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INFLUENCE OF ΛNN FORCES AND THE NUCLEAR CORE SIZE ON THE $^7_\Lambda$Li LEVEL SPECTRUM

V.N.Fetisov[1)]

Lebedev Physical Institute RAS
117924 Moscow, Russia

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The effective zero-range ΛNN interaction with the Λ-spin dependence pecular to dispersive ΛNN forces is introduced to estimate its influence on the shell model spectra of light hypernuclei. The parameters of three-body and pair interactions are evaluated using the values of B_Λ for s-shell hypernuclei with A=4 and 5. It is shown that the description of the $^7_\Lambda\mathrm{Li}_{g.s.}$ doublet energy $\Delta E \simeq 689$ keV fitted with other hypernuclei gives the reduced radius $R \simeq 2.2$ fm for ^{6}Li bound in $^7_\Lambda$Li (for the free nucleus ^{6}Li $R \simeq 2.4$ fm) and the dispersive ΛNN force contribution to this energy ~ 250 keV. A more refined version of the $^7_\Lambda$Li level spectrum including ΛN and ΛNN forces and the compression of the ^{6}Li core is proposed.

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The current shell model calculations of hypernuclear spectra are based on the pair ΛN interaction and in doing so, the coupling of Λ and Σ channels is disregarded [1, 2]. However, it is of interest to estimate the contributions of ΛNN forces to the level spacing with the KEK-BNL hypernuclear γ-spectroscopy program in mind [3].

This work was stimulated by results [4] where the large contribution ($\sim 30\%$) of dispersive ΛNN (DF) forces to the $1^+, 0^+$ spin splitting $\Delta E_4(1^+, 0^+) \simeq 1.1$ MeV in $^4_\Lambda$H(He) has been obtained and by the discussion [5] of two forms of DF and their role in light hypernuclei as well.

Recently the $^7_\Lambda\mathrm{Li}_{g.s.}$ doublet splitting $\Delta E_7(3/2^+_1, 1/2^+) \simeq 689 \pm 4$ keV has been measured in the $(\pi^+, K^+\gamma)$ reaction in KEK [6]. This value exceeds the expected energy[2)] ~ 440 keV evaluated in the shell model with the spin-spin parameter of ΛN interaction $\Delta(B) \simeq 0.3$ MeV [2] describing hypernuclear levels in the vicinity of $^{10}_\Lambda$B. From this KEK result it is inferred that the empirical value of Δ(Li) in $^7_\Lambda$Li is nearly twice as large as that for more heavy hypernuclei.

1) e-mail: fetisov@sci.lebedev.ru

2) The identification of the $^{10}_\Lambda$B secondary γ-line with $E_\gamma = 442 \pm 2.1$ keV [7] ascribed [8] to $^7_\Lambda$Li is probably an open problem.

To elucidate the reasons of this difference we study in the next sections the dependence of Δ and the ${}^{7}_{\Lambda}$Li level splitting on the host nucleus (or hypernucleus) sizes and ΛNN forces.

For the p shell hypernuclei four potential parameters Δ, S_{Λ}, S_N and T (expressed here in MeV) corresponding to spin-spin, two spin-orbit and tensor parts of ΛN interaction determine the level positions [9]. The harmonic oscillator wave functions with a common h.o. parameter r_0 for the Λ particle and nucleons are accepted to estimate a scale of influence of nuclear sizes on Δ that is important parameter to produce a doublet splitting. In this case Δ is of the form

$$\Delta(r_0) = V_{\sigma}(a/\pi)^{3/2}\frac{(1 + ar_0^2 - \varepsilon)}{(1 + 2ar_0^2)^{5/2}}, \tag{1}$$

where $V_{\sigma} = V_s - V_t$ is a difference of singlet and triplet volume integrals for the ΛN Gaussian potential $\sim \exp(-ar^2)(1 - \varepsilon + \varepsilon P_x)$ with $a = 0.9376\ \mathrm{fm}^{-2}$ [10] and $\varepsilon = 0.25$ [4].

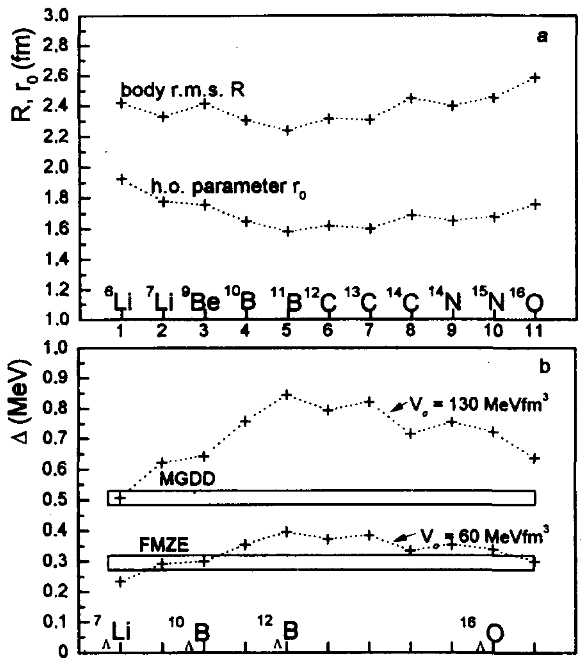

Fig.1. Nuclear body r.m.s. radii, oscillator parameters and Δ for the p shell hypernuclei

The body r.m.s. radii R and the values of $r_0({}^{A}Z)$ are shown for the p shell nuclei in Fig.1a. They were calculated with the Tassie – Barker correction on the center-of-mass motion [11] using nuclear charge radii and the proton radius ~ 0.8 fm [12]. One of last estimates of $\Delta \simeq 0.5$ have been obtained from a description of $\Delta E_4(1^+, 0^+)$ (the MGDD set of parameters) [1]. Within the Λ-nucleus model [10] and using ΔE_4 as well we find $V_{\sigma} \simeq 130\mathrm{MeV}\cdot\mathrm{fm}^3$ and too large $B_{\Lambda}({}^{5}_{\Lambda}\mathrm{He}) \simeq 6.9$ MeV. For this case eq. (1) gives the upper curve shown in Fig.1b. But these large values of Δ have been excluded [2] early by the BNL data [7]. If one accepts $\Delta(B) \simeq 0.3$ (the FMZE set of parameters) [2] and $r_0(B) \simeq 1.74$ fm then $V_{\sigma} \simeq 60$ MeV$\cdot$fm^3 and we have the lower curve in Fig.1b with $\Delta(\mathrm{Li}) \simeq 0.23$. The value $\Delta(\mathrm{Li}) \simeq 0.23$ is in a contradiction with the KEK data. Really the ${}^{7}_{\Lambda}$Li doublet splitting can be written with a good accuracy as

$$\Delta E_7(\frac{3^+}{2_1}, \frac{1^+}{2}) \simeq \frac{3}{2}\beta^2\Delta(\mathrm{Li}), \tag{2}$$

where $\beta = 0.992$ is the amplitude of the dominant ${}^{13}S$-state in the ${}^{6}\mathrm{Li}_{g.s.}$ wave function [13]. For $\Delta(\mathrm{Li}) = 0.23$ obtained for the free nucleus ^{6}Li ($r_0 = 1.92$ fm) the value $\Delta E_7 = 340$ keV is too small in comparison with $\Delta E_7^{exp} \simeq 689$ keV. Since Δ is a decreasing function of r_0 to reproduce $\Delta E_7^{exp.}$ the bound nucleus ^{6}Li should be compressed upto unrealistic small sizes ($R \simeq 1.8$ fm, $r_0 \simeq 1.45$ fm) in a strong contradiction with a scale of compression ($\Delta R/R \simeq 15\%$) predicted by the cluster model calculations [14]. These difficulties of the ${}^{7}_{\Lambda}$Li shell model with the pair ΛN forces leads us to consideration of the role of ΛNN forces.

Two types of ΛNN forces arise from a strong $\Lambda N \leftrightarrow \Sigma N$ coupling. First of them $V^{2\pi}_{\Lambda NN}$ is generated due to the sequential double pion exchange accompanied by $\Lambda\Sigma$ conversion [15] and the other one, so called dispersive forces $V^{D}_{\Lambda NN}$, occurs when one of baryons of the intermediate ΣN pair interacts with an additional nucleon (3π-exchange) [4, 5]. The direct employment of explicit expressions for ΛNN forces derived by the pion exchange formalism and given in ref. [5] is not well grounded in the shell model calculations especially due to their indeterminate behaviour at small distances between baryons that is strongly dependent from short range correlations. Because of the short range nature of three-body forces in a scale of hypernuclear sizes it seems to be reasonable to write them in a zero range approximation conserving the Λ-spin dependence pecular to dispersive forces

$$V_{\Lambda NN} = \delta(\mathbf{r}_\Lambda - \mathbf{r}_1)\delta(\mathbf{r}_\Lambda - \mathbf{r}_2)(t + t^s \boldsymbol{\sigma}_\Lambda(\boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_2)). \tag{3}$$

Here a constant t includes Λ-spin independent parts of the dispersive and 2π-exchange three-body forces which, as calculations show, have a tendency to compensate each other. The spin-dependent term of the potential (3) is not zero only in ${}^{13}S$-state of a nucleon pair. Obviously it gives no contribution to matrix elements for nuclear cores with the total spin S=0. These properties of eq.(3) reflects probably the suppression of $\Lambda\Sigma$ coupling seen explicitly for ${}^{5}_{\Lambda}$He in the two-channel formalism of the s shell hypernucleus description [16].

Four volume integrals $V_{s(t)}$ and t, t^s were determined using B_Λ in the $0^+, 1^+$ states of ${}^{4}_{\Lambda}$H(He) and $B_\Lambda({}^{5}_{\Lambda}\mathrm{He})$ within the Λ-nucleus model [10] with the Gaussian nuclear wave functions from equations

$$-\frac{\hbar^2}{2\mu}\frac{d^2\chi_0}{dr^2} + \{S\exp(-\frac{6}{R_3^2}r^2) - V(S)\exp(-\frac{3}{2R'^2_3}r^2)\}\chi_0 = (-2.22\ \mathrm{MeV})\chi_0, \tag{4}$$

$$-\frac{\hbar^2}{2\mu}\frac{d^2\chi_1}{dr^2} + \{P\exp(-\frac{6}{R_3^2}r^2) - aV(S)\exp(-\frac{3}{2R'^2_3}r^2)\}\chi_1 = (-1.1\ \mathrm{MeV})\chi_1, \tag{5}$$

$$\begin{aligned} -\frac{\hbar^2}{2\mu_\alpha}\frac{d^2\chi}{dr^2} \quad &+ \quad \{\frac{81}{16}(P + \frac{1}{3}S)\left(\frac{R_3}{R_\alpha}\right)^6 \exp(-\frac{4}{R_\alpha^2}r^2) - \\ &- \quad b\left(\frac{R'_3}{R'_\alpha}\right)^3 V(S)\exp(-\frac{3}{2R'^2_\alpha}r^2)\}\chi = (-3.12\ \mathrm{MeV})\chi, \end{aligned} \tag{6}$$

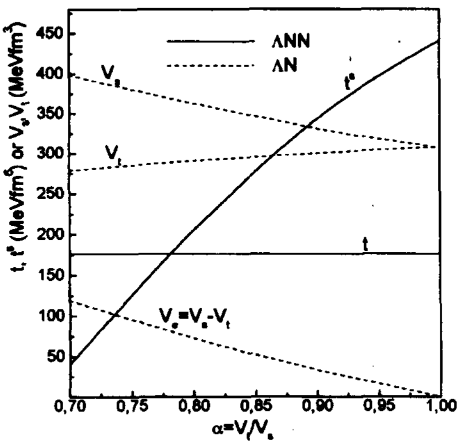

Fig.2. The volume integrals as the functons of α

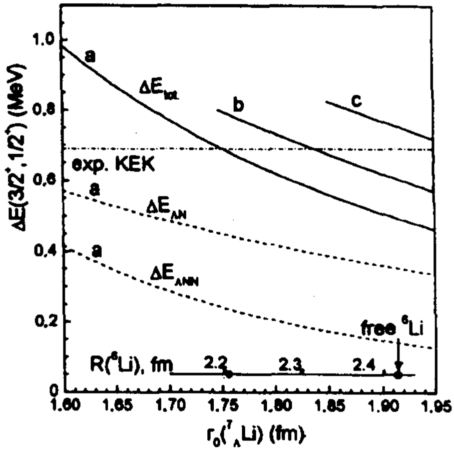

Fig.3.The dependence of the $^{7}_{\Lambda}$Li doublet splitting on r_0 or $R(^6\text{Li})$. The curves are obtained for $V_\sigma = 60$ MeVfm^3 (a), $V_\sigma = 90$ MeVfm^3 (b) and $V_\sigma = 130$ MeVfm^3 (c)

where χ are the radial functions of the Λ-particle;

$$S = \lambda(t - 2t^s), \quad P = \lambda(t + \frac{2}{3}t^s), \quad \lambda = \frac{3^{5/2}}{\pi^3 R_3^6}, \quad \alpha = V_t/V_s;$$

$$V(S) = \frac{3}{2}(1+\alpha)V_s\left(\frac{3}{2\pi R_3'^2}\right)^{3/2}, \quad a = \frac{1+5\alpha}{3(1+\alpha)}, \quad b = \frac{2(1+3\alpha)}{3(1+\alpha)};$$

$$R_3 = \bar{R}(^3\text{H},^3\text{He}) = 1.6 \text{ fm}, \quad R_\alpha = R(^4\text{He}) = 1.46 \text{ fm};$$

$$R'_{3(\alpha)} = (R^2_{3(\alpha)} + c)^{1/2}, \quad c = 1.6026 \text{ fm}^2.$$

The volume integrals as functions of the ratio $\alpha = V_t/V_s$ are shown in Fig.2. For $\alpha \simeq 0.67$ one finds t^s=0, $V_\sigma \simeq 130$ MeV·fm^3 and therefore all the value of ΔE_4 results from spin-spin ΛN forces. If $\alpha = 1$ then $V_\sigma = 0$, $t^s \simeq 440$ MeV·fm^6 and the total doublet splitting is described by DF. For the acceptable value $V_\sigma \simeq 60$ MeV·fm^3 derived from $\Delta(B) = 0.3$ we have $\alpha = 0.83$ and $t^s = 250$ MeV·fm^6.

Now the additional term originates from DF in the final formula for

$$\Delta E_7 = \beta^2 \left\{ \frac{3}{2} V_\sigma (a/\pi)^{3/2} \frac{(1 + a r_0(\text{Li})^2 - \varepsilon)}{(1 + 2a r_0(\text{Li})^2)^{5/2}} + \frac{41}{3^{7/2}\pi^3 r_0(\text{Li})^6} \cdot t^s \right\}. \qquad (7)$$

The curves marked by a in Fig. 3 shows the separate contributions to $\Delta E_{tot.}$ from ΛN and ΛNN forces as functions of r_0(Li) (or R) at $V_\sigma = 60$ MeV·fm^6. In this case the DF contribution to ΔE_4 is about $1/2\Delta E_4^{exp} \simeq 500$ keV. As Fig. 3 suggests there is a need to reduce the ^{6}Li radius up to $R \simeq 2.2$ fm ($\Delta R/R \simeq 8\%$) to reproduce ΔE_7^{exp}. The parameters used for the curve b give somewhat larger values of R and $\Delta(B) \simeq 0.4$ and the DF contribution to ΔE_4 is $\simeq 1/3\Delta E_4^{exp}$ which is just the value predicted in ref.[4]. Lastly, the curve c corresponds to the case when the whole of splitting ΔE_4 results from

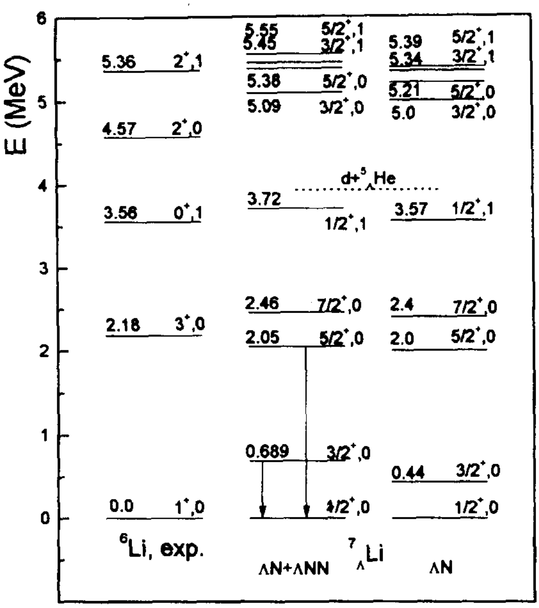

Fig.4.The measured γ-transitions and the new version of the ${}^7_\Lambda$Li spectrum

spin-spin ΛN forces and then one needs to increase (!) the ^{6}Li radius up to $R \simeq 2.5$ fm to be in agreement with the KEK data.

All the ${}^7_\Lambda$Li low-lying levels displayed in Fig.4 have been obtained with the Barker's NN interaction [13] using $V_\sigma = 60$ MeV·fm^3, $r_0 = 1.74$ fm (for compressed ^{6}Li, $\Delta = 0.3015$), $S_N = -0.4$, $S_\Lambda = -0.02$, $T = 0.02$ [2], $t = 176$ MeV·fm^6 and $t^s = 250$ MeV·fm^6. The spectrum marked by ΛN is given for $t = t^s = 0$.

The γ -deexcitation of the 2049 ± 2 keV $5/2^+$ level observed in KEK also is well reproduced with the spin-orbit parameter $-S_N = 0.4$ exceeding in four times $-S_N$ found for other p shell hypernuclei. The spectrum depends only on the sum $a + S_N$ where $a = -1.584$ is the Barker's single nucleon spin-orbit constant taking here for the free nucleus. It suggests that increasing $|S_N|$ should be rather associated with increasing $|a|$ ($\Delta a/a \simeq 20\%$) due to a decrease of the diffuseness of a single-nucleon potential under compression of ^{6}Li. This effect has been recognized recently for ${}^{12}_\Lambda$Be and ${}^{16}_\Lambda$C in the Skyrme-Hartree-Fock calculations [17].

In the framework of nuclear and hypernuclear models acceped here it is impossible to reproduce in consistent way the ${}^7_\Lambda$Li$_{g.s.}$ doublet splitting observed and the level positions in the s and p shell hypernuclei only with the pair ΛN interaction. A shell model description of this doublet splitting is possible: *i*) if one introduces the Λ-spin dependent dispersive ΛNN forces and *ii*) if the nuclear core response manifests itself in the compression of ^{6}Li. This conclusion is in line with the cluster model calculations in which a strong "glue-like" role of the Λ particle has been revealed [18, 14] and the dynamical contraction of ^{6}Li ($\sim 15\%$) has been predicted. In this work the r.m.s. radius of the bound ^{6}Li extracted from the doublet splitting is about 2.2 fm that correponds to its overall radial compression $\sim 8\%$.

The DF contribution to the $T = 0$ ground state doublet of ${}^7_\Lambda$Li is largest and it is equal to ~ 250 keV versus the 50 keV contribution to the $T = 1$ doublet in ${}^7_\Lambda$He. A large value of the induced spin-orbit parameter $|S_N|$ may be a signal of increasing the nucleon spin-orbit constant $|a|$ ($\sim 20\%$) due to the compression of ^{6}Li.

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1. D.J.Millener, A.Gal, C.B.Dover, and R.H.Dalitz, Phys. Rev. **C31**, 499 (1985).
2. V.N.Fetisov, L.Majling, J.Žofka, and R.A.Eramzhyan, Z. Phys. **A339**, 399 (1991).
3. H.Tamura, Nucl. Phys. **A639**, 83c (1998); H.Tamura, in *Abstr. of the Workshop on Strangeness Nucl. Phys.*, Seoul, Feb. 19-22, 1999, p. 123.
4. A.R.Bodmer and Q.N.Usmani, Nucl. Phys. **A477**, 621 (1988).
5. A.Gal, *Proc. of the LAMPF* (π, K) *Workshop*, Los Alamos, Oct. 11-13, 1990.
6. H.Tamura, private communication, January 1999; K.Tanida, in *Abstr. of the Workshop on Strangeness Nucl. Phys.*, Seoul, Feb. 19-22, 1999, p. 33.
7. R.E.Crien, S.Bart, M.May et al., Phys. Rev. **C41**, 1062 (1990).
8. L.Majling, J.Žofka, V.N.Fetisov, and R.A.Eramzhyan, Z. Phys. **A337**, 337 (1990).
9. A.Gal, J.M.Soper, and R.H.Dalitz, Ann. Phys. **63**, 53 (1971).
10. R.H.Dalitz and B.W.Downs, Phys. Rev. **111**, 967 (1958).
11. L.J.Tassie and F.C.Barker, Phys. Rev. **111**, 940 (1958).
12. R.C.Barrett and D.F.Jackson, *Nuclear Sizes and Structure*, Clarendon press, Oxford, 1977.
13. F.C.Barker, Nucl. Phys. **83**, 418 (1966).
14. E.Hiyama, M.Kamimura, K.Miyzaki, and T. Motoba, to be published in Phys. Rev. **C59**, No. 5 (1999).
15. R.Bhaduri, B.Loiseau, and Y.Nogami, Ann. Phys. **44**, 67 (1967).
16. B.F.Gibson, I.R.Afnan, J.A.Carlson, and D.R.Lehman, Prog. Theor. Phys. Suppl. **117**, 339 (1994).
17. T.Yu.Tretyakova and D.E.Lanskoy, to be published in Eur. Phys. J.
18. T.Motoba, H.Bandō, and K.Ikeda, Prog. Theor. Phys. **70**, 189 (1983).