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SUPPRESSION OF INCOHERENT SCATTERING OF MÖSSBAUER RADIATION BY NUCLEI IN REVERSING MAGNETIC FIELD

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The effect of magnetic field reversal on the incoherent scattering of Mössbauer radiation by nuclei is analyzed. A suppression of this process is shown to arise after such reversal. This effect is analyzed both from the quantum and classical points of view.

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One of the obstacles in realization of γ-lasers at Mössbauer isotopes is great ratio of the conversion-electron width to the coherent radiation one [1]. Therefore recent discovery by Shvyd'ko et al. [2,3] of short-time enhancement of the coherent radiation channel of the isotopes ^{57}Fe, caused by abrupt alteration of the magnetic field direction, seems to be very promising. They have been investigating time dependence of the Mössbauer transmission through a weak ferromagnetic crystal of $FeBO_3$ after reversal of the field. A short flash of the transmitted beam intensity has ben observed just after this reversal, which was followed by attenuating oscillations. Duration of this transient process is of the order of the nuclear lifetime τ_N. Similar reversals have been realized in many other experiments on Mössbauer absorption by soft ferromagnets placed in an external radio-frequency (rf) magnetic field (see the surveys [4-7]). The rf field gives rise to periodical reversals of the crystal magnetization, which ensure periodical jumps of the field at the nuclei ^{57}Fe between two values $+\mathbf{h}_0$ and $-\mathbf{h}_0$. The corresponding theory has been built in [8-13]. A strict explanation of the transient effect observed by Shvyd'ko et al. [2,3] has been provided by [14]. Besides, it was predicted also a suppression of the conversion-electron yield caused by the reversal of the magnetic field. In present article we consider a suppression of the incoherent scattering of γ-quanta in the same situation and clarify physical reasons of the effect.

Let the magnetic field at the Mössbauer nucleus in a crystal reverse at $t = 0$ from $+\mathbf{h}_0$ to $-\mathbf{h}_0$. At $t \to -\infty$ the incident γ-quantum has wave vector $\mathbf{k}$, polarization $\mathbf{e}_\lambda$ and frequency $\omega = E/\hbar$. In the initial state of the scatterer $|\alpha>$ the nucleus is described by the wave function $\psi^N_{I_gM_g}(t)$, where I is the spin of the nucleus and M is its projection on the direction of $+\mathbf{h}_0$, the crystal is described by $|\{v^0_s\}$, where $\{v^0_s\}$ stands for the initial set of phonons. The phononless energy distribution of incident γ-rays is determined by

$$w^{(0)}_e(E) = \frac{(\Gamma/2\pi)e^{-2W_e}}{(E-E_0)^2+(\Gamma/2)^2}, \tag{1}$$

where e^{-2W_e} is the Debye-Waller factor of the emitter, E_0 and Γ are respectively the energy and width of the level of the emitting nucleus. The scatterer in the final state is

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described by $|\beta\rangle = \psi^N_{I_g M'_g}(t)|\{v'_s\}$, while the γ-quantum has wave vector $\mathbf{k}'$ and polarization $\mathbf{e}'_{\lambda'}$. We assume the following resonant condition to be fulfilled:

$$E \simeq E_0^+ = E'_0 + \hbar\alpha_{eg}, \qquad \hbar|\alpha_{eg}| \gg \Gamma, \tag{2}$$

where E'_0 is the resonant energy of the absorbing nucleus,

$$\alpha_{eg} = (\gamma_g M_g - \gamma_e M_e)h_0/\hbar, \tag{3}$$

and γ_κ is the gyromagnetic ratio of the nucleus in the ground ($\kappa = g$) or excited ($\kappa = e$) state. In this case only the isolated nuclear transition $M_g \to M_e$ is generated in the field $+\mathbf{h}_0$ and $-M_g \to -M_e$ in the field $-\mathbf{h}_0$.

The electromagnetic wave scattered by the j-th nucleus, having initial spin projection $+M_g$, into the β-th channel (in units $i(2\pi\hbar\omega)^{1/2}$) is given by (see also [14])

$$\mathbf{E}_{sc}(\mathbf{r},t)^j_{\alpha\to\beta} = \sum_{\lambda'} \mathbf{e}'_{\lambda'} f_{\alpha\beta}(\mathbf{k},\mathbf{e}_\lambda;\mathbf{k}',\mathbf{e}'_{\lambda'})^N_j \frac{1}{r}\times$$

$$\times\left\{(1-\theta(t^*))\, e^{-i\omega' t^*} + \theta(t^*) e^{-i\omega_0'^- t^* - \Gamma t^*/2\hbar}\right\}, \tag{4}$$

where $t^* = t - r/c$ is the retarded time, $\omega_0'^{\pm} = E'_0/\hbar \pm (\gamma_g M'_g - \gamma_e M_e)h_0/\hbar$ are the resonant frequencies of transitions $\pm M_e \to \pm M'_g$ in the field $\mathbf{h}_0$,

$$\theta(x) = \begin{cases} 1, & x > 0, \\ 0, & x < 0, \end{cases} \tag{5}$$

$f_{\alpha\beta}$ is the scattering amplitude of γ-quanta by the Mössbauer isotope $+M_g$ in the constant field $\mathbf{h}_0$. In slow-collision approximation [15] it is given by

$$f_{\alpha\beta}(\mathbf{k},\mathbf{e}_\lambda;\mathbf{k}',\mathbf{e}'_{\lambda'})^N_j = -\langle\{v'_s\}|e^{-i\mathbf{k}'\mathbf{u}_j}|\{v^0_s\}\rangle\langle\{v^0_s\}|e^{i\mathbf{k}\mathbf{u}_j}|\{v^0_s\}\rangle\times$$

$$\times\frac{c^{-2}\langle e|\hat{j}^N_{\lambda'}(\mathbf{k}')|g'\rangle^*\langle e|\hat{j}^N_\lambda(\mathbf{k}')|g\rangle}{E - E_0^+ + i\Gamma/2}, \tag{6}$$

where $\mathbf{u}_j$ is the displacement of the j-th nucleus from its equilibrium position and $\hat{j}^N_\lambda(\mathbf{k})$ are the Fourier components of the nuclear current density operators (see e.g. [10, 14]).

The wave scattered by the nucleus being initially in the state $-M_g$ will be [14]

$$\mathbf{E}_{sc}(\mathbf{r},t)^j_{-\alpha\to-\beta} = \sum_{\lambda'} \mathbf{e}'_{\lambda'} f_{-\alpha,-\beta}(\mathbf{k},\mathbf{e}_\lambda;\mathbf{k}',\mathbf{e}'_{\lambda'})^N_j \frac{1}{r}\times$$

$$\times\left\{e^{-i\omega' t^*} - e^{-i\omega_0'^+ t^* - \Gamma t^*/2\hbar}\right\}\theta(t^*), \tag{7}$$

where $f_{-\alpha,-\beta}$ is defined by Eq. (6) with e, g replaced by $-e, -g$.

The flux density of γ-rays scattered by the $\pm M_g$ nucleus is

$$j_{sc}(\omega,t)(\pm) = c\sum_{M'_g}\ \sum_{\{v'_s\},\{v^0_s\}} g(\{v^0_s\})|\mathbf{E}_{sc}(\mathbf{r},t) \pm \alpha \to \pm\beta|^2, \tag{8}$$

where $g(\{v_s^0\})$ is the Gibbs distribution over the initial states of the lattice, and $\mathbf{E}_{sc}(\mathbf{r},t)$ is taken in units $i(2\pi\hbar\omega)^{1/2}$. The corresponding instantaneous differential cross section for scattering of γ-quanta by the isotope ^{57}Fe with $I_g = 1/2$ in an unpolarized target will be

$$\sigma(\omega,t) = \frac{1}{2}\left(\sigma^{(+)}(\omega,t) + \sigma^{(-)}(\omega,t)\right), \tag{9}$$

where

$$\sigma^{(\pm)}(\omega,t) = \frac{1}{c} j_{sc}(\omega,t)^{(\pm)} r^2 \tag{10}$$

are the cross sections at the nuclei being initially in the states $\pm M_g$. The incoherent scattering cross section by the whole target is proportional to (10).

It is useful to introduce the following notations:

$$x = 2(E - E_0^+)/\Gamma, \quad \tau = \Gamma t^*/\hbar, \quad x_0 = 2(E_0 - E_0^+)/\Gamma, \tag{11}$$

where x_0 is the detuning parameter and τ is the time in units of the nuclear lifetime $\tau_N = \hbar/\Gamma$. Then substituting (4) and (7) into (8)–(10), one obtains

$$\sigma^{(+)}(\omega,t) \sim \frac{1}{x^2+1}\left\{(1-\theta(\tau)) + e^{-\tau}\theta(\tau)\right\} \tag{12}$$

and

$$\sigma^{(-)}(\omega,t) \sim \frac{1}{x^2+1}\left\{(1 + e^{-\tau} - 2\cos(x\tau/2)\, e^{-\tau/2}\right\}\theta(\tau). \tag{13}$$

These cross sections must be averaged over the energy distribution of incident γ-quanta:

$$\bar{\sigma}^{(\pm)}(t) = \int_0^\infty w_e(E)\sigma^{(\pm)}(\omega,t)dE. \tag{14}$$

Substitution of (12), (13) into (14) gives the experimentally measured cross sections

$$\bar{\sigma}^{(+)}(t) = \bar{\sigma}^{(+)}\left\{(1-\theta(\tau)) + e^{-\tau}\theta(\tau)\right\} \tag{15}$$

and

$$\bar{\sigma}^{(-)}(t) = \bar{\sigma}^{(+)}\left\{1 - \left[\cos(x_0\tau/2) + (2/x_0)\sin(x_0\tau/2)\right] e^{-\tau}\right\}\theta(\tau), \tag{16}$$

where $\bar{\sigma}^{(+)}$ is a standard incoherent scattering cross section of γ-quanta in the stationary case:

$$\bar{\sigma}^{(+)} \sim \frac{1}{x_0^2+4}. \tag{17}$$

From (12),(13) and (15),(16) we see that prior to the field reversal the scattering proceeds only at the nuclei $+M_g$. After the reversal these nuclei continue to decay generating the exponentially attenuating wave concentrated at the resonant energy ω_0^-, which corresponds to de-excitation transition $M_e \to M_g'$ in the field $-\mathbf{h}_0$. On the contrary, the $-M_g$ nuclei begin to absorb incident radiation only at $t > 0$. But their contribution to the radiation yield grows gradually from zero value at $t = 0$ to $\bar{\sigma}^{(+)}$ at $t >> \tau_N$. Aside of the exact resonance ($x_0 \neq 0$) the function $\bar{\sigma}^{(-)}(\tau)$ oscillates with the period $\delta\tau = 4\pi/|x_0|$. In the case of exact resonance ($x_0 = 0$) this cross section becomes a monotonically growing function of the time:

$$\bar{\sigma}^{(-)}(t) = \bar{\sigma}^{(+)}\left\{1 - (1+\tau)e^{-\tau}\right\}\theta(\tau). \tag{18}$$

In an unpolarized target with isotopes ^{57}Fe the averaged cross section for exact resonance is

$$\bar{\sigma}(t) = \bar{\sigma}(0)\left\{1 - \tau e^{-\tau}\theta(\tau)\right\}, \tag{19}$$

where $\bar{\sigma}(0) = \bar{\sigma}^{(+)}/2$ is the value of $\bar{\sigma}(t)$ prior to the reversal.

The function (19) in the interval $0 \leq \tau < \infty$ is a sum of monotonically decreasing ($\bar{\sigma}^{(+)}(t)$) and increasing ($\bar{\sigma}^{(-)}(t)$) functions. Therefore it looks like a well with a minimum at the point $\tau = 1$, where it equals $0.67\bar{\sigma}(0)$. This means a suppression of the incoherent process during the time $\sim \tau_N$.

For better understanding of the results we will model the nucleus by a classical harmonic oscillator interacting with the classical electromagnetic wave. Namely, we will consider the point particle with charge q and mass m, vibrating with the eigenfrequency $\omega_0(t)$, which coincides with the frequency of transition in the nucleus. For the nuclei $-M_g$ this eigenfrequency takes the value ω_0^- at $t < 0$ and ω_0^+ at $t > 0$. For the nuclei with opposite orientation it is respectively ω_0^+ and ω_0^-. The external electromagnetic wave $\mathbf{E}(t) = \mathbf{E}_0 \cos\omega t$ acts on the oscillator with the force $\mathbf{f}(t) = \mathbf{f}_0 \cos\omega t$, where $\mathbf{f}_0 = q\mathbf{E}_0$. Let the coordinate of the particle along $\mathbf{E}_0$ be $z(t)$. Its vibrations are described by the Newton equation, which contains both the friction force $-\gamma\dot{z}$ and the radiation reaction proportional to the third derivative of z [16]. The latter leads only to addition of the radiation damping to γ, therefore it is omitted here for brevity. Then the motion equation in the complex form reads (see also [17])

$$\ddot{z} + \gamma\dot{z} + \omega_0^2(t)z = (f_0/m)e^{i\omega t}, \tag{20}$$

As in the quantum case we demand that

$$0 \leq |\omega - \omega_0^+| \sim \gamma, \quad \gamma \ll \omega_0^+, |\omega^+ - \omega^-|, \tag{21}$$

that is only vibrations with the frequency ω_0^+ are resonating with the external force.

First we will consider the oscillator to model the nucleus $-M_g$ starting to move at ($t > 0$), when $\omega(t) = \omega_0^+$. In this case we must solve Eq.(20) with initial condition $z(0) = \dot{z}(0) = 0$. In the approximation (21) it has the following solution:

$$z(t)^- = A\left\{e^{i\omega_0^+ t - \gamma t/2} - e^{i\omega t}\right\}, \tag{22}$$

where the complex amplitude is

$$A = \frac{f_0/2m\omega_0}{\omega - \omega_0^+ + i\gamma/2}. \tag{23}$$

The oscillator similar to $+M_g$ nuclei at $t < 0$ performs forced vibrations with the amplitude A and frequency ω. After the reversal ($t > 0$), when frequency of the external force deviates essentially from new eigenfrequency ω_0^- of the oscillator, it is described then by Eq. (20) with $f_0 \approx 0$. The at $t > 0$ the solution is

$$z(t)^+ = Ae^{i\omega_0^- t - \gamma t/2}. \tag{24}$$

The classical electromagnetic wave emitted by the vibrating charged particle

$$\mathbf{E}_{sc}(\mathbf{r}, t) \sim \ddot{\mathbf{z}}(t) \tag{25}$$

depends on time and frequency as predicted by quantum calculations (4) and (7).

Thus contributions into the scattered radiation from the nuclei with different orientation have quite different behavior. While $+M_g$ nuclei at $t > 0$ give rise to the attenuating wave with carrier frequency $\omega_0'^- q$, the $-M_g$ nuclei produce both the transient attenuating wave with frequency $\omega_0'^+$ and the stationary one with frequency ω'. Respectively, for $+M_g$ nuclei the intensity of scattered radiation exponentially decreases at $t > 0$ and for $-M_g$ those it grows during the time of the order of τ_N. Their summation in unpolarized targets lead to appearance of the well in the curve describing time dependence of the incoherent radiation yield. Thus the incoherent channel is suppressed since the nucleus does not react instantaneously on the alteration of external conditions. Its response time coincides with the nuclear lifetime τ_N. It is noteworthy that this suppression effect is not related to enhancement of the coherent radiative channel caused by interference of three coherent scattered waves with frequencies ω, ω^+, and ω^-.

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