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FERMIONIC ATOM LASER

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We consider an output coupling of magnetically trapped two species fermi gas to a untrapped species that can be done using *rf* or optical Raman transitions. The process can be used to produce an intense output beam of fermionic atoms once the device reaches a threshold for zero temperature case. For finite temperatures there is no threshold as the output current grows smoothly. The behaviour that recalls optical convenient and cavity quantum electrodynamics lasers puts us on to the idea to call the device as *fermionic atom laser*.

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Since the experimental realisation of Bose – Einstein condensation (BEC) [1] there still exists a great interest in properties of trapped degenerate gases [2]. Although most attention is being paid to alkali bosons recently some work both experimental and theoretical was done on fermi gases [3 – 5]. Another current direction researchers are pursuing is the atom laser, or boser, idea which got a number of proposals [6] and had been shown to be realizable experimentally [7]. The idea is to couple out the formed condensates to get an output coherent beam of atoms. Other bosers proposed are the excitonic [8] and exciton-polariton [9] lasers. Atomic parametric oscillator which produces correlated atomic beams was proposed in [10].

Up to now most attention was paid only to the bosonic atom lasers. That is obviously due to a great progress in the BEC area. However there exists yet another interesting possibility to get coherent type of atomic waves. Recently in a series of papers Stoof, Houbiers et al. [4] and Baranov et al. [5] discussed a possibility of formation of BCS states and of superfluidity in atomic ^{6}Li in a magnetic trap. Although the formation of cooper pairs was not yet observed experimentally their estimates show that the process is rather possible. Accepting the possibility of formation of the fermionic pairs (in the momentum space) one can wonder what would happen in the case of their coupling out of the trap.

In this paper we present a simple scheme for the output coupling. We will show that the problem is akin to the case of electron tunneling between a superconductor and a normal metal and will state that the device possesses a threshold at zero temperature while the device behaves as a thresholdless one at finite temperatures. We then point out an analogy between the device and cavity quantum electrodynamics (QED) lasers [11].

Some of the boser proposals exploited the Born – Markov approximation that was shown as being failed for the case of an atom laser [12]. Very recently a non-Markovian stochastic Schrödinger equation has been derived [13] that opens up the way for further investigation of the atom lasers, being open quantum systems coupled to a finite number of output oscillators as is the case. Although a detailed microscopic theory which should include quantum fluctuations present would provide a deeper understanding of the process, here we present a simple theory without accounting fully quantum fluctuations to have an idea what is going on in the fermionic atom laser case. However in the present paper we do not make the markovian approximation but just work within an Hamiltonian approach which was applied to the superconductive tunneling problem [14]. Another simplification we do is considering the problem in an homogeneous approximation assuming that the magnetic trap fields are homogeneous so that particles are simply placed in a potential well box. We will trace out an analogy between the case and the electron tunneling [15] and will also consider finite temperatures [16, 17].

Let us consider two species of fermions confined in a trap which interact to each other by two-body collisions (s-wave scattering). The Hamiltonian with one output channel reads

$$H = \sum_{\sigma=+,-} \int d^3\mathbf{r} \left(\psi_\sigma^\dagger(\mathbf{r}) \left[-\frac{\hbar^2}{2M}\nabla^2 + V_\sigma(\mathbf{r}) - \mu_\sigma \right] \psi_\sigma(\mathbf{r}) - \right.$$

$$- \frac{g}{2}\psi_\sigma^\dagger(\mathbf{r})\psi_{-\sigma}^\dagger(\mathbf{r})\psi_{-\sigma}(\mathbf{r})\psi_\sigma(\mathbf{r}) \Big) + \int d^3\mathbf{r} \left(\psi_o^\dagger(\mathbf{r}) \left[-\frac{\hbar^2}{2M}\nabla^2 + V_o(\mathbf{r}) - \mu_o \right] \psi_o(\mathbf{r}) + \right.$$

$$\left. + \sum_\sigma \lambda_\sigma \left(\psi_o^\dagger(\mathbf{r})\psi_\sigma(\mathbf{r}) + \text{h.c.} \right) \right). \tag{1}$$

Here $\psi_\sigma^\dagger(\mathbf{r})$ and $\psi_\sigma(\mathbf{r})$ are the creation and annihilation field operators for the two fermionic species ($\sigma = \pm$) and $\psi_o(\mathbf{r})$ describes the output field with chemical potential μ_o and λ_σ coupling constants; $V_\sigma(\mathbf{r})$ are the trapping potentials,$V_o(\mathbf{r})$ is the repelling potential, μ_σ the chemical potentials, M the mass of the particles and g the interaction parameter.

The Hamiltonian describes, e.g. trapped atomic ^{6}Li (fermion) gas. The two trapped levels correspond to $|6\rangle$ and $|5\rangle$ ones [4] and we choose transitions to the $|3\rangle$ untrapped level as the output channel. The transitions from trapped levels to the untrapped one can be provided e.g. by *rf* fields as had been used for the bosons' output coupling [7].

Now we expand the annihilation and creation operators for the second quantized field in trap eigenfunctions as:

$$\psi_\sigma(\mathbf{r}) = \sum_n v_{n\sigma}(\mathbf{r}) a_{n\sigma}, \quad \psi_o(\mathbf{r}) = \sum_k v_{ko}(\mathbf{r}) b_k \tag{2}$$

and after linearisation that introduces the gap Δ and assuming the functions $v_{n\sigma}(\mathbf{r})$ and $v_{ko}(\mathbf{r})$ are known, we present the Hamiltonian of our model in the form

$$H = \sum_{\sigma=+,-} \theta_{n\sigma} a_{n\sigma}^\dagger a_{n\sigma} + \sum_k \theta_{ko} b_k^\dagger b_k +$$

$$+ \sum_{nn'} (\delta_{nn'o} a_{n+}^\dagger a_{n'-}^\dagger + \delta_{nn'o}^* a_{n+} a_{n'-}) + \sum_{nk\sigma} (T_{nk\sigma} a_{n\sigma}^\dagger b_k + \text{h.c.}) \tag{3}$$

where

$$\theta_{n\sigma} = \int d^3\mathbf{r} v_{n\sigma}^*(\mathbf{r}) \left[-\frac{\hbar^2}{2M}\nabla^2 + V_\sigma(\mathbf{r}) - \mu_\sigma \right] v_{n\sigma}(\mathbf{r}),$$

$$\theta_{ko} = \int d^3\mathbf{r} v_{ko}^*(\mathbf{r}) \left[-\frac{\hbar^2}{2M}\nabla^2 + V_o(\mathbf{r}) - \mu_o \right] v_{ko}(\mathbf{r}),$$

$$T_{nk\sigma} = \lambda_\sigma \int d^3\mathbf{r} v_{n\sigma}^*(\mathbf{r}) v_{ko}(\mathbf{r}), \quad \delta_{nn'o} = \Delta \int d^3\mathbf{r} v_{n+}^*(\mathbf{r}) v_{n'-}^*(\mathbf{r}.) \tag{4}$$

After the diagonalization procedure the Hamiltonian takes the following form

$$H = \sum_{n\sigma=+,-} \omega_{n\sigma} c^\dagger_{n\sigma} c_{n\sigma} + \sum_k \theta_{ko} b^\dagger_k b_k + \sum_{nk\sigma} (\alpha_{nk\sigma} b^\dagger_k c_{n\sigma} + \beta^*_{nk\sigma} b^\dagger_k c^\dagger_{-n,-\sigma} + \text{h.c.}), \quad (5)$$

where we have defined

$$\alpha_{nk\sigma} = u_n T^*_{nk\sigma}, \quad \beta_{nk\sigma} = v_n T_{nk\sigma}, \quad \omega_{n\sigma} = \theta_{n\sigma}(|u_n|^2 + |v_n|^2) + 2\delta_{nno}(u^*_n v_n + \text{c.c.}) \quad (6)$$

and

$$a_{n,+} = u_n c_{n,+} + v^*_n c^\dagger_{-n,-}, \qquad a^\dagger_{-n,-} = -v_n c_{n,+} + u^*_n c^\dagger_{-n,-}. \quad (7)$$

For the model under consideration one has [4]

$$|u_n|^2 = \frac{1}{2}\left(1 + \frac{\xi_n}{\sqrt{\xi_n^2 + |\Delta|^2}}\right), \quad |u_n|^2 + |v_n|^2 = 1, \quad (8)$$

where $\xi_n = \epsilon_n - \epsilon_F$ is the energy difference from the "mean" fermi level defined as $\epsilon_F = (\mu_+ + \mu_-)/2$, here $\mu_\pm$ are the fermi levels of the two species; the cooper particles have the dispersion relation $\omega_{n\sigma} = -m_\sigma \delta\epsilon_F + \sqrt{\xi_n^2 + |\Delta|^2}$ with $m_\pm = \pm 1/2$ and $\delta\epsilon_F = \mu_+ - \mu_-$ is the difference between the fermi levels of the species.

For the fermionic output coupler we distinguish two different cases with the two species chemical potentials being equal ($\mu_+ = \mu_- \equiv \mu$) and nonequal ($\mu_+ \neq \mu_-$) to each other. Here we consider only the case of equal chemical potentials that gives the highest critical BCS temperature [4].

We now proceed with the Hamiltonian (5) to calculate the output intensity or current of the fermionic beam that is the mean of the derivative of number of fermions coupled out $< \dot{N}_o >$ where $N_o \equiv \sum_k b^\dagger_k b_k$. One can carry out the standard procedure [14, 15] to calculate the quantity and will arrive at the formula

$$\left\langle \dot{N}_o \right\rangle \propto \sum_{nk} |T_{nk}|^2 \left(u_n^2 \left[N_F(E_{n\sigma}) - N_F(E_k)\right] \delta(E_{n\sigma} - E_k - \delta E) + \right.$$

$$\left. + v_n^2 \left(\left[N_F(-E_{n\sigma}) - N_F(E_k)\right] \delta(E_{n\sigma} + E_k + \delta E)\right)\right) \quad (9)$$

where $N_F(E_k)$ is the Fermi distribution for noninteracting particles $N_F(E_k) = 1/[\exp(\beta E_k) + 1]$ with energy spectrum E_k and the inverse temperature $\beta = 1/k_B T$. Here n runs for the trap level states with dispersion relation $E_{n\sigma} = \sqrt{\xi_n^2 + |\Delta|^2}$ ($\delta\epsilon_F = 0$) and k for the output free particle dispersion $E_k = (\hbar k)^2/2M$ (we assume the potentials $V_+(\mathbf{x})$ and $V_-(\mathbf{x})$ to be space independent so they just shift the chemical potentials $\mu_\pm$). δE is the difference ("bias") between the Fermi levels of in-trap and out-of-trap fermions, which is proportional to the magnetic field. Note that here $\delta E > 0$ so that the second term in the braces vanishes.

The Eq.(9) can be seen as the Fermi's golden rule applied for the transitions from the trap states to the output so that $N_F(E_{n\sigma})[1 - N_F(E_k)] - [1 - N_F(E_{n\sigma})] N_F(E_k) = N_F(E_{n\sigma}) - N_F(E_k)$ accounts for the probability of particle to transit from the occupied trap level $E_{n\sigma}$ to the unoccupied output level E_k and the summing includes all possible transitions. The Eq.(9) accounts for transitions of both the particles and "antiparticles" (holes) with energies $\pm E_{n\sigma}$ respectively. Note that $N_F(-E_{n\sigma}) = 1 - N_F(E_{n\sigma})$.

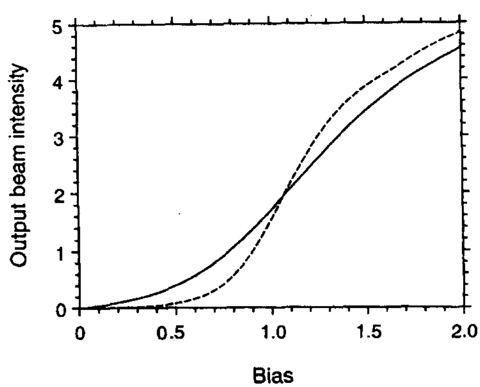
Let us first consider zero temperature case. In this case the Fermi distribution $N_F(E)$ becomes a step function being equal to one below the fermi level and zero above that level. The case with equal chemical potentials is very similar to the superconductor-normal metal electron tunneling process [15 − 17]. For that case the Eq.(9) simplifies to become

$$\left\langle \dot{N}_o \right\rangle \propto \sqrt{(\delta E)^2 - |\Delta|^2} \cdot \theta \left(\delta E - |\Delta|\right). \tag{10}$$

Here $\theta\left(\delta E - |\Delta|\right)$ is Heviside's step function.

There is a threshold here, corresponding to the gap value $\delta E = |\Delta|$. It can be explained as a value of the energy difference equal to the "bound energy" of the formed fermionic pair: one of the paired particles should get energy for output coupling enough that the partner particle could jump over the gap. The value of the gap for the trapped fermions is of the order of the BCS transition temperature T_c which is about 30 nK [4]. The value of the δE which is the difference between the two fermi levels can be in principle adjusted to the value of the gap, however it needs the two species to be trapped by an extremely weak magnetic field. A lower limit of the magnetic field was estimated to be $B \gg \frac{a_{hf}}{\mu_e} \simeq 0.011\,\mathrm{T}$ [4], where a_{hf} is the hyperfine constant and μ_e the electron magneton. On the other hand, the energy difference which can be achieved is about $\delta E \sim 10^{-5} - 10^{-4}\,\mathrm{K}$. So a possibility to observe the threshold is questionable, however it exists in principle. Once a larger transition temperature will be achieved the device possesses the threshold.

In the case of finite temperatures Eq.(9) gives a smooth output current dependence on the applied field (see Figure). There is no threshold here since the current grows smoothly when the field increases. The reason is that for finite temperatures there always exist particles at the excited levels so that unpaired fermions at the fermi level can be coupled out for any value of the bias. This case is reminiscent of cavity QED lasers with spontaneous photons being emitted into cavity mode [11]. So that the thermal fluctuations at finite temperatures which create particles at upper levels in trapped fermi gas play the same role as spontaneous emission induced noise in the case of cavity QED lasers resulting in thresholdless lasing.



Fermionic atom laser's output $< \dot{N}_o >$ (arbitrary units) as function of the bias δE (scaled in units of the gap Δ) for $T/T_c = 0.5$ (full curve) and $T/T_c = 0.25$ (dashed curve) temperatures for the case of equal chemical potentials of trapped species $\mu_+ = \mu_-$: the mass of ^{6}Li atom $M = 10^{-26}\,\mathrm{kg}$; the fermi momentum $k_F = 0.42 \cdot 10^6 \mathrm{m}^{-1}$; the trap size $L = 15\,\mu\mathrm{m}$; the gap is $\Delta = 0.9 \cdot 10^{-30}\,\mathrm{J}$

The consideration can be extended to the cases with both traps containing fermions in BCS states that would be an analog of superconductor-superconductor electron tunneling [15 − 17] or with traps containing noninteracting fermi gases without pairing that is the normal metal – normal metal tunneling case. Imagine also a case of two traps when one of the traps contains cooper pairs and the other one two species of interacting fermions. The process of particle exchange between the traps would be similar to a model describing Anderson impurity embedded in a superconductor that had recently been solved exactly for the zero temperature case [18]. All these processes can serve as tests of fundamental quantum physics phenomena. Actually, there exists an interesting possibility to get Schroedinger cat states for the case of two trapped BCS fermi gases in the direct analogy with a Josephson junction case proposed in [19].

In conclusion, we have considered output coupling of trapped fermionic pairs. For zero temperature case we have found a threshold, a critical value of the trapping magnetic field, above which the output current of fermions begins to grow. For finite temperatures the output beam grows smoothly without a threshold transition. Both threshold and thresholdless behaviors remind optical lasers: convenient lasers and cavity QED or microlasers, so that we called the device as fermionic atom laser.

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