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PIEZOMAGNETISM AND STRESS INDUCED PARAMAGNETIC MEISSNER EFFECT IN MECHANICALLY LOADED HIGH-T_c GRANULAR SUPERCONDUCTORS

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Two novel phenomena in a weakly coupled granular superconductor under an applied stress are predicted based on recently suggested piezophase effect (a macroscopic quantum analog of the piezoelectric effect). Namely, we consider the existence of stress induced paramagnetic moment in zero applied magnetic field (piezomagnetism) and its influence on a low-field magnetization (leading to a mechanically induced paramagnetic Meissner effect). The conditions under which these effects can be experimentally measured in high-T_c granular superconductors are discussed.

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Despite the fact that granular superconductors have been actively studied for decades, they continue contributing to the variety of intriguing and peculiar phenomena (both fundamental and important for potential applications) providing at the same time a useful tool for testing new theoretical ideas [1]. To give just a few recent examples, it is sufficient to mention paramagnetic Meissner effect (PME) [2,3] originated from a cooperative behavior of weak-links mediated orbital moments in high-T_c granular superconductors (HTGS). Among others are recently introduced thermophase [4,5] and piezophase [6] effects suggesting, respectively, a direct influence of a thermal gradient and an applied stress on phase difference between the adjacent grains. Besides, two dual effects in HTGS, an appearance of magnetic field induced electric polarization (magnetoelectric effect [7]) and existence of electric field induced magnetization (converse magnetoelectric effect [8]) have been predicted.

In this Letter we discuss a possibility of two other interesting effects which are expected to occur in a granular material under mechanical loading. Specifically, we predict the existence of stress induced paramagnetic moment in zero applied magnetic field (piezomagnetism) and its influence on a low-field magnetization (leading to a mechanically induced PME).

The possibility to observe tangible piezoeffects in mechanically loaded grain boundary Josephson junctions (GBJJs) is based on the fact that under plastic deformation, grain boundaries (GBs) (which are the natural sources of weak links in HTGS), move rather rapidly via the movement of the grain boundary dislocations (GBDs) comprising these GBs [9–11]. Using the above evidence, in Ref.[6] a *piezophase* response of a single GBJJ (created by GBDs strain field ϵ_d acting as an insulating barrier of thickness l and height U in a SIS-type junction with the Josephson energy $J \propto e^{-l\sqrt{U}}$) to an applied stress was considered. To understand how piezoeffects manifest themselves through GBJJs, let us invoke an analogy with the so-called *thermophase effect* [4,5] (a quantum mechanical alternative for the conventional thermoelectric effect) in JJs. In essence, the thermophase effect assumes a direct coupling between an applied temperature drop ΔT and the resulting phase difference $\Delta\phi$ through a JJ. When a rather small temperature gradient is applied to a JJ, an entropy-carrying normal current $I_n = L_n \Delta T$ (where L_n is the thermoelectric coefficient) is generated through such a junction. To satisfy the constraint dictated by the Meissner effect, the resulting supercurrent $I_s = I_c \sin[\Delta\phi]$ (with $I_c = 2eJ/h$ being the Josephson critical current) develops a phase difference through a weak link. The normal current is locally canceled by a counterflow of supercurrent, so that the total current through the junction $I = I_n + I_s = 0$. As a result, supercurrent $I_c \sin[\Delta\phi] = -I_n = -L_n \Delta T$ generates a nonzero phase difference leading to the linear thermophase effect [4,5] $\Delta\phi \simeq -L_{tp}\Delta T$ with $L_{tp} = L_n/I_c(T)$.

By analogy, we can introduce a *piezophase effect* (as a quantum alternative for the conventional piezoelectric effect) through a JJ [6]. Indeed, a linear conventional piezoelectric effect relates induced polarization P_n to an applied strain ϵ as [12] $P_n = d_n\epsilon$, where d_n is the piezoelectric coefficient. The corresponding normal piezocurrent density is $j_n = dP_n/dt = d_n\dot{\epsilon}$ where $\dot{\epsilon}(\sigma)$ is a rate of plastic deformation which depends on the number of GBDs of density ρ and a mean dislocation rate v_d as follows [13] $\dot{\epsilon}(\sigma) = b\rho v_d(\sigma)$ (where b is the absolute value of the appropriate Burgers vector). In turn, $v_d(\sigma) \simeq v_0(\sigma/\sigma_m)$. To meet the requirements imposed by the Meissner effect, in response to the induced normal piezocurrent, the corresponding Josephson supercurrent of density $j_s = dP_s/dt = j_c \sin[\Delta\phi]$ should emerge within the contact. Here $P_s = -2enb$ is the Cooper pair's induced polarization with n the pair number density, and $j_c = 2ebJ/\hbar V$ is the critical current density. The neutrality conditions ($j_n + j_s = 0$ and $P_n + P_s =$const) will lead then to the linear piezophase effect $\Delta\phi \simeq -d_{pp}\dot{\epsilon}(\sigma)$ (with $d_{pp} = d_n/j_c$ being the piezophase coefficient) and the concomitant change of the pair number density under an applied strain, viz., $\Delta n(\epsilon) = d_{pn}\epsilon$ with $d_{pn} = d_n/2eb$.

Given the markedly different mechanisms and scales of stress induced changes in defect-free thin films [14] and weak-links-ridden ceramics [15], it should be possible to experimentally register the suggested here piezophase effects (see below).

To adequately describe magnetic properties of a granular superconductor, we employ a model of random three-dimensional (3D) overdamped (JJ) array which is based on the well known tunneling Hamiltonian [16,17]

$$\mathcal{H} = \sum_{ij}^{N} J(r_{ij})[1 - \cos\phi_{ij}], \tag{1}$$

where $\{i\} = \mathbf{r}_i$ is a 3D lattice vector, N is the number of grains (or weak links), $J(r_{ij})$ is the Josephson coupling energy with $\mathbf{r}_{ij} = \mathbf{r}_i - \mathbf{r}_j$ the separation between the grains; the gauge invariant phase difference is defined as $\phi_{ij} = \phi_{ij}^0 - A_{ij}$, where $\phi_{ij}^0 = \phi_i - \phi_j$ with ϕ_i being the phase of the superconducting order parameter, and

$$A_{ij} = \frac{2\pi}{\Phi_0}\int_i^j \mathbf{A}(\mathbf{r})\cdot d\mathbf{l}$$

is the frustration parameter with $\mathbf{A}(\mathbf{r})$ the electromagnetic vector potential which involves both external fields and possible self-field effects (see below).

In the present paper, we consider a long-range interaction between grains [5,7,8,16,17] (with $J(r_{ij}) = J$) and model the true short-range behavior of a HTGS sample through the randomness in the position of the superconducting grains in the array (see below).

According to the above-discussed scenario, under mechanical loading the superconducting phase difference will acquire an additional contribution $\delta\phi_{ij}(\sigma) = -B\boldsymbol{\sigma}\cdot\mathbf{r}_{ij}$, where $B = d_n\dot{\epsilon}_0/\sigma_m j_c b$ with $\dot{\epsilon}_0 = b\rho v_0$ being the maximum deformation rate and the other parameters defined earlier. If, in addition to the external loading, the network of superconducting grains is under the influence of an applied frustrating magnetic field $\mathbf{H}$, the total phase difference through the contact reads

$$\phi_{ij}(\mathbf{H},\boldsymbol{\sigma}) = \phi_{ij}^0 + \frac{\pi}{\Phi_0}(\mathbf{r}_{ij}\wedge\mathbf{R}_{ij})\cdot\mathbf{H} - B\boldsymbol{\sigma}\cdot\mathbf{r}_{ij}, \tag{2}$$

where $\mathbf{R}_{ij} = (\mathbf{r}_i + \mathbf{r}_j)/2$.

To neglect the influence of the self-field effects [7,8,17] in a real material, the corresponding Josephson penetration length λ_J must be much larger than the junction (or grain) size. Specifically, this condition will be satisfied for short junctions with the size $d \ll \lambda_J$, where $\lambda_J = \sqrt{\Phi_0/4\pi\mu_0 j_c\lambda_L}$ with λ_L being the grain London penetration depth and j_c its Josephson critical current density. In particular, since in HTGS $\lambda_L \simeq 150\,\mathrm{nm}$, the above criterium will be well met for $d \simeq 1\mu\mathrm{m}$ and $j_c \simeq 10^4\mathrm{A/m}^2$ which are the typical parameters for HTGS ceramics [1]. Likewise, to ensure the uniformity of the applied stress σ, we also assume that $d \ll \lambda_\sigma$, where λ_σ is a characteristic length over which σ is kept homogeneous.

When the Josephson supercurrent $I_{ij}^s = I_c\sin\phi_{ij}$ circulates around a set of grains, it induces a random magnetic moment $\boldsymbol{\mu}_s$ of the Josephson network [16]

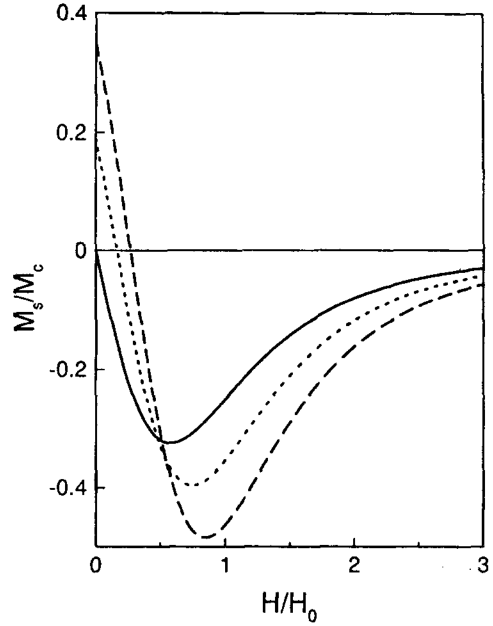
$$\boldsymbol{\mu}_s \equiv -\frac{\partial\mathcal{H}}{\partial\mathbf{H}} = \sum_{ij} I_{ij}^s(\mathbf{r}_{ij}\wedge\mathbf{R}_{ij}), \tag{3}$$

which results in the stress induced net magnetization

$$\mathbf{M}_s(\mathbf{H},\boldsymbol{\sigma}) \equiv \frac{1}{V} < \boldsymbol{\mu}_s >= \int_0^\infty d\mathbf{r}_{ij}d\mathbf{R}_{ij}f(\mathbf{r}_{ij},\mathbf{R}_{ij})\boldsymbol{\mu}_s, \tag{4}$$

where V is a sample's volume and f the joint probability distribution function (see below). To capture the very essence of the superconducting piezomagnetic effect, in what follows we assume for simplicity that an *unloaded sample* does not possess any spontaneous magnetization at zero magnetic field (that is $M_s(0,0) = 0$) and that its Meissner response to a small applied field H is purely diamagnetic (that is $M_s(H,0) \simeq -H$). According to Eq.(2), this condition implies $\phi_{ij}^0 = 2\pi m$ for the initial phase difference with $m = 0, \pm1, \pm2, ...$

In order to obtain an explicit expression for the piezomagnetization, we consider a site positional disorder that allows for small random radial displacements. Namely, the sites in a 3D cubic lattice are assumed to move from their equilibrium positions according to the normalized distribution function $f(\mathbf{r}_{ij}\mathbf{R}_{ij}) \equiv f_r(\mathbf{r}_{ij})f_R(\mathbf{R}_{ij})$. For simplicity we assume an exponential distribution law for the distance between grains, $f_r(\mathbf{r}) = f(x)f(y)f(z)$ with $f(x_j) = (1/d)e^{-x_j/d}$ (which reflects a short-range character of the Josephson coupling in granular superconductors [5]), and some short range distribution for the dependence of the center-of-mass probability $f_R(\mathbf{R})$ (around some constant value D).



The reduced magnetization M_s/M_c as a function of the reduced applied magnetic field H/H_0, according to Eq.(5) for different values of reduced applied stress: $\sigma/\sigma_c = 0$ (solid line), $\sigma/\sigma_c = 0.01$ (dotted line), and $\sigma/\sigma_c = 0.05$ (dashed line)

Taking the applied stress along the x-axis, $\boldsymbol{\sigma} = (\sigma, 0, 0)$, normally to the applied magnetic field $\mathbf{H} = (0, 0, H)$, we get finally

$$M_s(H,\sigma) = -M_0(\sigma)\frac{H_{tot}(H,\sigma)/H_0}{[1 + H_{tot}^2(H,\sigma)/H_0^2]^2}, \tag{5}$$

for the induced transverse magnetization (along the z-axis), where $H_{tot}(H,\sigma) = H - -H^*(\sigma)$ is the total magnetic field with $H^*(\sigma) = [\sigma/\sigma_0(\sigma)]H_0$ being a stress-induced contribution. Here, $M_0(\sigma) = I_c(\sigma)SN/V$ with $S = \pi dD$ being a projected area around the Josephson contact, $H_0 = \Phi_0/S$, and $\sigma_0(\sigma) = \sigma_m[j_c(\sigma)/j_d](b/d)$ with $j_d = d_n\dot{\epsilon}_0$ and $\dot{\epsilon}_0 = b\rho v_0$ being the maximum values of the dislocation current density and the plastic deformation rate, respectively. According to the recent experiments [15], the tunneling dominated critical current I_c (and its density j_c) in HTGS ceramics was found to exponentially increase under compressive stress, viz. $I_c(\sigma) = I_c(0)e^{\beta\sigma}$ with $\beta \simeq 1/\sigma_m$. Specifically, the critical current at $\sigma = 9\,\mathrm{kbar}$ was found to be three times higher its value at $\sigma = 1.5\,\mathrm{kbar}$, clearly indicating a weak-links-mediated origin of the phenomenon (in the best defect-free thin films this ratio is controlled by the stress induced modifications of the carrier number density and practically never exceeds a few percents [14]). Strictly speaking, the critical current will also change (decrease) with applied magnetic field. However, for the fields under discussion (see below) in the first approximation this effect can be neglected. In view of Eq.(5), dependence of I_c on σ will lead to a rather strong piezomagnetic effects. Indeed, Figure shows changes of the initial stress-free diamagnetic magnetization M_s/M_c (solid line) under an applied stress σ/σ_c. Here $M_c \equiv M_0(0)$ and $\sigma_c \equiv \sigma_0(0)$ (see below for estimates). As we see, already relatively small values of an applied stress render a low field Meissner phase strongly paramagnetic (dotted and dashed lines) simultaneously increasing the maximum of the magnetization and shifting it towards higher magnetic fields. According to Eq.(5), the initially diamagnetic Meissner effect turns paramagnetic as soon as the piezomagnetic contribution $H^*(\sigma)$

exceeds an applied magnetic field H. To see whether this can actually happen in a real material, let us estimate a magnitude of the piezomagnetic field H^*. Typically [2,3,18], for HTGS ceramics $S \approx 10\mu m^2$, leading to $H_0 \simeq 1G$. To estimate the needed value of the dislocation current density j_d, we turn to the available experimental data. According to Ref.[10], a rather strong polarization under compressive pressure $\sigma/\sigma_m \simeq 0.1$ was observed in YBCO ceramic samples at $T = 77\,K$ yielding $d_n = 10^2\,C/m^2$ for the piezo-electric coefficient. Usually [6,9,11], for GBJJs $\dot{\epsilon}_0 \simeq 10^{-2}\,s^{-1}$, and $b \simeq 10\,nm$ leading to $j_d = d_n\dot{\epsilon}_0 \simeq 1\,A/m^2$ for the maximum dislocation current density. Using the typical values of the critical current density $j_c(\sigma) = 10^4\,A/m^2$ (found [15] for $\sigma/\sigma_m \simeq 0.1$) and grain size $d \simeq 1\,\mu m$, we arrive at the following estimate of the piezomagnetic field $H^* \simeq 10^{-2}H_0$. Thus, the predicted stress induced paramagnetic Meissner effect should be observable for applied magnetic fields $H \simeq 10^{-2}H_0 \simeq 0.01\,G$ which correspond to the region where the original PME was first registered [2,3].

In turn, the piezoelectric coefficient d_n is related to a charge Q in the GBJJ as [18] $d_n = (Q/S)(d/b)^2$. Given the above-obtained estimates, we get $Q \simeq 10^{-13}C$ for an effective charge accumulated by the GBs. Notice that the above values of the aplied stress σ and the resulting effective charge Q correspond (via the so-called electroplastic effect [18]) to an equivalent applied electric field $E = b^2\sigma/Q \simeq 10^7 V/m$ at which rather pronounced electric-field induced effects in HTGS have been recently observed [1,8].

Besides, according to Ref.[15] the Josephson projected area S slightly decreases under pressure thus leading to some increase of the characteristic field $H_0 = \Phi_0/S$. In view of Eq.(5), it means that a smaller compression stress is needed to actually reverse the sign of the induced magnetization M_s. Furthermore, if an unloaded granular superconductor already exhibits the PME, due to the orbital currents induced spontaneous magnetization resulting from an initial phase difference $\phi^0_{ij} = 2\pi r$ in Eq.(2) with fractional r (in particular, $r = 1/2$ corresponds to the so-called [2, 3] π-type state), then according to our scenario this effect will either be further enhanced by applying a compression (with $\sigma > 0$) or will disappear under a strong enough extension (with $\sigma < 0$). Though a particular form of $M_s(H, \sigma)$ obtained in this paper may slightly change (due mainly to neglected here field dependence of the critical current and self-field effects), the above-estimated range of accessible parameters still suggests quite an optimistic possibility to observe the predicted effects experimentally in HTGS ceramics.

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