П И С Ь М А
В ЖУРНАЛ ЭКСПЕРИМЕНТАЛЬНОЙ И ТЕОРЕТИЧЕСКОЙ ФИЗИКИ

ОСНОВАН В 1965 ГОДУ *ТОМ 69, ВЫПУСК 3*
ВЫХОДИТ 24 РАЗА В ГОД *10 ФЕВРАЛЯ, 1999*

Pis'ma v ZhETF, vol.69, iss.3, pp.161 - 165

ABELIAN DYONS IN THE MAXIMAL ABELIAN PROJECTION OF SU(2) GLUODYNAMICS

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Submitted 21 December 1998

Correlations of the topological charge Q, the electric current J^e and the magnetic current J^m in $SU(2)$ lattice gauge theory in the Maximal Abelian projection are investigated. It occurs that the correlator $\ll QJ^eJ^m \gg$ is nonzero for a wide range of values of the bare charge. It is shown that: *(i)* the abelian monopoles in the Maximal Abelian projection are dyons which carry *fluctuating* electric charge; *(ii)* the sign of the electric charge $e(x)$ coincides with that of the product of the monopole charge $m(x)$ and the topological charge density $Q(x)$.

PACS: 11.15.Ha, 12.38.Aw

There are several approaches to the confinement problem in Quantum Chromodynamics (QCD) [1]. The most popular is the model of the dual superconducting vacuum [2]: the vacuum is supposed to be a media of condensed magnetic charges (monopoles). This model naturally explains the confinement phenomena. During the last decade the method of abelian projections [3] has been successfully used in lattice calculations in order to show that the vacuum of gluodynamics behaves as a dual superconductor (see e.g. the reviews [4]). Mostly, the numerical simulations were performed in the so called Maximal Abelian (MaA) projection [5].

An oldest and rather popular model of the QCD vacuum is the instanton-anti-instanton medium (see [6] and the references therein). It is not clear whether the confinement phenomenon can be explained within this approach [7]. However, the instanton-based models may have some relation to the dual superconductor model, since the instantons and monopoles are interrelated, as demonstrated analytically in [8] and numerically in [9, 10]. One can expect that the instantons may affect the properties of the abelian monopoles in the MaA projection. Indeed, it has been shown by numerical calculations [10] that the abelian monopole becomes the abelian dyon in the field of a single instanton. In this paper we study the properties of the electric charge of the abelian monopoles in the real vacuum of lattice $SU(2)$ gluodynamics.

In Section 1 we discuss the properties of abelian monopoles in a self-dual non-abelian field, and we introduce the lattice notation. The results of numerical simulations in the real and the cooled vacuum are described in Section 2.

1. Abelian Monopoles and (Anti-) Self-Dual Fields. The (anti-) self-dual configuration of the $SU(2)$ gauge field is defined as follows:

$$F_{\mu\nu}(A) = \pm\frac{1}{2}\varepsilon_{\mu\nu\alpha\beta}F_{\alpha\beta}(A) \equiv \pm{}^{*}F_{\mu\nu}\,, \tag{1}$$

where $F_{\mu\nu}(A) = \partial_{[\mu,}A_{\nu]} + i[A_\mu, A_\nu]$. In Ref. [10] the abelian monopole in the MaA projection of the lattice gluodynamics was studied in the self-dual field of the instanton. It was found that the current of the abelian monopole is accompanied by the electric current.

The correlation of the electric and the magnetic currents in the field of the instanton can be qualitatively explained as follows [10]. The MaA projection is defined [5] by the minimization of the functional $R[A^{\Omega}(x)]$ over the gauge transformations $\Omega(x)$, where $R[A] = \int d^4x[(A^1_\mu)^2 + (A^2_\mu)^2]$. Therefore, in this projection the off-diagonal gauge fields $A^{\pm}_\mu = A^1_\mu \pm iA^2_\mu$ are suppressed with respect to the diagonal gauge field A^3_μ. Thus, the commutator term $0.5\mathrm{Tr}(\sigma^3[A_\mu, A_\nu]) = \varepsilon^{3bc}A^b_\mu A^c_\nu$ in $F^3_{\mu\nu}$ is small compared to the abelian field-strength $f_{\mu\nu}(A) = \partial_{[\mu,}A^3_{\nu]}$. Therefore, in the MaA projection eq.(1) yields:

$$f_{\mu\nu}(A) \approx \pm{}^{*}f_{\mu\nu}(A)\,. \tag{2}$$

Due to eq.(2), the abelian monopole currents must be correlated with the electric currents:

$$J^e_\mu = \partial_\nu f_{\mu\nu}(A) \approx \pm\partial_\nu{}^{*}f_{\mu\nu}(A) = J^m_\mu\,. \tag{3}$$

Therefore, in the MaA projection the abelian monopoles are dyons on the background of (anti-) self-dual $SU(2)$ fields.

In the present publication we study the correlation of electric and magnetic currents in the vacuum of the $SU(2)$ lattice gluodynamics in the MaA projection. The definition of the abelian monopole current on the lattice is [11]:

$$J^m_\mu(y) = \frac{1}{4\pi}\sum_{\nu\lambda\rho}\varepsilon_{\mu\nu\lambda\rho}[\bar\theta_{\lambda\rho}(x+\hat\nu) - \bar\theta_{\lambda\rho}(x)]. \tag{4}$$

Here the function $\bar\theta_{\mu\nu}$ is the normalized plaquette angle $\theta_{\mu\nu}$: $\bar\theta_{\mu\nu} = \theta_{\mu\nu} - 2\pi k_{\mu\nu} \in (-\pi;\pi]$, $k_{\mu\nu} \in \mathbb{Z}$. The monopole current $J^m_\mu(x)$ is attached to the links of the dual lattice. One can easily show that the monopole currents are quantized, $J^m_\mu \in \mathbb{Z}$, and conserved, $\partial_\mu J^m_\mu = 0$.

The lattice electric current is defined as

$$K^e_\mu(x) = \frac{1}{2\pi}\sum_{\nu}[\bar\theta_{\mu\nu}(x) - \bar\theta_{\mu\nu}(x-\hat\nu)]. \tag{5}$$

In the continuum limit, this equation corresponds to the usual definition: $K^e_\mu = \partial_\nu f_{\mu\nu}$. The electric currents K^e_μ are defined on the original lattice. These are conserved, i.e., $\partial_\mu K^e_\mu = 0$, but, contrary to the magnetic currents, are not quantized, i.e., $K^e_\mu \in R$.

In order to calculate the correlators of the electric and the magnetic currents, one has to define the electric current on the dual lattice or the magnetic current on the original lattice. We use the following definition of the electric current J^e_μ on the *dual* lattice:

$$J^e_\mu(y) = \frac{1}{16} \sum_{x \in {}^*C(y,\mu)} \left[K^e_\mu(x) + K^e_\mu(x-\hat{\mu})\right] , \tag{6}$$

where the summation in the r.h.s. is over the eight vertices x of the 3-dimensional cube ${}^*C(y,\mu)$, to which the current $J^e_\mu(y)$ is dual. As in eq.(4), the point y lies on the dual lattice and the vertices x belong to the original lattice. The current J^e_μ defined by eq.(6) has the standard continuum limit: $J^e_\mu = \partial_\nu f_{\mu\nu}$.

We use the simplest definition for the topological charge density:

$$Q(x) = \frac{1}{2^9\pi^2} \sum_{\mu_1,\mu_2,\mu_3,\mu_4=-4}^{4} \varepsilon^{\mu_1,\mu_2,\mu_3,\mu_4} \mathrm{Tr}[U_{\mu_1\mu_2}(x) U_{\mu_3\mu_4}(x)] , \tag{7}$$

where $U_{\mu_1\mu_2}(x)$ is the plaquette matrix. On the dual lattice the topological charge density $Q(y)$ corresponding to the monopole current $J^m_\mu(y)$ is defined by averaging over the sites nearest to the current $J^m_\mu(y)$:

$$Q(y) = \frac{1}{8} \sum_x Q(x) ; \tag{8}$$

the summation here is the same as in eq.(6).

The simplest (connected) correlator of the electric and the magnetic currents is: $\ll J^m_\mu J^e_\mu \gg = < J^m_\mu J^e_\mu > - < J^m_\mu >< J^e_\mu > \equiv < J^m_\mu J^e_\mu >$, $< J^m_\mu > = < J^e_\mu > = 0$ is due to the Lorentz invariance. The correlator $< J^m_\mu J^e_\mu >$ is zero due to the opposite parities of the operators J^m and J^e.

The nonvanishing correlator is $\ll J^m_\mu(y) J^e_\mu(y) Q(y) \gg$ which is both Lorentz and parity invariant. Due to equalities $< J^m_\mu(y) > = < J^e_\mu(y) > = < Q(y) > = 0$ the connected correlator is:

$$G = \ll J^m_\mu(y) J^e_\mu(y) Q(y) \gg = < J^m_\mu(y) J^e_\mu(y) Q(y) > . \tag{9}$$

The density of electric and magnetic charges strongly depends on β. To compensate this dependence we consider the normalized correlator $\bar{G}$:

$$\bar{G} = \frac{1}{\rho^e \rho^m} < J^m_\mu(y) J^e_\mu(y) q(y) > , \tag{10}$$

where

$$\rho_{m,e} = \frac{1}{4V} \sum_l < |J^{m,e}_l| > , \qquad q(x) = \frac{Q(y)}{|Q(y)|} \equiv \mathrm{sign}\, Q(x) ,$$

V being the lattice volume (total number of sites).

2. Magnetic and Electric Currents: Numerical Study. We perform the numerical simulations on the 8^4 lattice with periodic boundary conditions. We thermalize lattice fields using the standard heat bath algorithm. All correlators for each value of β were calculated on 100 statistically independent configurations. To fix the MaA projection we use the overrelaxation algorithm of Ref. [12].

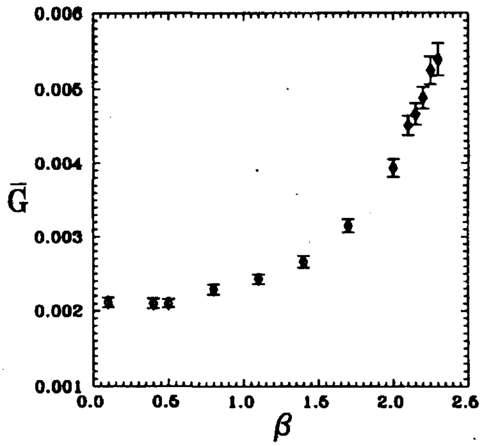

Fig.1. The correlator $\bar{G}$, eq.(10), vs. β

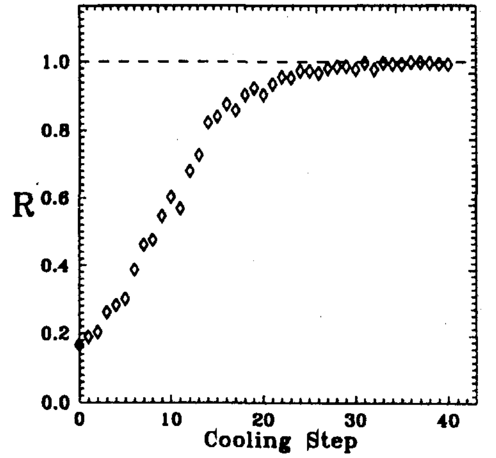

Fig.2. The ratio R, eq.(11), vs. the number of cooling steps at $\beta = 2.2$

The correlator $\bar{G}$ given by eq.(10) vs. β is shown in Fig.1. Since the product of electric and magnetic currents is correlated with the topological charge, we see that the abelian monopole carries the electric charge which depends on the topological charge density at the abelian monopole position.

We have found that the correlator $\bar{G}$ grows during the cooling of the field configurations. This means that the strongest correlation of the electric and the magnetic charges is observed in (anti-) self-dual fields (e.g., for the instanton configuration).

In order clarify how our results are related to those of Ref. [10], we study the correlator

$$R = \frac{< J^m_\mu(y) J^e_\mu(y) q(y) >}{< |J^m_\mu(y) J^e_\mu(y) q(y)| >} \tag{11}$$

in the cooled vacuum. The correlator R *vs.* the number of the cooling steps n is shown in Fig.2 at $\beta = 2.2$. The plateau $R = 1$ at $n > 25$ corresponds to the classical instanton configuration studied in Ref. [10]. In the real (not cooled) vacuum, the field configurations are not self–dual, and we have $R < 1$ at $n = 0$.

Our results show that the abelian monopoles in the MaA projection of $SU(2)$ gluodynamics carry a fluctuating electric charge. The sign of the electric charge is equal to that of the product of the topological charge density and the magnetic charge. The large electric charge is in the (anti-) self–dual vacuum, while in the real (not cooled) vacuum the induced charge is smaller.

M.I.P. acknowledges the kind hospitality of the Theoretical Department of the Kanazawa University, where a part of this work has been done. F.V.G. is grateful for the kind hospitality of the Theoretical Physics Department of the Free University of Amsterdam. This work was supported by the grants INTAS-RFBR-95-0681, RFBR-96-02-17230a and RFBR-96-15-96740.

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