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THE $\rho^{\pm} - \rho^0$ MASS SPLITTING PROBLEM

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It is discussed the problem of the $\rho^{\pm} - \rho^0$ mass splitting. It is suggested to use the $\phi \to \rho\pi \to 3\pi$ decay to measure the $\rho^{\pm} - \rho^0$ mass splitting.

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In the framework of the $SU(3)$ theory with the U – spin invariance of electromagnetic interactions, taking into account the ideal $\omega - \phi$ mixing and ignoring $\rho^0 - \phi$ mixing for the Okubo – Zweig – Iizuki rule reasons, it was obtained [1] for the $\rho^0 - \omega$ mixing

$$-\mathrm{Re}\left(\Pi_{\rho^0\omega}\right) = \left(m^2_{K^{*\pm}} - m^2_{K^{*0}}\right) - \left(m^2_{\rho^{*\pm}} - m^2_{\rho^{*0}}\right) . \qquad (1)$$

The advent of quantum chromodynamics did not affect Eq. (1) for the U-spin invariance of isospin symmetry breaking interactions was not affected. But, now we perceived the importance of the $u - d$ quark mass splitting in the isospin symmetry breaking, see, for example, review [2]. Eq. (1) is correct to terms caused by both isospin symmetry breaking interactions and $SU(3)$ symmetry breaking interactions ("semi-strong interactions"). It means that corrections up to 25% to Eq. (1) are possible.

Particle Data Group [3] gives for the $K^{*\pm} - K^{*0}$ mass splitting

$$m_{K^{*0}} - m_{K^{*\pm}} = 6.7 \pm 1.2 \ \ \mathrm{MeV} \qquad (2)$$

and for the $\rho^{\pm} - \rho^0$ mass splitting

$$m_{\rho^0} - m_{\rho^{\pm}} = 0.1 \pm 0.9 \ \ \mathrm{MeV}. \qquad (3)$$

But the $\rho^{\pm} - \rho^0$ mass splitting can be calculated with Eq. (1) taking into account the well specified $\omega \to \pi^+\pi^-$ decay [3].

Really, as was first pointed by Glashow [4] the ω meson decays into $\pi^+\pi^-$ via the $\rho^0 - \omega$ mixing, see also, for example, [5 – 9],

$$B(\omega \to \pi^+\pi^-) = \frac{\Gamma\left(\rho^0 \to \pi^+\pi^- \, ; \, m_\omega\right)}{\Gamma_\omega} \left| \frac{\Pi_{\rho^0\omega}}{m^2_\omega - m^2_{\rho^0} - i \cdot m_\omega \left(\Gamma_\omega(m_\omega) - \Gamma^0_\rho(m_\omega)\right)} \right|^2 . \qquad (4)$$

As known [5 – 9] one can ignore $Im\left(\Pi_{\rho^0\omega}\right)$. Besides, the interference pattern of the ρ^0 and ω mesons in the $e^+e^- \to \pi^+\pi^-$ reaction and in the $\pi^+\pi^-$ photoproduction on nuclei

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shows [5-9] that $\mathrm{Re}\,(\Pi_{\rho^0\omega}) < 0$. So, taking into account $B(\omega \to \pi^+\pi^-) = 0.0221 \pm 0.003$ [3] one gets

$$-\mathrm{Re}\,(\Pi_{\rho^0\omega}) = -(3.91 \pm 0.27) \cdot 10^{-3}\mathrm{GeV}^2\,. \qquad (5)$$

From Eqs. (1), (2) and (5) follows

$$m_{\rho^0} - m_{\rho^\pm} = 5.26 \pm 1.41 \ \ \mathrm{MeV}\,. \qquad (6)$$

This result is a puzzle. First, this mass splitting is considerable and contrary to Eq. (3). Second, it is largely of electromagnetic origin also as the $\pi^\pm - \pi^0$ splitting but has the opposite sign. The ρ^0 meson is heavier than the $\rho^\pm$ one!

If to consider the Eq. (1) as the linear one[3] then $m_{\rho^0} - m_{\rho^\pm} = 4.1 \pm 1.2$ MeV and the situation does not change essentially. Certainly, it may be that corrections to Eq. (1) are important, but the current theoretical understanding of the vector meson mass splitting in the isotopical multiplets is far from being perfect, see, for example, [2, 10-12].

As for Eq. (3), it stems from [13] where the $\tau^- \to \nu_\tau\pi^-\pi^0$ data [13] are fitted in combination with the $e^+e^- \to \pi^+\pi^-$ ones [14], which have the same, excluding $\rho^0 - \omega$ mixing, production mechanism. But a combined fit of different experiments is open to a loss of sizable systematic errors. That is why the problem of an alternative experimental measurement of the $\rho^\pm - \rho^0$ mass splitting is ambitious enough. But this task is a considerable challenge for it is practically meaningless to compare different experiments with the different ρ production mechanisms for the large width of the ρ meson. The point is that our current knowledge of hadron production mechanisms is far from being perfect and generally in the resonance region we have a spectrum

$$\frac{dN}{dE} \sim \frac{f(E)}{(E - E_R)^2 + \Gamma^2/4}\,, \qquad (7)$$

where $f(E)$ is a poorly varying in resonance region unknown function[4] which can shift the visible peak up to a few MeV from E_R. Really, let take into account two first terms of expansion of $f(E)$ in the resonance region

$$f(E) = f_0 + (E - E_R)\,f_1 + \dots\,. \qquad (8)$$

Then the shift of the visible peak

$$\Delta E_R = \frac{\Gamma^2}{8} \cdot \frac{f_1}{f_0}\,. \qquad (9)$$

So, if $f_1 = \pm f_0/4.72\Gamma = \pm 1.4 f_0$ GeV^{-1}, $\Gamma = 151$ MeV, then

$$\Delta E_R = \pm 4\,\mathrm{MeV}\,. \qquad (10)$$

Certainly, one can use other than $e^+e^- \to \pi^+\pi^-$ and $\tau^- \to \nu_\tau\pi^-\pi^0$ different processes with the same $\rho^\pm$ and ρ^0 production mechanism, for example, $a_1^-(1260) \to \rho^-\pi^0 \to$

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[3] The linear relation occurs, for example, in the heavy vector meson chiral lagrangian [11].

[4] Efficiencies of registration can play a role of such functions for different processes with the same ρ production mechanism.

$\to \pi^-\pi^0\pi^0$ and 5) $a_1^-(1260) \to \rho^0\pi^- \to \pi^+\pi^-\pi^-$, the advantage of which is the absence of the $\rho^0-\omega$ mixing. But in this case the problem of different experimental systematic errors also exists.

It seems to us that the most adequate process for the aim under discussion is the $\phi \to \rho^+\pi^- + \rho^-\pi^+ + \rho^0\pi^0 \to \pi^+\pi^-\pi^0$ decay. Indeed, the charged and neutral ρ mesons are produced in the one reaction with the same mechanisms. Already now Spherical Neutral Detector (SND) and Cryogenic Magnetic Detector-2 at the e^+e^- collider VEPP-2M in Novosibirsk have collected $\sim 10^7$ ϕ mesons each that is $\sim 10^6$ $\phi \to \rho\pi \to 3\pi$ decays each. With the ϕ factory DAΦNE in Frascati, two orders of magnitude larger statistics will be collected.

The differential cross section of the $e^+e^- \to \pi^+(k_+)\pi^-(k_-)\pi^0(k)$ reaction can be written in the symmetrical form [15, 16]

$$\frac{d\sigma}{dm_+^2\,dm_-^2\,dm^2 d\cos\vartheta_N d\varphi} =$$

$$= \frac{\alpha^2|\mathbf{k}_+|^2|\mathbf{k}_-|^2\sin^2\vartheta_{+-}\sin^2\vartheta_N}{128\pi^2 s^2}|F|^2\delta(m_+^2+m_-^2+m^2-s-2m_{\pi^+}^2-m_{\pi^0}^2)\,, \qquad (11)$$

where $m_+^2 = (k_+ + k)^2$, $m_-^2 = (k_- + k)^2$, $m^2 = (k_+ + k_-)^2$, $s = (k_+ + k_- + k)^2$, ϑ_N is the angle between the normal to the production plane and the e^+e^- beam direction in the center mass system, ϑ_{+-} is the angle between the directions of the π^+ and π^- momenta in the center mass system.

The formfactor F of the $\gamma^* \to \rho\pi$ decay with taking into account the $\rho^0-\omega$ mixing has the form

$$F = A_\rho(s,m_+)\frac{2g_{\rho\pi\pi}(m_+)}{D_{\rho^+}(m_+)}\exp\{i\cdot\delta(m_+)\} + A_\rho(s,m_-)\frac{2g_{\rho\pi\pi}(m_-)}{D_{\rho^-}(m_-)}\exp\{i\cdot\delta(m_-)\}+$$

$$+A_\rho(s,m)\frac{2g_{\rho\pi\pi}(m)}{D_{\rho^0}(m)}\exp\{i\cdot\delta(m)\}\left(1+\frac{A_\omega(s)}{A_\rho(s,m)}\frac{\Pi_{\rho^0\omega}}{D_\omega(m)}\exp\{-i\cdot\delta(m)\}\right)\,, \qquad (12)$$

where $D_V(x)$ is a propagator of a V meson, in the simplest case $D_V(x) = m_V^2 - x^2 - i\cdot x\Gamma_V(x)$, $\Gamma_\rho(x) = \left(g_{\rho\pi\pi}^2(x)/6\pi\right)\left(q_\pi^3(x)/x^2\right)$, to a good accuracy one can consider that propagators of the $\rho^\pm$ and ρ^0 mesons differ by values of the masses $m_{\rho^\pm}^2$ and $m_{\rho^0}^2$ only, $\delta(x)$ is a phase due to the triangle singularity (the Landau anomalous thresholds) [17].

At the ϕ meson energy $|A_\omega(s)/A_\rho(s\,,\,m)| \simeq 0.02$, that is the $\rho^0-\omega$ mixing effects are negligible. As the energy ($\sqrt{s}$) increases the interference between terms in Eq. (12) decreases and is inessential at $\sqrt{s} = 1.5-2$ GeV, that is a circumstance favorable for the aim under consideration, but the statistics in this energy region is poor, besides, the $\rho^0-\omega$ effects in this energy region are expected to be considerable [16, 17].

By itself the $J/\psi \to \rho\pi \to 3\pi$ decay stands. Generally speaking, it is possible to select the adequate statistics in the future for $B(J/\psi \to \rho\pi) = (1.28 \pm 0.1)\cdot 10^{-2}$. The interference between the terms in Eq. (12) is practically absent here, but the $\rho^0-\omega$ mixing effects can essentially prevent the measurement of the $\rho^\pm-\rho^0$ mass splitting $B(J/\psi \to \rho^0\pi^0 = (4.2 \pm 0.5)\cdot 10^{-3})$ and $B(J/\psi \to \omega\pi^0 = (4.2 \pm 0.6)\cdot 10^{-4})$, especially

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5) A.M. Zaitsev, private communication.

for the relative phase of the amplitudes of the $J/\psi \to \rho^0\pi^0$ and $J/\psi \to \omega\pi^0$ decays is unknown. The taking into account of the effects of the heavy ρ' mesons in the $J/\psi \to 3\pi$ decay one can find in [18].

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