

P-VORTICES, NEXUSES AND EFFECTS OF GAUGE COPIES

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We perform the careful study of the gauge copies problem for the direct center projection in $SU(2)$ lattice gauge theory. Our results indicate that this gauge is not appropriate for the investigation of the center vortices. We also show that the point-like objects, nexuses, are important for the confinement dynamics.

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1. The old idea about the role of the center vortices in confinement phenomena [1] has been revived recently with the use of lattice regularization. Both gauge invariant [2] and gauge dependent [3] approaches were developed. The gauge dependent studies were done in a particular gauge, named center gauge. Such gauge leaves intact center group local gauge invariance. It is believed that gauge dependent P-vortices defined on the lattice plaquettes are able to locate thick gauge invariant center vortices and thus provide the essential evidence for the center vortex picture of confinement. So far 3 different center gauges have been used in practical computations: the indirect center gauge [3], the direct center gauge [4] and the Laplacian center gauge [5]. It is known that the first two of these gauges suffer from gauge copies problem. Many results supporting the above mentioned role of P-vortices were obtained in the direct center gauge. Recently the following feature of this gauge has been discovered [6]: there are gauge copies which correspond to higher maxima of the gauge fixing functional F (see below for definition) than usually obtained and at the same time these new gauge copies produce P-vortices evidently with no center vortex finding ability since projected Wilson loops have no area law. It has been argued in [7] that one can still use direct center gauge to locate center vortices if one uses gauge fixing algorithm avoiding "bad" copies of [6]. Below we subject this statement to the careful check. Another goal of our paper is to investigate properties of recently introduced new objects called nexuses [8, 9] or center monopoles [10]. One can define nexus in $SU(N)$ gauge theory as 3D object formed by N center vortices meeting at the center, or nexus, with the zero (mod N) net flux. We use P-vortices in the center projection to define nexuses in $SU(2)$ lattice gauge theory.

2. Direct center gauge is defined by the maximization of the following functional of the lattice gauge field $U_{n,\mu}$ [4]:

$$F(U) = \frac{1}{4V} \sum_{n,\mu} \left(\frac{1}{2} \text{Tr} U_{n,\mu} \right)^2 = \frac{1}{4V} \sum_{n,\mu} \frac{1}{4} (\text{Tr}_{adj} U_{n,\mu} + 1), \quad (1)$$

with respect to local gauge transformations, and can be considered as Landau gauge for adjoint representation; V is the lattice volume. Condition (1) fixes the gauge up to $Z(2)$ gauge transformation. Fixed configuration can be decomposed into $Z(2)$ and coset parts:

$U_{n,\mu} = Z_{n,\mu} V_{n,\mu}$, where $Z_{n,\mu} = \text{signTr} U_{n,\mu}$. Plaquettes constructed from $Z_{n,\mu}$ field have values ± 1 . Those of them taking values -1 compose the so called P-vortices. P-vortices form closed surfaces in 4D space. Some evidence has been collected, that P-vortices in the direct center gauge can serve to locate gauge invariant center vortices. It has been reported [4] that projected Wilson loops computed via linking number of the static quarks trajectories and P-vortices have area law with the string tension $\sigma_{Z(2)}$ very close to the string tension of the nonabelian theory $\sigma_{SU(2)}$. This fact has been called center dominance. Another important observation was that the density of P-vortices scales as a physical quantity [4, 11]. We inspect these statements using careful gauge fixing procedure.

The most common method to fix the gauge of the type (1) is the relaxation algorithm which makes maximization iteratively site by site. The relaxation is made more effective with the help of the overrelaxation. It is known that another algorithm – simulated annealing – is more effective and very useful when gauge copies problem becomes severe [12]. Here we do not employ simulated annealing and apply gauge fixing procedure explained in details in ref. [4]. We call it RO (relaxation-overrelaxation) procedure.

The main problem of the direct center gauge fixing is that the functional $F(U)$ (1) has many local maxima. We call configurations corresponding to these local maxima gauge copies. They are lattice Gribov copies in fact. It is well known that for some gauge conditions which are formulated as the maximization of a nonlocal functional (e.g. Landau, Coulomb and Maximal Abelian gauges) the gauge dependent quantities depend strongly on the local maxima picked up, while to find out the global maximum is impossible. Thus it is necessary to approach the global maximum as close as possible. We follow the following procedure proposed and checked in [12]: for given configuration we generate N_{cop} gauge equivalent copies applying random gauge transformations, and fix the gauge for each gauge copy using the RO procedure. After that we compute the gauge dependent quantity X on the gauge copy corresponding to the highest maximum of (1), $F_{max}(N_{cop})$. Averaging over statistically independent gauge field configurations and varying N_{cop} we obtain the function $X(N_{cop})$ and extrapolate it to $N_{cop} \rightarrow \infty$ limit. This should provide a good estimation for X computed on the global maximum unless the algorithm in use does not permit to reach the global maximum or its vicinity (the situation we met also in the present study). The main difference of the present study from the calculations performed earlier is that we use the higher value of the gauge copies ($1 \leq N_{cop} \leq 20$) than it was used in refs. [3, 4, 7, 11] and make careful analysis of N_{cop} dependence. Due to that our results differ drastically from those reported previously [3, 4, 7, 11].

Separately we compute observables using the modified (LRO) gauge fixing procedure [6]: every configuration has been first fixed to Landau gauge, and then the RO algorithm for the direct center gauge has been applied. In this case the effect of large number of gauge copies, N_{cop} , is not very important, we confirm the results of ref. [6].

Note that there exists another proposal [13] for the general gauge fixing procedure which is free of gauge copies problem. In some particular limit this procedure corresponds to the search of the global maximum [12]. There is also a class of gauge conditions [5], [14] which do not suffer from the gauge copies problem.

3. Our computations have been performed on lattice $L^4 = 12^4$ for $\beta = 2.3; 2.4$ and $L^4 = 16^4$ for $\beta = 2.5$. For $\beta = 2.3, 2.4$ ($\beta = 2.5$) we study 100 (50) statistically independent gauge field configurations. Using the described above gauge fixing procedure

we calculate the various observables as functions of the number of randomly generated gauge copies N_{cop} ($1 \leq N_{cop} \leq 20$).

(i) We confirm the conclusion of ref. [6] that gauge copies generated via LRO procedure have higher maxima of $F(U)$ and thus are closer to the global maximum of $F(U)$. We found that $F_{max}^{LRO}(N_{cop}) > F_{max}^{RO}(N_{cop})$ for any value of N_{cop} , at any considered value of β .

(ii) We find that LRO procedure gives copies with significantly lower density, ρ , of P-vortices than RO procedure. We use the standard definition:

$$\rho = \frac{1}{12V} \sum_{n;\mu>\nu} (1 - Z_{n,\mu\nu}).$$

Thus gauge copies generated by RO and LRO procedures are indeed different even in the limit $N_{cop} \rightarrow \infty$.

(iii) The difference between LRO and RO procedure results can be qualitatively explained as follows. Fixing the Landau gauge we get the configuration almost without P-vortices, the subsequent RO procedure substantially increases the number of P-vortices but percolating cluster does not appear. The original gauge field configuration contains a lot of P-vortices and the local RO procedure is not able to remove all large (and even wrapping) clusters of P-vortices. The field configuration after application of LRO procedure contains many small P-vortex clusters; the field configuration after application of RO procedure contains one large percolating cluster. It seems that this cluster is responsible for the area law behavior of the projected Wilson loops (see below).

(iv) The most important observable is the $Z(2)$ -projected Creutz ratio $\chi(I)$ which we calculate using the procedure suggested in refs. [3, 4]. $\chi(I)$ is defined through the projected Wilson loops, $W_{Z(2)}(C) = \exp\{i\pi\mathcal{L}(\Sigma_P, C)\}$. Here $\mathcal{L}(\Sigma_P, C)$ is the 4D linking number of the closed surface, Σ_P , formed by P-vortex and closed loop C .

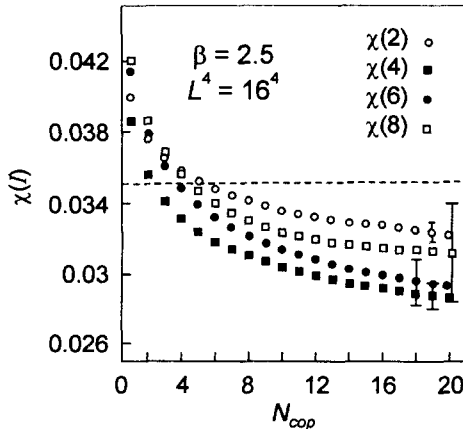


Fig.1. The dependence of the Creutz ratios $\chi(I)$ on the number of gauge copies N_{cop} for $\beta = 2.5, L^4 = 16^4$. The error bars are shown for one value of N_{cop} only and they are characteristic for the other data points. The dashed line corresponds to the nonabelian string tension, $\sigma_{SU(2)}$

In Fig.1. we show the dependence of $\chi(I)$ on N_{cop} for $\beta = 2.5$. It occurs that this dependence is nicely fitted by the function $C_1 + C_2/\sqrt{N_{cop}}$. The reason for such dependence is still to be understood. In Table we give the ratio¹⁾ $\sigma_{Z(2)}/\sigma_{SU(2)}$. $\sigma_{Z(2)}$ is computed from $\chi(I)$ for $3 \leq I \leq 4$ data at 12^4 lattice and for $3 \leq I \leq 6$ data at 16^4

¹⁾ The data for $\sigma_{SU(2)}$ are taken from [15].

lattice. For $N_{cop} = 3$ (number of gauge copies used in [4]) $\sigma_{Z(2)}$ is close to $\sigma_{SU(2)}$. But it becomes significantly lower for $N_{cop} \rightarrow \infty$. Thus RO procedure results strongly depend on N_{cop} . It is important that $\sigma_{Z(2)}$ is 20–30 % lower than $\sigma_{SU(2)}$ for $N_{cop} \rightarrow \infty$. This implies that even if one restricts oneself to RO procedure as it is suggested in [7], one cannot conclude that P-vortices indeed well locate *all* center vortices.

(v) For gauge copies generated by LRO procedure we confirm the result of [6] that $\chi(I)$ is zero within statistical errors for any value of N_{cop} .

The comparison of $\sigma_{Z(2)}$, $\sigma_{SU(2)}$ and ρ for RO gauge fixing center projection

N_{cop}	$\sigma_{Z(2)}/\sigma_{SU(2)}$			$2\rho/\sigma_{SU(2)}a^2$		
	$\beta = 2.3$	$\beta = 2.4$	$\beta = 2.5$	$\beta = 2.3$	$\beta = 2.4$	$\beta = 2.5$
3	0.94(2)	0.93(2)	0.98(2)	1.30(1)	1.51(1)	1.74(1)
20	0.87(2)	0.80(2)	0.83(3)	1.27(1)	1.42(1)	1.61(2)
∞	0.82(3)	0.71(3)	0.71(3)	1.24(1)	1.33(2)	1.49(2)

(vi) In Table we also show the ratio $2\rho/\sigma_{SU(2)}a^2$ (ρ is the density of P-vortices). As it is claimed in ref. [11] in case of the uncorrelated plaquettes carrying P-vortices 2ρ coincides with the dimensionless string tension, $\sigma_{SU(2)}a^2$. The results presented in Table show that the density of P-vortices does not scale correctly, and therefore ρ is not proportional to $\sigma_{SU(2)}a^2$.

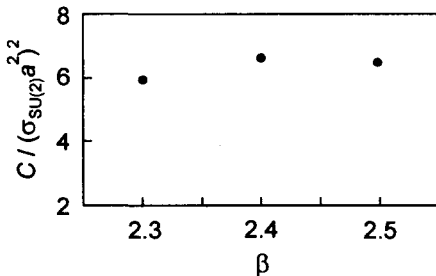


Fig.2. The β -dependence of the ratio of the nexus condensate, C , to the $SU(2)$ string tension in lattice units [15]

(vii) We also investigate the properties of the point-like objects, called nexuses. On the 4D lattice we have the conserved currents of nexuses, defined after the center projection. We calculate the phase, s_l , of the $Z(2)$ link variable: $Z_l = \exp(i\pi s_l)$, $s_l = 0, 1$. Then we define the plaquette variable $\sigma_P = ds \bmod 2$, ($\sigma_P = 0, 1$). The nexus current (or center monopole current [10]) is then defined as $*j = \frac{1}{2}\delta^* \sigma_P$. These currents live on the surface of the P-vortex (on the dual 4D lattice) and P-vortex flux goes through positive and negative nexuses in alternate order. The important characteristic of the cluster of currents is the condensate, C , defined [16] as the percolation probability. As it is shown in ref. [10] the condensate C of the nexus currents is the order parameter for the confinement – deconfinement phase transition. We found that C is nonzero for the gauge copies obtained via RO procedure (when the projected Wilson loops have the area law). C is zero (in the thermodynamic limit $L \rightarrow \infty$) for gauge copies obtained using LRO procedure (when the projected Wilson loops have no area law). It is interesting that for RO procedure C seems to scale as the physical quantity with the dimension $(mass)^4$. This is illustrated in Fig.2, where we plot the β -dependence of the ratio $C/(\sigma_{SU(2)}a^2)^2$. Thus these new

objects might be important degrees of freedom for the description of the nonperturbative effects.

(viii) It is important to perform the same calculations for the indirect center gauge [3] and for the Laplacian center gauge [5].

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