РОССИЙСКАЯ АКАДЕМИЯ НАУК

П И С Ь М А
В ЖУРНАЛ ЭКСПЕРИМЕНТАЛЬНОЙ И ТЕОРЕТИЧЕСКОЙ ФИЗИКИ

ОСНОВАН В 1965 ГОДУ *ТОМ 71, ВЫПУСК 7*
ВЫХОДИТ 24 РАЗА В ГОД *10 АПРЕЛЯ, 2000*

Pis'ma v ZhETF, vol.71, iss.7, pp.401 - 406 © 2000 April 10

THE $\omega(782) \to 5\pi$ DECAY

N.N.Achasov[1)] *A.A.Kozhevnikov*[1)]

Laboratory of Theoretical Physics, Sobolev Institute for Mathematics
630090 Novosibirsk-90, Russia

Submitted 6 March 2000

The partial widths of the decays $\omega \to 2\pi^+2\pi^-\pi^0$ and $\pi^+\pi^-3\pi^0$ are evaluated and their excitation curves in e^+e^- annihilation are obtained.

PACS: 11.30.Rd, 12.39.Fe, 13.30.Eg

At present, the unconventional, from the point of view of the chiral pion dynamics, sources of soft pions are feasible. Indeed, the progress in increasing the luminosity of low energy e^+e^- colliders (ϕ factories) [1] could offer the naturally controlled sources of soft pions. Other possible source of such pions could be the intense photon beams [2], provided the sufficiently low invariant mass regions of the many pion systems are isolated. The yield of pions is considerably enhanced when they are produced through proper vector resonance states. Then, choosing the many pion decays of sufficiently low lying resonances, one can obtain the soft pions in quantities sufficient for testing the predictions of chiral models that include vector mesons.

The decay $\omega(782) \to 5\pi$ whose final state pions possess the momenta $|\mathbf{q}_\pi| \simeq 74$ MeV, is just of this kind. The latter value is sufficiently small to expect the manifestation of chiral dynamics in the most clean form. By this we mean that the higher derivative and loop terms in the effective Lagrangian are severely suppressed. The present paper is devoted to the evaluation of the partial width of this decay and plotting its excitation curve in e^+e^- annihilation.

The $\rho\pi$ sector is considered here on the basis of the Weinberg Lagrangian [3] revived later as the Lagrangian of hidden local symmetry (HLS) [4]. The former looks as

$$\begin{aligned}\mathcal{L} &= -\frac{1}{4}\left(\partial_\mu\rho_\nu - \partial_\nu\rho_\mu + g[\rho_\mu \times \rho_\nu]\right)^2 + \frac{m_\rho^2}{2}\left[\rho_\mu + \frac{[\pi \times \partial_\mu\pi]}{2gf_\pi^2(1+\pi^2/4f_\pi^2)}\right]^2 + \\ &+ \frac{(\partial_\mu\pi)^2}{2\left(1+\pi^2/4f_\pi^2\right)^2} - \frac{m_\pi^2\pi^2}{2(1+\pi^2/4f_\pi^2)},\end{aligned} \quad (1)$$

[1)] e-mail: achasov@math.nsc.ru, kozhev@math.nsc.ru where $\boldsymbol{\pi}$, m_π and $\boldsymbol{\rho}_\mu$, m_ρ stand for the isovector fields of π and ρ mesons and their masses, respectively, and $f_\pi = 92.4$ MeV is the pion decay constant. Cross stands for the vector product of the isovector quantities defined on the isotopic space. The $\rho\rho\rho$ coupling constant g and the $\rho\pi\pi$ coupling constant $g_{\rho\pi\pi}$ are related to the ρ mass and pion decay constant f_π via the parameter of hidden local symmetry a as [4]

$$g = m_\rho / f_\pi \sqrt{a}, \quad g_{\rho\pi\pi} = \sqrt{a} m_\rho / 2 f_\pi. \tag{2}$$

Note that $a = 2$, if one demands the universality condition $g = g_{\rho\pi\pi}$ to be satisfied. Then the so called Kawarabayashi-Suzuki-Riazzuddin-Fayyazuddin relation $2g^2_{\rho\pi\pi} f^2_\pi / m^2_\rho = 1$ [5] arises which beautifully agrees with experiment. The $\rho\pi\pi$ coupling constant resulting from this relation is $g_{\rho\pi\pi} = 5.89$. The inclusion of the interaction of the $\omega(782)$ with the $\rho\pi$ state is achieved upon adding the term induced by the anomalous Lagrangian of Wess and Zumino [4, 6],

$$\mathcal{L}_{\omega\rho\pi} = \frac{N_c g^2}{8\pi^2 f_\pi} \varepsilon_{\mu\nu\lambda\sigma} \partial_\mu \omega_\nu \left(\boldsymbol{\pi} \cdot \partial_\lambda \boldsymbol{\rho}_\sigma \right), \tag{3}$$

where ω_ν stands for the ω meson field, $N_c = 3$ is the number of colors.

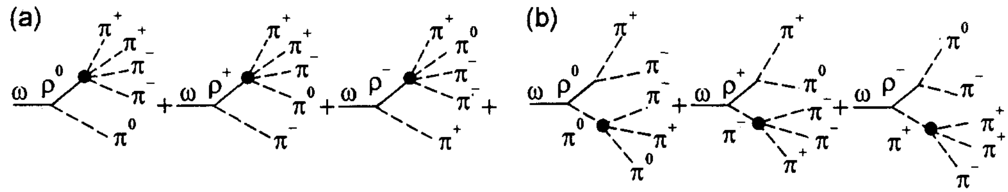

Fig.1. The diagrams describing the amplitudes of the decay $\omega \to \pi^+\pi^-\pi^+\pi^-\pi^0$. The shaded circles in the set (a) refer to the whole $\rho \to 4\pi$ amplitudes Eq. (5). The shaded circles in the set (b) refer to the effective $\pi \to 3\pi$ vertices given by Eq. (6). The symmetrisation over momenta of identical pions is meant. The diagrams for the decay $\omega \to \pi^+\pi^-\pi^0\pi^0\pi^0$ are obtained from those shown upon the evident replacements

One may convince oneself that the $\omega \to \rho\pi \to 5\pi$ decay amplitude unambiguously results from the Weinberg Lagrangian Eq. (1) and the anomaly induced Lagrangian (3). This amplitude is represented by the diagrams shown in Fig.1. Its general expression is expected to be cumbersome. However, it can be considerably simplified upon noting that the small pion momenta permit one to use the nonrelativistic expressions,

$$\begin{aligned}
M[\rho^0 \to \pi^+(q_1)\pi^+(q_2)\pi^-(q_3)\pi^-(q_4)] &\simeq -\frac{g_{\rho\pi\pi}}{2f^2_\pi}(\varepsilon, q_1 + q_2 - q_3 - q_4), \\
M[\rho^0 \to \pi^+(q_1)\pi^-(q_2)\pi^0(q_3)\pi^0(q_4)] &\simeq -\frac{g_{\rho\pi\pi}}{4f^2_\pi}(\varepsilon, q_1 - q_2), \\
M[\rho^+ \to \pi^+(q_1)\pi^+(q_2)\pi^-(q_3)\pi^0(q_4)] &\simeq \frac{g_{\rho\pi\pi}}{4f^2_\pi}(\varepsilon, q_1 + q_2 - 2q_4), \qquad (4) \\
M[\rho^+ \to \pi^+(q_1)\pi^0(q_2)\pi^0(q_3)\pi^0(q_4)] &\simeq \frac{g_{\rho\pi\pi}}{f^2_\pi}(\varepsilon, q_1),
\end{aligned}$$

for the $\rho \to 4\pi$ decay amplitudes in the diagrams Fig.1a. Here ε, q_A $(A = 1, 2, 3, 4)$ stand for the ρ meson polarization and pion momentum four-vectors.[2] The above expressions are valid with the accuracy 5% in the 4π mass range relevant for the present purpose. Likewise, the expression for the combination $D_\pi^{-1} M(\pi \to 3\pi)$ standing in the expression for the diagrams in Fig.1b can be replaced, with the same accuracy, by $-(8m_\pi^2)^{-1}$ times the nonrelativistic $\pi \to 3\pi$ amplitudes. The latter look as

$$\begin{aligned} M(\pi^+ \to \pi^+\pi^+\pi^-) &= -2m_\pi^2/f_\pi^2, \\ M(\pi^+ \to \pi^+\pi^0\pi^0) &= -m_\pi^2/f_\pi^2, \\ M(\pi^0 \to \pi^+\pi^-\pi^0) &= -m_\pi^2/f_\pi^2, \\ M(\pi^0 \to \pi^0\pi^0\pi^0) &= -3m_\pi^2/f_\pi^2. \end{aligned} \tag{5}$$

Note that, in the nonrelativistic limit, the $\rho \to 4\pi$ decay amplitudes depend on the HLS parameter a only through an overall factor $g_{\rho\pi\pi}/f_\pi^2 = \sqrt{a}m_\rho/2f_\pi^3$, while the $\pi \to 3\pi$ amplitudes do not depend on it at all.

Yet even the simplified expressions for the $\omega \to 5\pi$ amplitudes are not easy to use for evaluation of the branching ratios. To go further, one should note the following. The invariant mass of the 4π system on which the contribution of the diagrams shown in Fig.1a depends, changes in very narrow range 558 MeV $< m_{4\pi} <$ 642 MeV. Hence, one can set it in all the ρ propagators, with the accuracy 20% in width, to the equilibrium value $\overline{m_{4\pi}^2}^{1/2} = 620$ MeV evaluated for the pion energy $E_\pi = m_\omega/5$ which gives the dominant contribution. The same is true for the invariant mass of the pion pairs on which the ρ propagators entering the diagrams Fig.1b depend. This mass varies in the narrow range 280 MeV $< m_{2\pi} <$ 360 MeV. With the same accuracy, one can set it to $\overline{m_{2\pi}^2}^{1/2} = 295$ MeV in all relevant propagators. On the other hand, the amplitude of the process $\omega \to \rho^0\pi^0 \to (2\pi^+2\pi^-)\pi^0$ corresponding to the first diagram in Fig.1a is

$$M_{\omega\to\rho^0\pi^0\to(2\pi^+2\pi^-)\pi^0} = 4\frac{N_c g_{\rho\pi\pi} g^2}{8(2\pi)^2 f_\pi^3}\varepsilon_{\mu\nu\lambda\sigma} q_\mu \epsilon_\nu (q_1 + q_2)_\lambda \frac{q_{4\sigma}}{D_\rho(q - q_4)}, \tag{6}$$

where the momentum assignment is $\pi^+(q_1)\pi^+(q_2)\pi^-(q_3)\pi^-(q_5)\pi^0(q_4)$. Hereafter q_μ, ϵ_μ stand for the four-vectors of momentum, polarization, respectively, of the $\omega(782)$, and

$$D_\rho(q) = m_\rho^2 - q^2 - im_\rho^2\left(\frac{s - 4m_\pi^2}{m_\rho^2 - 4m_\pi^2}\right)^{3/2}\frac{\Gamma_{\rho\pi\pi}(m_\rho)}{\sqrt{q^2}} \tag{7}$$

is the inverse propagator of the $\rho(770)$. The other relevant amplitude corresponding to the first diagram in Fig.1b is

$$M_{\omega\to\rho^0\pi^0\to(\pi^+\pi^-)(\pi^+\pi^-\pi^0)} = \frac{N_c g_{\rho\pi\pi} g^2}{8(2\pi)^2 f_\pi^3}\varepsilon_{\mu\nu\lambda\sigma} q_\mu \epsilon_\nu (1 + P_{12})(1 + P_{35})\frac{q_{1\lambda} q_{3\sigma}}{D_\rho(q_1 + q_3)}, \tag{8}$$

where P_{ij} interchanges the momenta q_i and q_j. Then, taking into account the above consideration concerning the invariant masses, one can show that

$$M_{\omega\to 2\pi^+2\pi^-\pi^0} \approx \frac{5}{2}M_{\omega\to\rho^0\pi^0\to(2\pi^+2\pi^-)\pi^0}\left[1 - \frac{D_\rho(\overline{m_{4\pi}^2})}{2D_\rho(\overline{m_{2\pi}^2})}\right]. \tag{9}$$

[2] Our notation for the scalar product of two four-vectors a and b is $(a, b) = a_0b_0 - \mathbf{ab}$.

The same treatment shows that

$$M_{\omega\to\pi^+\pi^-3\pi^0}\approx\frac{5}{2}M_{\omega\to\rho^+\pi^-\to(\pi^+3\pi^0)\pi^-}\left[1-\frac{D_\rho(\overline{m^2_{4\pi}})}{2D_\rho(\overline{m^2_{2\pi}})}\right], \tag{10}$$

where

$$M_{\omega\to\rho^+\pi^-\to(\pi^+3\pi^0)\pi^-}=-4\frac{N_c g_{\rho\pi\pi}g^2}{8(2\pi)^2 f_\pi^3}\frac{\varepsilon_{\mu\nu\lambda\sigma}q_\mu\epsilon_\nu q_{1\lambda}q_{2\sigma}}{D_\rho(q-q_2)}, \tag{11}$$

and the momentum assignment is $\pi^+(q_1)\pi^-(q_2)\pi^0(q_3)\pi^0(q_4)\pi^0(q_5)$. The second term in square brackets of Eqs. (9) and (10) approximates the contribution of the diagrams shown in Fig.1b. The numerical values of $\overline{m^2_{4\pi}}^{1/2}$ and $\overline{m^2_{2\pi}}^{1/2}$ found above are such that the correction factor in parentheses of Eqs. (9) and (10) amounts to 20% in magnitude. In what follows, the above correction will be taken into account as an overall factor of 0.64 in front of the branching ratios of the decays $\omega\to5\pi$. When making this estimate, the imaginary part of the ρ propagators in square brackets of Eq. (8) and (9) is neglected. This assumption is valid with the accuracy better than 1% in width.

The evaluation of the partial widths valid with accuracy 20% can be obtained upon taking the amplitude of each considered decays as 5/2 times the $\rho\pi$ production state amplitude with the subsequent decay $\rho\to4\pi$, and calculate the partial width of the latter with the help of Eq. (5). One obtains

$$\begin{aligned}B_{\omega\to2\pi^+2\pi^-\pi^0} &= \left|1-\frac{D_\rho(\overline{m^2_{4\pi}})}{2D_\rho(\overline{m^2_{2\pi}})}\right|^2\left(\frac{5}{2}\right)^2\frac{2}{\pi\Gamma_\omega}\int_{4m_{\pi^+}}^{m_\omega-m_{\pi^0}}dm\times\\ &\times\frac{m^2\Gamma_{\omega\to\rho^0\pi^0}(m)\Gamma_{\rho\to2\pi^+2\pi^-}(m)}{|D_\rho(m^2)|^2}=1.1\cdot10^{-9},\end{aligned} \tag{12}$$

where $\Gamma_{\omega\to\rho^0\pi^0}(m)=g^2_{\omega\rho\pi}q^3(m_\omega,m,m_{\pi^0})/12\pi$, and $g_{\omega\rho\pi}=N_cg^2/8\pi^2f_\pi=14.3\ \mathrm{GeV}^{-1}$. Note also the a^{-1} dependence of the $\omega\to5\pi$ width on the HLS parameter a. The branching ratio $B_{\omega\to\pi^+\pi^-3\pi^0}$ is obtained from Eq. (12) upon inserting the lower integration limit to $m_{\pi^+}+3m_{\pi^0}$, $m_{\pi^0}\to m_{\pi^+}$ in the expression for the momentum q and substitution of the $\rho^+\to\pi^+3\pi^0$ decay width corrected for the mass difference of charged and neutral pions. Of course, the main correction of this sort comes from the phase space volume of the final 4π state. One obtains

$$B_{\omega\to\pi^+\pi^-3\pi^0}=8.5\cdot10^{-10}. \tag{13}$$

As is pointed out in Ref. [4], the inclusion of the direct $\omega\to\pi^+\pi^-\pi^0$ vertex reduces the 3π decay width of the ω by 33%. This implies that one should use the suppression factor $\simeq0.75$ instead of 0.64, which results in the increase of the above branching ratios by the factor of 1.17.

The numerical value of the $\omega\to5\pi$ decay width changes by the factor of two when varying the energy within $\pm\Gamma_\omega/2$ around the ω mass. In other words, the dependence of this partial width on energy is very strong. This is illustrated by Fig.2 where the $\omega\to5\pi$ excitation curves in e^+e^- annihilation are plotted. The mentioned strong energy dependence of the partial width results in the asymmetric shape of the ω resonance and the shifting of its peak by +0.7 MeV. As is seen from Fig.2, the peak value of the 5π state production cross section is about 1.5-2.0 femtobarns. Yet the decays $\omega\to5\pi$ can

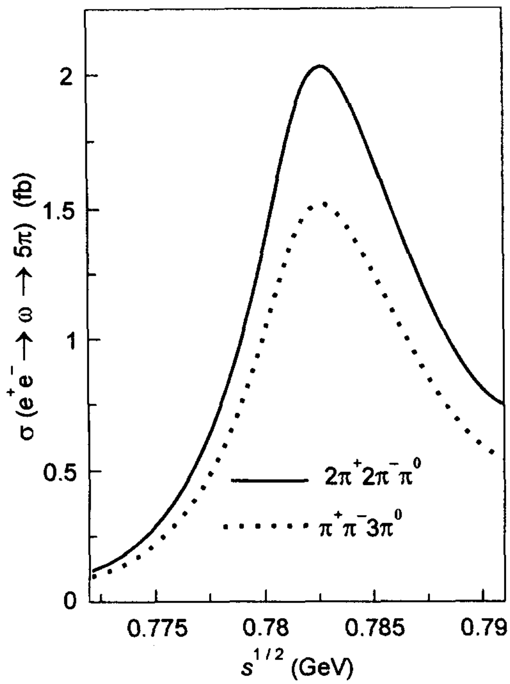

Fig.2. The $\omega \to 5\pi$ excitation curves in e^+e^- annihilation in the vicinity of the ω resonance

be observable on e^+e^- colliders. Indeed, with the luminosity $L = 10^{33}\text{cm}^{-2}\cdot\text{s}^{-1}$ near the ω peak, which seems to be feasible, one may expect about 2 events per week for the considered decays to be detected at these colliders.

The strong energy dependence of the five pion partial width of the ω implies that the branching ratio at the ω mass, $B_{\omega\to5\pi} = \Gamma_{\omega\to5\pi}/\Gamma_\omega$, evaluated above, is slightly different from that determined by the expression

$$B^{\rm aver}_{\omega\to5\pi}(E_1,E_2) = \frac{2}{\pi}\int_{E_1}^{E_2} dE \frac{E^2\Gamma_\omega B_{\omega\to5\pi}(E)}{(E^2-m_\omega^2)^2+(m_\omega\Gamma_\omega)^2}. \quad (14)$$

Taking $E_1 = 772$ MeV and $E_2 = 792$ MeV, one finds $B^{\rm aver}_{\omega\to2\pi^+2\pi^-\pi^0}(E_1,E_2) = 9.0\cdot10^{-10}$ and $B^{\rm aver}_{\omega\to\pi^+\pi^-3\pi^0}(E_1,E_2) = 6.7\cdot10^{-10}$ to be compared to Eq. (12) and (13), respectively. In particular, the quantity $B^{\rm aver}_{\omega\to2\pi^+2\pi^-\pi^0}(E_1,E_2)$ is the relevant characteristics of this specific decay mode in photoproduction experiments. The Jefferson Lab "photon factory" [2] could also be suitable for detecting the five pion decays of the ω. However, in view of the suppression of the ω photoproduction cross section by the factor of 1/9 as compared with the ρ one, the total number of ω mesons will amount to $7\cdot10^8$ per nucleon. Hence, the increase of intensity of this machine by the factor of 50 is highly desirable, in order to observe the decay $\omega\to5\pi$ and measure its branching ratio. Evidently, the ω photoproduction on heavy nuclei is preferable in view of the dependence of the cross section on atomic weight A growing as $A^{0.8-0.95}$ [7].

Together with the e^+e^- annihilation experiments, the study of the photoproduction of the five pion states on heavy nuclei would also allow to measure the corresponding partial width of the $\omega(782)$. The comparison with theoretical expectations presented here would give the possibility of testing the predictions of chiral models in the situation where the decay amplitudes are determined by very low pion momenta.

We are grateful to G.N.Shestakov for discussion. The present work is supported in part by the grant Russian Foundation for Basic Research – INTAS IR-97-232.

---

1. A.N.Skrinsky, in: *Proceedings of the Workshop on Physics and Detectors for DAΦNE'95*, Frascati, April 4-7, 1995; Eds. R.Baldini, F.Bossi, G.Capon et al., Frascati Physics Series v. IV, p. 3; G.Vignola, ibid. p. 19.
2. A.R.Dzierba, in: *Proceedings from Jefferson Lab/NCSU, Workshop on Hybrids and Photoproduction Physics*, North Carolina State University, November 13-15, 1997, p. 661.
3. S.Weinberg, Phys. Rev. **166**, 1568 (1968).
4. M.Bando, T.Kugo, S.Uehar et al., Phys. Rev. Lett. **54**, 1215 (1985); M.Bando, T.Kugo, and K.Yamawaki, Nucl. Phys. **B259**, 493 (1985); Progr. Theor. Phys. **73**, 1541 (1985); Phys. Rep. **164**, 217 (1988).
5. K.Kawarabayashi and M.Suzuki, Phys. Rev. Lett. **16**, 255 (1966); Riazzuddin and Fayyazuddin, Phys. Rev. **147**, 1071 (1966).
6. J.Wess and B.B.Zumino, Phys. Lett. **37B**, 95 (1971).
7. D.W.G.S.Leith, in: *Hadronic Interactions of Electrons and Photons*, Academic Press, London and New York, 1971, p.195.