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DYNAMICAL CORRELATIONS OF TWO-DIMENSIONAL VORTEX-LIKE DEFECTS

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We examine high-order dynamical correlations of defects (quantum vortices, disclinations etc) in thin films starting from the Langevin equation for the defect motion. We demonstrate that dynamical correlation functions F_{2n} of vorticity or disclinicity behave as $F_{2n} \sim y^2/r^{4n}$ where r is the characteristic scale and y is the renormalized fugacity. Therefore below the Berezinskii – Kosterlitz – Thouless transition temperature F_{2n} are characterized by anomalous scaling exponents. The behavior strongly differs from the normal law $F_{2n} \sim F_2^n$ occurring for simultaneous correlation functions, the non-simultaneous correlation functions appear to be much larger. The phenomenon resembles intermittency in turbulence.

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It is well known that defects like quantum vortices, spin vortices, dislocations and disclinations play an essential role in physics of low-temperature phases of thin films. Berezinskii [1] and then Kosterlitz and Thouless [2] recognized that there is a class of phase transitions in $2d$ systems related to the defects. The main idea of their approach is that in $2d$ the defects can be treated as point objects interacting like charged particles. The low-temperature phase corresponds to a fluid constituted of bound uncharged defect-antidefect pairs, which is an insulator, whereas the high-temperature phase contains free charged particles and can be treated as plasma. Correspondingly, in the low-temperature phase the correlation length is infinite whereas in the high-temperature phase it is finite. A huge number of works is devoted to different aspects of the problem, see, e.g., the surveys [3 – 7]. The scheme proposed by Kosterlitz and Thouless can be applied to superfluid and hexatic films and planar $2d$ magnetics. It admits a generalization for crystalline films, see Refs. [8, 9]. There are also applications to superconductive materials, especially to high-T_c superconductors, see, e.g., Ref. [10].

The dynamics of the films in the presence of the defects was considered in the papers [11, 12]. In the works a complete set of equations is formulated describing both motion of the defects and hydrodynamic degrees of freedom. Then, to obtain macroscopic dynamic equations, an averaging over an intermediate scale was performed. At the procedure the "current density" related to the defects was substituted by an expression proportional to the average "electric field" and to gradients of the temperature and of the chemical potential. The resulting equations perfectly correspond to the problems solved in the works [11, 12]. Unfortunately, at the procedure an information concerning high-order correlations of the defect motion is lost. That is the motivation for our work where these high-order correlations are examined.

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Static properties of the system of the vortex-like defects in thin films can be described quite universally. The starting point of the description is the free energy $\mathcal{F}$ associated with the defects

$$\mathcal{F}/T = -\sum_{i\neq j}\beta\, n_i n_j \ln\left(\frac{|x_i - x_j|}{a}\right) - \sum_j \ln y\,, \tag{1}$$

where the subscripts i, j label defects, x_i are positions of the defects, $n_j = \pm 1$ are "charges" of the defects, a is a cutoff parameter of the order of the size of the defect core, the coupling constant β and the fugacity y are dimensionless T-dependent factors. The expression (1) is correct for quantum vortices in superfluid films, for disclinations in hexatic films, and for spin vortices in $2d$ planar magnets. For dislocations in crystalline films the expression (1) has to be slightly modified [8].

The presence of the pairs in the system leads to non-trivial "dielectric" properties of the medium. As a result the interaction between the charges is modified, the effect can be described in terms of a scale-dependent coupling constant β [2]. The dependence can be found in the framework of the scheme proposed by Kosterlitz [13]. Excluding the pairs with separations from a to r we come to renormalized values of the parameters β and y which obey the following renorm-group equations

$$\frac{d\beta}{d\ln(r/a)} = -cy^2\,, \qquad \frac{dy}{d\ln(r/a)} = (2-\beta)y\,, \tag{2}$$

where c is a numerical factor of order unity. In the low-temperature phase, the coupling constant β tends to a constant on large scales. The asymptotic value of β is larger than 2, the critical value $\beta_c = 2$ corresponds to the transition temperature. In the asymptotic region, where β can be treated as r-independent, the renormalized fugacity y remains r-dependent. Its asymptotic behavior can easily be extracted from Eq. (2): $y \propto r^{2-\beta}$. We see that in the low-temperature phase y tends to zero as scale increases. Thus the inequality $y \ll 1$ is satisfied for large scales in the low-temperature phase and probably in some region of scales above T_c.

We consider correlation functions

$$F_{2n}(t_1,\ldots,t_{2n};x_1,\ldots,x_{2n}) = \langle\rho(t_1,x_1)\ldots\rho(t_{2n},x_{2n})\rangle\,, \tag{3}$$

of the "charge density"

$$\rho(x) = \sum_j n_j\delta\left(x - x_j\right)\,. \tag{4}$$

For the superfluid films the "charge density" (4) is proportional to the vorticity $\mathrm{curl}\, v_s$. In statics, one gets the estimate [14]

$$F_2(r) \sim y^2(r)/r^4\,, \tag{5}$$

where $r = |x_1 - x_2|$. For high-order simultaneous correlation functions the normal estimate $F_{2n} \sim F_2^n$ is valid at the condition $y \ll 1$ [15].

Following Ref. [11] we accept the following stochastic equation

$$\frac{dx_{j,\alpha}}{dt} = -\frac{D}{T}\left[\frac{\partial\mathcal{F}}{\partial x_{\alpha j}} + n_j\gamma\epsilon_{\alpha\beta}\frac{\partial\mathcal{F}}{\partial x_{\beta j}}\right] + \xi_{j,\alpha}\,, \tag{6}$$

determining the trajectory of the j-th vortex in a superfluid film. Here $\mathcal{F}$ is the free energy (1), D is a diffusion coefficient, γ is a dimensionless parameter and ξ_j are Langevin forces with the correlation function

$$\langle \xi_{i,\alpha}(t_1)\xi_{j,\beta}(t_2)\rangle = 2D\delta_{ij}\delta_{\alpha\beta}\delta(t_1 - t_2) \,. \tag{7}$$

The equation (6) can be derived in the spirit of the procedure proposed by Hall and Vinen for the $3d$ superfluid, see Ref. [16]. Huber [17] argued that the same equation is correct for spin vortices in planar $2d$ magnetics. We believe that at $\gamma = 0$ the equation (6) is applicable to the dynamics of disclinations in hexatic films like membranes, freely suspended films and Langmuir films. We should also take into account annihilation and creation processes. They are characterized by the creation rate $\bar{R}(r)$ which is a probability density for a defect-antidefect pair with the separation r to be created per unit time per unit area and by the annihilation rate $R(r)$ which is a probability for a defect-antidefect pair to annihilate per unit time if the pair is separated by the distance r. Really, both $\bar{R}(r)$ and $R(r)$ are nonzero only if r is of the order of the core size a.

To examine the correlation functions (3) we use the Doi technique [18] enableing one to treat systems of classical particles where creation and annihilation processes occur. The Doi technique is formulated in terms of the creation $\hat{\psi}$ and annihilation ψ operators which satisfy the same commutation rules as ones for Bose-particles, we should introduce the fields $\psi_\pm$, $\hat{\psi}_\pm$ corresponding to defects and to antidefects. An effective Hamiltonian can be written via the fields. Then one can formulate a conventional diagrammatic expansion in terms of R, $\bar{R}$, β and propagators of defects. Extracting blocks with small separations between the defects, one can pass to renormalized quantities. Details of the procedure will be published elsewhere [19]. Here we formulate only results. The fugacity is expressed via the renormalized values of the creation and annihilation rates as

$$\bar{R}(x) = \frac{y^2}{r^4}R(x) \,. \tag{8}$$

The renormalized value of the annihilation rate is

$$R(x) = 8\pi\beta\delta(x) \,. \tag{9}$$

The renormalized values of β and y obey the same renorm-group equations (2) as in statics. For D and γ we get the following renorm-group equations

$$\frac{dD}{d\ln(r/a)} \sim -y^2 \,, \qquad \frac{d\gamma}{d\ln(r/a)} \sim y^2 \,, \tag{10}$$

analogous to the equation (2) for β. We conclude that corrections to D and γ are small (and irrelevant) due to the small value of the fugacity.

Next, we can examine correlation functions (3) rewriting the charge density (4) as

$$\rho = \hat{\psi}_+\psi_+ - \hat{\psi}_-\psi_- \,. \tag{11}$$

One should distinguish between contributions related to different numbers k of the defect-antidefect pairs passing trough the points x_m at given times t_m. They can be estimated as

$$F_{2n} \sim y^{2k}(r_*)r_*^{-4n} \,, \tag{12}$$

where we assume that all space separations are of the order r_* and all time intervals are of the order r_*^2/D. We see that the ratio of the expression (12) contains the $2k$-th power of a dimensionless small parameter y. Thus we conclude that the leading contribution to F_{2n} is related to a single defect-antidefect pair, that corresponds to $k = 1$. The situation is illustrated in Fig.1. Though the contribution associated with a number of defect-antidefect pairs contains an additional huge entropy factor it has also an additional small factor associated with small probability to observe defect-antidefect pairs with separations larger than the core radius, the smallness is accounted for the strong Coulomb attraction. In the large-scale limit when β is saturated we have $F_{2n} \propto r_*^{-4(n-1)-2\beta}$. If some space separations among $|r_i - r_j|$ and/or some time intervals differ strongly then one can formulate some simple rules. For example, if one of the time intervals τ is much larger than all values of $|r_i - r_j|^2/D$ then the correlation function behaves as $F_{2n} \propto \tau^{-\beta}$.

For the pair correlation function we have the same estimate (5) as in statics. The high-order correlation functions are much larger than their normal estimates via the pair correlation function. Namely, in accordance with Eqs. (5), (12) we have

$$F_{2n}/F_2^n \sim y^{-2n+2} \gg 1 \,. \tag{13}$$

Let us explain in terms of our scheme the origin of the estimate $F_{2n} \sim F_2^n$ for the simultaneous correlation functions. This estimate corresponds to $k = n$ in Eq. (12). The reason is that two defects cannot pass simultaneously through $2n$ points and at least $k = n$ defect-antidefect pairs should be taken to obtain a non-zero contribution to the simultaneous correlation function F_{2n}. The situation is illustrated in Fig.2. Thus we have two different regimes: for simultaneous and for non-simultaneous correlation functions. To establish the boundary between the regimes we should consider small time intervals where the single-pair contribution is finite but small. The smallness is associated with diffusive "smearing". The characteristic time where the simultaneous regime passes into the non-simultaneous one can be estimated from $Dt \sim r^2/|\ln[y(r)]|$, where r is the characteristic space separation.

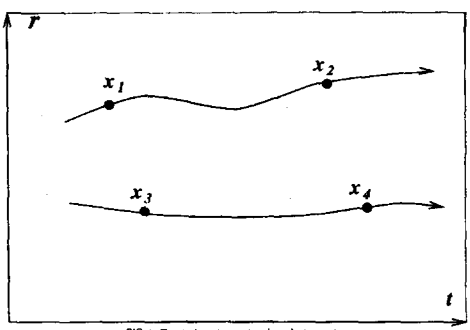

Fig.1. Two trajectories passing through given points

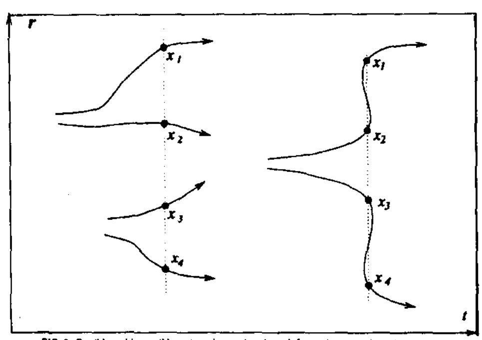

Fig.2. Possible and impossible trajectories passing through four points at a given time moment.

The considered effect resembles intermittency in turbulence (see, e.g., Ref. [20]) leading to large r-dependent factors in the ratios like (13) in the velocity correlation functions of a turbulent flow. However, for the defects the large r-dependent factors are related to the ultraviolet cutoff parameter whereas for intermittency in turbulence the large r-dependent factors are related to the infrared (pumping) scale. Our situation is thus closer to the inverse cascade (see Ref. [21]) realized on scales much larger than the pumping length. Our results can also be compared with non-trivial tails of probability distribution functions in the physics of disorder materials, see, e.g., Refs. [22, 23].

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