-x^2}{2x}\Big]\ln z_0+$$
$$+\frac{\alpha}{2\pi}\Big[\frac{2(1-x+x^2)}{x}\big(2\ln(1-x)l_u+4F(1-x)-4\ln(1-x)-\ln^2(1-x)\big)-\frac{1+x^2}{x}-2x\ln x+$$
$$+\frac{\pi^2}{6x}(3+6x^3)+(2-2x+3x^2)\ln^2x+\frac{-3+8x-8x^2+6x^3}{x}L_{i2}(1-x)\Big]\Big\}(\mu\nu p_1p_2)\ , \qquad (19)$$

where $x=\omega_1/\varepsilon$ is the energy fraction of hard photon, $z_0=\varepsilon^2\theta_0^2/m^2$ and θ_0 is a limited angle that defines collinear region $(\mathbf{k}_1\parallel\mathbf{p}_1)$: $\widehat{\mathbf{k}_1\mathbf{p}_1}<\theta_0$. Function F on the right side of Eq.(19) is defined as follows

$$F(z)=\int\limits_1^{1/z}\frac{dx}{x}\ln(1-x)\ .$$

The corresponding contribution into one-loop corrected leptonic tensor due to region $(\mathbf{k}_1 \parallel \mathbf{p}_2)$ reads

$$\frac{\alpha}{4\pi^2}\int\frac{d^3k_1}{\omega_1}P_{\mu\nu}(\mathbf{k}_1 \parallel \mathbf{p}_2) = i\eta\frac{\alpha}{2\pi}\Big\{\big[\frac{1+(1+y)^2}{y}\ln\tilde{z}_0 - \frac{2(1+y)}{y}\big](1+\frac{\alpha}{2\pi}\rho)+$$
$$+\frac{\alpha}{2\pi}\big[\frac{1+(1+y)^2}{y}((\ln\tilde{z}_0 - 2l_u)\ln(1+y) - 2F(1+y)) + \frac{2-y^2}{2y}\big]\ln\tilde{z}_0+$$
$$+\frac{\alpha}{2\pi}\big[\frac{2(1+y)}{y}(2\ln(1+y)l_u + 4F(1+y) - 4\ln(1+y) - \ln^2(1+y) - y\ln^2 y) - \frac{1}{y}+$$
$$+\frac{\pi^2}{6y}(3+10y+8y^2) - \frac{3+6y+4y^2}{y}L_{i2}(1+y)\big]\Big\}(\mu\nu p_1p_2),\ \tilde{z}_0 = z_0Y^2,\ y=\frac{\omega_1}{\varepsilon_2}\ . \quad (20)$$

Two last lines on the right sides of both Eq.(19) and Eq.(20) appear due to contribution of tensor $R^{(m)}_{\mu\nu}$. Therefore we see that term, proportional to the electron mass squared on (13), contributes in collinear kinematics with next-to-leading (via terms containing l_u) and next-next-to-leading accuracy. It cannot be reconstructed from result of Ref.[6] but can be important for a description of events in such experimental conditions when angle of hard photon in process (2) does not observed.

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