Pis'ma v ZhETF, vol.69, iss.3, pp.171 - 175 © 1999 February 10

THE POLARIZATION OF RADIATION IN SINGLE CRYSTALS IN THE SEMICLASSICAL APPROACH

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Submitted 21 December 1998

The radiation emission spectra of polarized photons emitted from charged particles in single oriented crystals are obtained in Bayer – Katkov – Strakhovenko semiclassical approach. The results of numerical calculations are presented in the region of small angles of incidence for which the coherent theory fails but magnetic bremsstrahlung region isn't yet achieved.

PACS: 41.60.-m, 78.70.-g

The processe of photon emission from charged particles at high energies in oriented single crystals is widely applied in experimental physics for the production of polarized photon beams. For sufficiently large incident angles ϑ_0 to crystal axis/planes $\vartheta_0 \gg \vartheta_v$, where ϑ_v is the characteristic angle given by $\vartheta_v = U_0/m$ (U_0 is the scale of axial/planar potential and m the electron mass) this process is well described in the theory of coherent bremsstrahlung (CB)[1]. This theory is constructed in the framework of the first Born approximation in crystalline potential and fails at small angles and very high energies.

Recently V.N. Bayer with co-authors developed a general theory for radiation emission from high-energy particles and pair production by high-energy photons in strong crystalline fields [2]. This theory is not restricted by the first Born approximation and is based on the semiclassical character of motion of ultrarelativistic particles in strong fields. For $\vartheta_0 \gg \vartheta_v$, this theory is reduced to the standard theory for CB. At very small angles $\vartheta_0 \ll \vartheta_v$, the semiclassical theory reproduces the results of constant field approximation (CFA). For intermediate angles, $\vartheta_0 \sim \vartheta_v$, the numerical calculations in the exact theory face serious mathematical difficulties and to obtain specific numerical results some approximation and modeling methods have been developed. First numerical results of the radiation emission spectra in the semiclassical theory were published in [3] (in the framework of additional modeling assumption); exact calculations were presented in [4]. In Ref. [5] an analytical method of calculations in the framework of semiclassical approach was developed and new numerical results were presented. These papers demonstrate an essential difference of exact spectra from the corresponding results of coherent theory at small angles $\vartheta_0 \leq \vartheta_v$. There is good agreement between first experimental results at small angles [6, 7] and these calculations.

It is very important to investigate the polarization phenomena at very high energies in oriented crystals, where the CB theory doesn't work. If we could find optimal crystal orientations, which bring to the large intensity enhancements of essentially polarized photons at large frequencies, it will give us an interesting possibility of producing high energy polarized photon beams.

In this paper the formulae for radiation emission spectra of polarized photons are derived in Bayer – Katkov – Strakhovenko semiclassical approach. We also present preliminary results of numerical calculations in the most interesting region of intermediate

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angles $\vartheta_0 \leq \vartheta_v$ for planar orientation of crystal, where maximal polarization effects are expected. To carry out the arising integrals over time of complicated oscillating functions we have elaborated the calculation algorithm and the special integration program, which is very effective especially at small angles.

We start from the general semiclassical formula [2] which gives the spectral distribution of polarized photon energy, averaged over the initial and summed over the final electron polarizations:

$$dE = \alpha \frac{d^3k}{(2\pi)^2} \int_{-\infty}^{\infty} dt_1 \int_{-\infty}^{\infty} dt_2 \frac{1}{4\varepsilon'^2} [(\varepsilon + \varepsilon')^2 (\mathbf{ev}_2)(\mathbf{e}^* \mathbf{v}_1) - \omega^2 (\mathbf{ev}_1)(\mathbf{e}^* \mathbf{v}_2) + \tag{1}$$

$$+\omega^2 (\mathbf{v}_1 \mathbf{v}_2 - 1 + \frac{1}{\gamma^2})(\mathbf{ee}^*)] e^{ik'(x_1 - x_2)}$$

where $\alpha = 1/137$ is fine structure constant, $k^\mu = (\omega, \mathbf{k})$ and e-stands, correspondingly, for photon 4-momenta and polarization vector; $\varepsilon, \varepsilon' = \varepsilon - \omega$ - stands for initial and final electron energies, $k'^\mu = k^\mu \varepsilon / \varepsilon'$, $\gamma = \varepsilon / m$; $\mathbf{r}_{1,2} = \mathbf{r}(t_{1,2})$ and $\mathbf{v}_{1,2} = \mathbf{v}(t_{1,2})$ are electron coordinates and velocities at the moment of time $t_{1,2}$, $x_{1,2} = (t_{1,2}, \mathbf{r}_{1,2})$.

We are going to find the radiation emission spectra of polarized photons, integrated over the emitted photon angles ϑ, φ. Therefore, the photon polarization vectors should be defined via the direction which remains fixed (keeping in mind smallness of $\vartheta, \vartheta \sim 1/\gamma$) after integration over ϑ, φ . We chose two independent polarization vectors in the following form:

$$\mathbf{e}_1 = \frac{[\mathbf{n}_2 \mathbf{n}]}{|[\mathbf{n}_2 \mathbf{n}]|}, \quad \mathbf{e}_2 = [\mathbf{n} \mathbf{e}_1], \tag{2}$$

where $\mathbf{n} = \mathbf{k}/\omega$, $\mathbf{n}_2$ is a unit vector transverse to the incident electron velocity $\mathbf{v}_0$, the exact directions of $\mathbf{n}_2$ can be fixed by experimental layout. We choose $\mathbf{n}_2$ transverse to the crystal axis, along which the incident electron is aligned. At $\vartheta \ll 1$ polarization vectors $\mathbf{e}_1$ and $\mathbf{e}_2$ have the same fixed directions for any φ , and the Stocks parameters defined in this vectors, correspond to the definite directions after integration over emitted photon angles.

Utilizing the method derived in ref.[2], we integrate (1) over photon angles ϑ, φ and go over to the intensity $I = dE/dt$. If longitudinal length of crystal is much higher than radiation formation length, after averaging over electron trajectories in crystal, we can neglect the intensity dependence on time [2]. Finally, we present the polarized photon emission spectra in terms of Stocks parameters ξ_1, ξ_2, ξ_3:

$$\frac{dI}{d\omega} = \frac{\alpha}{\pi} \frac{m^2 \omega}{\varepsilon^2} \int \frac{d^3 r_0}{V} F(\mathbf{r}_0, \vartheta_0) \left[\int_0^{\infty} \frac{d\tau}{\tau} [(T_0 + \right.$$

$$\left. + \gamma^2 \xi_1 T_1 + \gamma^2 \xi_3 T_3) \sin A_1 + \gamma^2 \xi_2 T_2 \cos A_1] - \frac{\pi}{2} \right]. \tag{3}$$

Where $\mathbf{r}_0$ is the entry point, V is crystal volume, integration over $\mathbf{r}_0$ gives the averaging over electron trajectories in crystal, $F(\mathbf{r}_0, \vartheta_0)$ is the coordinate distribution function at given value of ϑ_0. The quantities of $A_1 = A_1(\tau)$ and $T_i = T_i(\tau), i = 0, 1, 2, 3$, in (3) are defined as follows:

$$A_1 = \frac{m^2 \omega}{2\varepsilon\varepsilon'} \tau \left[1 + \gamma^2 \left[\frac{1}{\tau} \int_{-\tau/2}^{\tau/2} \mathbf{v}_\perp^2(t)\, dt - \mathbf{a}^2{}_\perp(\tau) \right] \right], \tag{4}$$

$$T_0 = 1 + \frac{1}{4}\varphi(\varepsilon)\gamma^2(\mathbf{v}_{1\perp} - \mathbf{v}_{2\perp})^2 \ , \tag{5}$$

$$T_1 = -2a_x a_y + a_y(v_{1x} + v_{2x}) + a_x(v_{1y} + v_{2y}) - (v_{1x}v_{2y} + v_{1y}v_{2x}), \tag{6}$$

$$T_2 = \frac{\varphi(\varepsilon)}{2}(\mathbf{a}[\mathbf{v}_1\mathbf{v}_2]), \tag{7}$$

$$T_3 = a_y^2 - a_x^2 + a_x(v_{1x} + v_{2x}) - a_y(v_{1y} + v_{2y}) - (v_{1x}v_{2x} - v_{1y}v_{2y}), \tag{8}$$

where

$$\mathbf{a}(\tau) = \frac{1}{\tau}\int_{-\tau/2}^{\tau/2}\mathbf{v}(t)dt, \quad \varphi(\varepsilon) = \frac{\varepsilon'}{\varepsilon} + \frac{\varepsilon}{\varepsilon'}, \quad \mathbf{v}_\perp = (v_x, v_y), \quad \mathbf{v}_1 = \mathbf{v}(-\tau/2), \quad \mathbf{v}_2 = \mathbf{v}(\tau/2).$$

The Stocks parameters and projections of vectors correspond to the frame defined by following unit vectors:

$$\hat{\mathbf{e}}_x = [\mathbf{n}_2\mathbf{v}_0] \ , \ \hat{\mathbf{e}}_y = \mathbf{n}_2 \ , \ \hat{\mathbf{e}}_z = \mathbf{v}_0 \ . \tag{9}$$

The formulae for polarized photon emission spectra in semiclassical approach were derived earlier in ref.[2] and an example of numerical calculations was presented in [3]. Our results for unpolarized photon (T_0) and circular polarized photon (T_2) coincide with [2]. For linear polarization (T_1 and T_3) our formulae are completely different and the results of our numerical calculations differ from [3] even qualitatively. The difference is originated in the non proper choice of the photon polarization vectors in [2] [2)].

For further calculations we shall use the rectilinear trajectory approximation which is valid, strongly speaking, for $\vartheta_0 \gg \vartheta_c$, where ϑ_c is the Lindhard channeling angle [10]. We present the numerical calculations at $\varepsilon = 150$ GeV for $< 001 >$ aligned single diamond crystal at temperature $T = 293\,\mathrm{K}$, ϑ_0 is the angle of the vector $\mathbf{v}_0$ to this axis, and φ_0 is the angle of the $\mathbf{v}_0$ projection onto the plane $\{001\}$ to the plane $\{1\bar{1}0\}$. The calculations are carried out for uniform distribution. We make use of Moliere potential of isolated atom. On Fig.1 the graduate changing of intensity and polarization spectra are presented for decreasing values of φ_0 at fixed ϑ_0 (transfer to purely planar orientation). The behavior of the unpolarized intensity coincides, in general, with results of Ref.[4]. The shape of the spectra is determined by the competition of coherent effects, which are determined by high time region $\tau \geq 1/q_z$ in the integrals over time in (3), and magnetic bremsstrahlung effects, which come from low time region $\tau \ll 1/q_z$. As compared with the CB results, the intensity of coherent peaks decreases, the peaks change their form and move down to low frequencies ω. One can also see the arising of new peaks and deeps [4]. At the same time, when φ decreases, magnetic bremsstrahlung effects begin to contribute and bring to overall increasing of intensity. They are stronger displayed at higher electron energies and lower radiated frequencies. In the wide range of angles the radiation has an essential linear polarization. The circular polarization doesn't vanish but practically is negligible ($\xi_2 \leq 0.01$). As in the CB theory, the spectral distribution of the degree of polarization repeats, in general, the structure of the intensity distribution, i.e. peaks of intensity and peaks of polarization are situated at the same frequencies.

[2)] When the E-print version of this paper was published [8], V.M. Strakhovenko informed us that in the updated English edition of their book [9] the polarized photon emission spectra have been corrected and now coincide with our corresponding Eqs.(6) – (8).

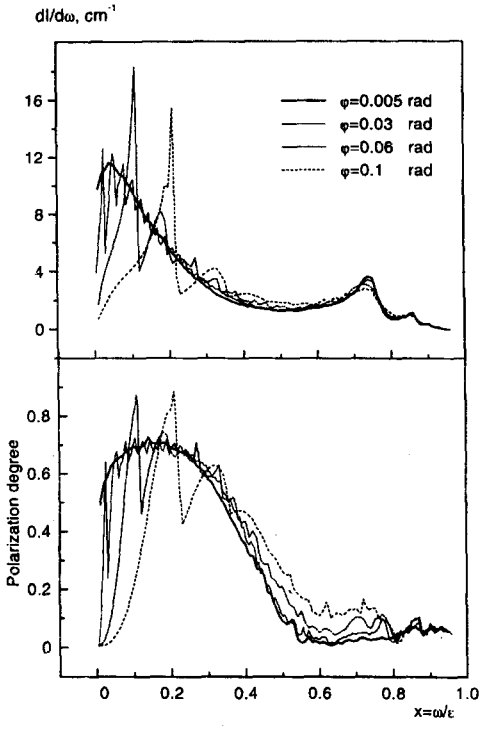

Fig.1. Changes of intensity and polarization spectra at decreasing of φ_0 at fixed ϑ_0

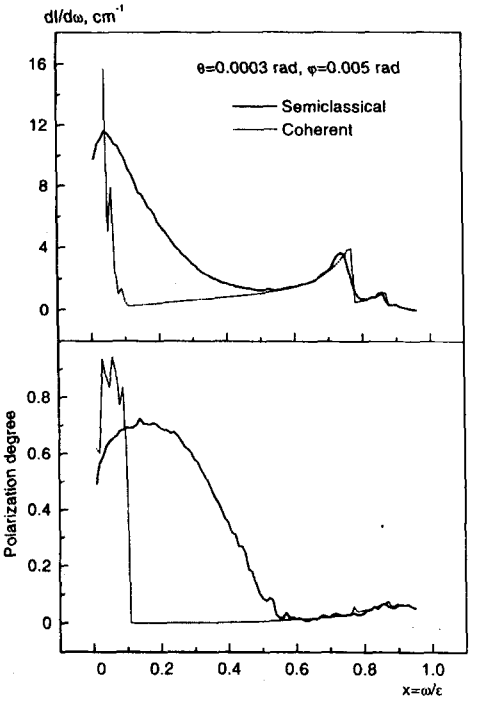

Fig.2. Comparison the spectral distributions of intensity and degree of polarization in coherent [1] and semiclassical [2] approaches at small incident angles

It is worth noting that in the most interesting region of large $x \geq 0.5 - 0.6$ the photon emission spectra and polarization could be satisfactory described in (non modified) coherent theory up to rather small angles. On Fig.2 we present the comparison of semiclassical and coherent approaches.

When $\varphi_0 \to 0$ at fixed ϑ_0, the reciprocal vectors q_z which contribute to sums in Eq.(3), form different groups, so that each vector q_z from the given group tends to the same limit, $q_z \to q_z^{(i)}$, $(i = 1, 2, 3, ...)$. The first group of vectors which tend to zero, $q_z^{(i)} = 0$, determine the magnetic bremsstrahlung contribution in the range of small ω. This contribution brought to the large maximum of the degree of linear polarization (0.6-0.8) at $x = \omega/\varepsilon = 0.1 \div 0.3$. The radiation is polarized in $\{110\}$ plane ($\xi_1 \sim 0$). The groups of the vectors, which tend to nonvanishing limits, ($q_z^{(i)} \neq 0$) correspond to a number of high frequency peaks. For sufficiently large frequencies ($x \geq 0.6$), the shape and position of these peaks can be approximately described in coherent theory (Fig.2). The magnitude of the peaks is determined, approximately, as a sum of coherent contributions of reciprocal vectors with given $q_z^{(i)}$. For polarized spectra, these contribution could be of different sign, or some contribution could vanish. Therefore, the polarization in the region of high ω peaks strongly depends on crystal orientation and is relatively small. For instance, for $< 001 >$ aligned crystal, in the region of main high frequency peak, the individual contributions practically compensate each other and degree of polarization is very small. For $< 110 >$ orientation, in the range of similar peak the main contributions

are not compensated, and degree of polarization appear to be essentialy larger (Fig.3). High-energy photon pronounced maxima in the radiation emission have been observed experimentally at CERN [6] in diamond and Si crystals. A discussion is going on as to the possibility of utilizing this effect for producing high-energy polarized photon beams.

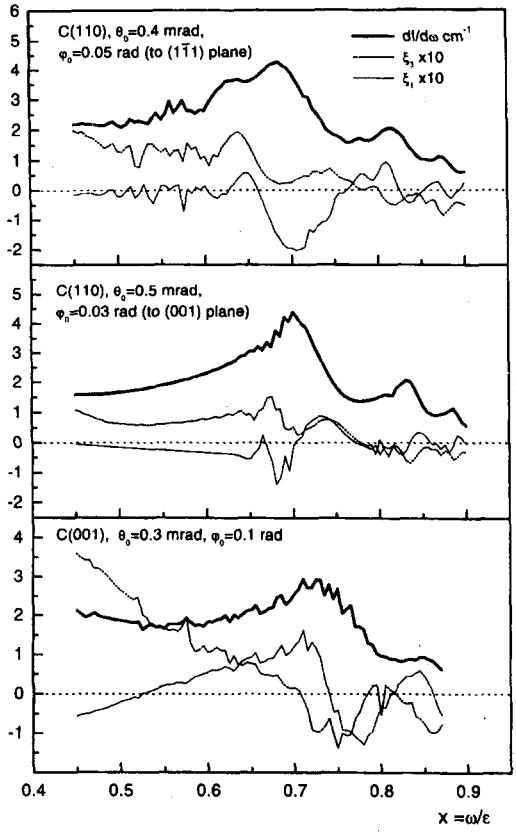

Fig.3. The spectral distribution of intensity and Stocks parameters in the range of high-energy photon peaks for $< 001 >$ and $< 110 >$ oriented diamond single crystal at $\varepsilon = 150$ Gev

More precise calculations for different crystal orientations in the wide range of incident electron angles and energies we are going to present in subsequent papers.

This work was supported by the ISTC grant, Project A-099. One of us (N.T) is grateful to Prof. R.Avakian for invitation to participate in this Project.

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