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# DIFFRACTIVE $S$ AND $D$ WAVE VECTOR MESONS IN DEEP INELASTIC SCATTERING

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We derive helicity amplitudes for diffractive leptoproduction of the $S$ and $D$ wave states of vector mesons. We predict a dramatically different spin dependence for production of the $S$ and $D$ wave vector mesons. We find very small $R = \sigma_L/\sigma_T$ and abnormally large higher twist effects in production of longitudinally polarized $D$ wave vector mesons.

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Diffractive vector meson production $\gamma^* + p \to V + p'$, in deep inelastic scattering (DIS) at small $x = (Q^2 + m_V^2)/(W^2 + Q^2)$ is a testing ground of ideas on the QCD pomeron exchange and light-cone wave function (LCWF) of vector mesons ([1−5], for the recent review see [6]). (For the kinematics see figure, $Q^2 = -q^2$ and $W^2 = (p+q)^2$ are standard DIS variables). The ground state vector mesons, $V = \rho^0, \omega^0, \varphi^0, J/\Psi, \Upsilon$ are usually supposed to be the $S$ wave spin-triplet $q\bar{q}$ states. However, all the previous theoretical calculations used the $V\bar{q}q$ vertex $\phi_V V_\mu \bar{q}\Gamma_\mu q$ with the simplest choice $\Gamma_\mu = \gamma_\mu$, which corresponds to a certain mixture of the $S$ and $D$ wave states, and any discussion of the impact of the $D$ wave admixture in the literature is missing (here $V_\mu$ is the vector meson polarization vector and $\phi_V$ is the vertex function related to the vector meson LCWF as specified below).

We report here a derivation of helicity amplitudes for diffractive production of pure $S$ and $D$ wave $q\bar{q}$ systems for small to moderate momentum transfer $\mathbf{\Delta}$ within the diffraction cone. Understanding production of $D$ wave states is a topical issue for several reasons. First, the $D$ wave admixture may affect predictions for the ratio $R = \sigma_L/\sigma_T$, in which there is a persistent departures of theory from the experiment. To this end we recall that the nonperturbative long-range pion-exchange between light quarks and antiquarks [7] is a natural source of $S$-$D$ mixing in the ground state $\rho^0$ and $\omega^0$ mesons. Second, different spin properties of $S$ and $D$ wave production may facilitate as yet unresolved $D$ wave vs. $2S$ wave assignment of the $\rho'(1480)$ and $\rho'(1700)$ and of the $\omega'(1420)$ and $\omega'(1600)$ mesons.

In our analysis we rely heavily upon the derivation [8] of amplitudes of the $s$-channel helicity conserving (SCHC) and non-conserving (SCHNC) transitions, albeit in slightly different notations. We predict a dramatically different spin dependence for production of the $S$ and $D$ wave states, especially the $Q^2$ dependence of $R = \sigma_L/\sigma_T$ which derives from the anomalously large higher twist effects in the SCHC amplitude for production of longitudinally polarized vector mesons. Our technique can be readily generalized to higher excited states, $3^-$ etc., leptoproduction of which is interesting for the fact that they cannot be formed in $e^+e^-$ annihilation.

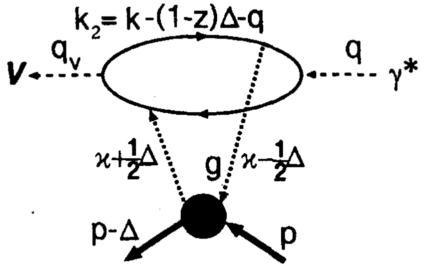


Fig.1: One of the four Feynman diagrams for the vector meson production $\gamma^* p \to Vp'$ via QCD two-gluon pomeron exchange

A typical leading log(1/x) (LL(1/x)) pQCD diagram for vector meson production is shown in figure. We use the standard Sudakov expansion of all the momenta in the two lightcone vectors

$$p' = p - q\frac{p^2}{s}, \quad q' = q + p'\frac{Q^2}{s}$$

such that $q'^2 = p'^2 = 0$ and $s = 2p' \cdot q'$, and the two-dimensional transverse component: $k = zq' + yp' + k_\perp, \kappa = \alpha q' + \beta p' + \kappa_\perp, \quad \Delta = \gamma p' + \delta q' + \Delta_\perp$ (with the exception of $\mathbf{r}$ which is a 3-dimensional vector, see below, hereafter $\mathbf{k}, \mathbf{\Delta}, ...$ always stand for 2-dimensional $k_\perp, \Delta_\perp$ etc.). The diffractive helicity amplitudes take the form

$$A^{S,D}_{\lambda_V \lambda_\gamma}(x, Q^2, \mathbf{\Delta}) = is\frac{C_F N_c c_V \sqrt{4\pi\alpha_{em}}}{2\pi^2} \int_0^1 \frac{dz}{z(1-z)} \int d^2\mathbf{k} \psi_{S,D}(z, \mathbf{k}) \times$$
$$\times \int \frac{d^2\kappa}{\kappa^4} \alpha_S(\max\left\{\kappa^2, \mathbf{k}^2 + \overline{Q}^2\right\}) I^{S,D}_{\lambda_V \lambda_\gamma}(\gamma^* \to V) \left(1 + i\frac{\pi}{2}\frac{\partial}{\partial \log x}\right) \mathcal{F}(x, \kappa, \mathbf{\Delta}), \qquad (1)$$

where $\lambda_V, \lambda_\gamma$ stand for helicities, $m$ is the quark mass, $C_F = (N_c^2 - 1)/2N_c$ is the Casimir operator, $N_c = 3$ is the number of colors, $c_V = 1/\sqrt{2}, 1/3\sqrt{2}, 1/3, 2/3$ for the $\rho^0, \omega^0, \phi^0, J/\Psi$ mesons, $\alpha_{em}$ is the fine structure constant, $\alpha_S$ is the strong coupling and $\overline{Q}^2 = m^2 + z(1-z)Q^2$ is the relevant hard scale. To the LL(1/x) the lower blob is related to the unintegrated gluon density matrix $\mathcal{F}(x, \kappa, \mathbf{\Delta})$ [5, 9, 10]. For small $\mathbf{\Delta}$ within the diffraction cone

$$\mathcal{F}(x, \kappa, \mathbf{\Delta}) = \frac{\partial G(x, \kappa^2)}{\partial \log \kappa^2} \exp(-\frac{1}{2}B_{3\mathbb{P}}\mathbf{\Delta}^2), \qquad (2)$$

where $\partial G/\partial \log \kappa^2$ is the conventional unintegrated gluon structure function and, modulo to a slow Regge growth, the diffraction cone $B_{3\mathbb{P}} \sim 6\,\mathrm{GeV}^{-2}$ [5].

In the light-cone formalism [11], one first computes the production of an on-mass shell $q\bar{q}$ pair of invariant mass $M$ and total momentum $q_M$. This amplitude is projected onto the state $(q\bar{q})_J$ of total angular momentum $J = 1$ using the running longitudinal and the usual transverse polarization vectors

$$V_L = \frac{1}{M}\left(q' + \frac{\mathbf{\Delta}^2 - M^2}{s}p' + \Delta_\perp\right), \qquad V_T = V_\perp + \frac{2(\mathbf{V}_\perp \cdot \mathbf{\Delta})}{s}(p' - q'), \qquad (3)$$

such that $(V_T V_L) = (V_T q_M) = (V_L q_M) = 0$. Then the resulting upper blob $I(\gamma^* \to V)$ is contracted with the radial LCWF of the $q\bar{q}$ Fock state of the vector meson,

$$\psi_{S,D}(z,\mathbf{k}) = \psi_{S,D}(\mathbf{r}^2) = \frac{\phi_{S,D}(\mathbf{r}^2)}{M^2 - m_V^2}\,. \tag{4}$$

Here $r = 1/2(k_2 - k_1)$, which in the rest frame is the relative 3-momentum in the $q\bar{q}$ pair, $r = (0,\mathbf{r}) = (0,\mathbf{k},k_z)$, $r^2 = -\mathbf{r}^2$, and

$$M^2 = 4(m^2 + \mathbf{r}^2) = \frac{m^2 + \mathbf{k}^2}{z(1-z)}\,.$$

To conform to this procedure, all the occurrences of the vector meson mass $m_V$ in $I_{\lambda_V \lambda_\gamma}$ of ref.[8] must be replaced by $M$.

A useful normalization of the radial LCWF's $\psi_{S,D}(\mathbf{r}^2)$ is provided by the $V \to e^+e^-$ decay constant, $\langle 0|J_\mu^{em}|V\rangle = f c_V \sqrt{4\pi\alpha_{em}} V_\mu$:

$$f_S = \frac{N_c}{(2\pi)^3}\int d^3\mathbf{r}\,\frac{8}{3}(M+m)\psi_S(\mathbf{r}^2)\,,\quad f_D = \frac{N_c}{(2\pi)^3}\int d^3\mathbf{r}\,\frac{32}{3}\frac{\mathbf{r}^4}{M+2m}\psi_D(\mathbf{r}^2)\,. \tag{5}$$

The nice observation is that we need not go again through all the calculations of helicity amplitudes. Indeed, the spinor vertices $\Gamma_\mu^{S,D}$ for the pure $S$ and $D$ wave states can be readily obtained from the simplest $\Gamma_\mu = \gamma_\mu$ used in [8]. Following [11], it can be easily shown that

$$\Gamma_\mu^S = \gamma_\mu - \frac{2r_\mu}{M+2m} = \mathcal{S}_{\mu\nu}\gamma_\nu;\quad \mathcal{S}_{\mu\nu} = g_{\mu\nu} - \frac{2r_\mu r_\nu}{m(M+2m)}. \tag{6}$$

Here we made use of $r^\mu\gamma_\mu = m$ and $(q_M \cdot r) = 0$. Once the $S$ wave is constructed, the spinor structure for a $D$ wave state can be readily obtained by contracting the $S$ wave vertex with $3r_\mu r_\nu + g_{\mu\nu}\mathbf{r}^2$ with the result

$$\Gamma_\mu^D = \mathbf{r}^2\gamma_\mu + (M+m)r_\mu = \mathcal{D}_{\mu\nu}\gamma_\nu;\quad \mathcal{D}_{\mu\nu} = \mathbf{r}^2 g_{\mu\nu} + \frac{M+m}{m}r_\mu r_\nu. \tag{7}$$

Consequently, the answers for either $S$ or $D$ wave production amplitudes can be immediately obtained from the expressions given in [8] by substitutions $V_\mu^* \to V_\nu^*\mathcal{S}_{\nu\mu}$, $V_\mu^* \to V_\nu^*\mathcal{D}_{\nu\mu}$ for $S$ and $D$ wave states respectively.

In terms of diffractive amplitudes $\mathbf{\Phi}_1$ and $\Phi_2$ defined in [8], we find for $S$ wave vector mesons (here $T$ stands for the transverse polarization)

$$I_{0L}^S = -4QMz^2(1-z)^2\Phi_2\left[1 + \frac{(1-2z)^2 m}{2z(1-z)(M+2m)}\right],$$

$$I_{TT}^S = \Bigg\{(\mathbf{V}^*\mathbf{e})[m^2\Phi_2 + (\mathbf{k}\mathbf{\Phi}_1)] + (1-2z)^2(\mathbf{V}^*\mathbf{k})(\mathbf{e}\mathbf{\Phi}_1)\frac{M}{M+2m} - $$
$$-(\mathbf{e}\mathbf{k})(\mathbf{V}^*\mathbf{\Phi}_1) + \frac{2m}{M+2m}(\mathbf{V}^*\mathbf{k})(\mathbf{e}\mathbf{k})\Phi_2\Bigg\},$$

$$I_{0T}^S = -2z(1-z)(2z-1)M(\mathbf{e}\mathbf{\Phi}_1)\left[1 + \frac{(1-2z)^2 m}{2z(1-z)(M+2m)}\right] + \frac{Mm}{M+2m}(2z-1)(\mathbf{e}\mathbf{k})\Phi_2\,,$$

$$I_{TL}^S = 2Qz(1-z)(2z-1)(\mathbf{V}^*\mathbf{k})\Phi_2 \frac{M}{M+2m} \,. \tag{8}$$

Because the difference between $\Gamma_\mu^S$ and $\gamma_\mu$ is a relativistic correction, the results for the $S$ wave vector mesons differ from those found in [8] only by a small relativistic corrections $\propto \mathbf{r}^2/M^2$. The exceptional case is suppression of $I_{TL}^S$ by factor $M/(2m+M) \sim 0.5$.

We skip the twist expansion for $S$ wave amplitides, which can easily be done following [8], and proceed to the much more interesting case of $D$ wave mesons, for which

$$I_{0L}^D = -QMz(1-z)\left(\mathbf{k}^2 - \frac{4m}{M}k_z^2\right)\Phi_2 \,,$$

$$I_{TT}^D = \Bigg\{ (\mathbf{V}^*\mathbf{e})\mathbf{r}^2[m^2\Phi_2 + (\mathbf{k}\boldsymbol{\Phi}_1)] + (1-2z)^2(\mathbf{r}^2+m^2+Mm)(\mathbf{V}^*\mathbf{k})(\mathbf{e}\boldsymbol{\Phi}_1) -$$

$$-\mathbf{r}^2(\mathbf{ek})(\mathbf{V}^*\boldsymbol{\Phi}_1) - m(M+m)(\mathbf{V}^*\mathbf{k})(\mathbf{ek})\Phi_2 \Bigg\} \,,$$

$$I_{0T}^D = -\frac{2z-1}{2}M\left\{(\mathbf{e}\boldsymbol{\Phi}_1)(\mathbf{k}^2 - \frac{4m}{M}k_z^2) + m(M+m)(\mathbf{ek})\Phi_2\right\} \,,$$

$$I_{TL}^D = 2Qz(1-z)(2z-1)(\mathbf{V}^*\mathbf{k})(\mathbf{r}^2+m^2+Mm)\Phi_2 \,, \tag{9}$$

The novel features of these amplitudes are best seen in the twist expansion in inverse powers of the hard scale $\overline{Q}^2$. As it was noted in [8], in all cases but the double helicity flip the dominant twist amplitudes come from the leading $\log\overline{Q}^2$ ($\mathrm{LL}\overline{Q}^2$) region of $\mathbf{k}^2 \sim$ $\sim R_V^{-2}, \boldsymbol{\Delta}^2 \ll \kappa^2 \ll \overline{Q}^2$. The closer inspection of our $I_{\lambda_V\lambda_\gamma}^D$ shows that the seemingly leading interference with the dominant $S$ wave component in the photon always appears in the quadrupole combination $2k_z^2 - \mathbf{k}^2$. Since the integration over quark loop can be cast in form $d^3\mathbf{r}$, such quadrupole combinations vanish after angular integration. As a result, the abnormally large higher twist contributions $\propto M^2/(M^2+Q^2)$ with large numerical factors come into play and significantly modify the $Q^2$ dependence of amplitudes for production of longitudinally polarized vector mesons:

$$I_{0L}^D = -\frac{Q}{M}\frac{32\mathbf{r}^4}{15(M^2+Q^2)^2}\left(1 - 8\frac{M^2}{M^2+Q^2}\right)\kappa^2 \,, \tag{10}$$

$$I_{\pm\pm}^D = (\mathbf{V}^*\mathbf{e})\frac{32\mathbf{r}^4}{15(M^2+Q^2)^2}\left(15 + 4\frac{M^2}{M^2+Q^2}\right)\kappa^2 \,, \tag{11}$$

$$I_{\pm L}^D = -\frac{32\mathbf{r}^4}{15(M^2+Q^2)^2}\frac{24Q(\mathbf{V}^*\boldsymbol{\Delta})}{M^2+Q^2}\kappa^2 \,, \tag{12}$$

$$I_{L\pm}^D = \frac{32\mathbf{r}^4}{15(M^2+Q^2)^2}\frac{8(\mathbf{e}\boldsymbol{\Delta})}{M}\left(1 + 3\frac{M^2}{M^2+Q^2}\right)\kappa^2 \,, \tag{13}$$

$$I_{\pm\mp}^D = (\mathbf{V}^*\boldsymbol{\Delta})(\mathbf{e}\boldsymbol{\Delta})\frac{32\mathbf{r}^4}{15(M^2+Q^2)^2}\left(1 - \frac{96}{7}\frac{\kappa^2\mathbf{r}^2}{M^2(M^2+Q^2)}\right) \,. \tag{14}$$

In a close similarity to the $S$ wave case [8], the leading twist double-helicity flip amplitude is dominated by soft gluon exchange, the $\mathrm{LL}\overline{Q}^2$ component is of higher twist.

In order to emphasize striking difference between the $D$ wave and $S$ wave state amplitudes, we focus on nonrelativistic heavy quarkonia, where $M^2 \approx m_V^2$, although all the qualitative results hold for light vector mesons too. For the illustration purposes, we evaluated the ratios of helicity amplitudes, $\rho_{D/S} = f_S A^D / f_D A^S$, for the the harmonic oscillator wave functions:

$$
\begin{aligned}
\rho_{0L}(D/S) &= \frac{1}{5}\left(1 - 8\frac{m_V^2}{Q^2 + m_V^2}\right), \\
\rho_{\pm\pm}(D/S) &= 3\left(1 + \frac{4}{15}\frac{m_V^2}{Q^2 + m_V^2}\right), \\
\rho_{0\pm}(D/S) &= -\frac{1}{5}(m_V a_S)^2\left(1 + 3\frac{m_V^2}{Q^2 + m_V^2}\right), \\
\rho_{\pm L}(D/S) &= \frac{3}{40}(m_V a_S)^4 .
\end{aligned} \tag{15}
$$

First, $A_{0L}$ changes the sign at $Q^2 \sim 7m_V^2$. The ratio $R^D = \sigma_L/\sigma_T$ has thus a non-monotonous $Q^2$ behavior and $R^D \ll R^S$. Furthermore, $R^D \lesssim 1$ in a broad range of $Q^2 \lesssim 225m_V^2$.. Whereas in heavy quarkonia the $S$–$D$ mixing is arguably weak [12], in light $\rho^0, \omega^0$ even a relatively weak $S$-$D$ mixing could have a substantial impact on $R$. Second, all the $D$ wave amplitides, SCHC and SCHNC alike, with exception of the higher twist component of double-helicity flip, are proportional to $\mathbf{r}^4$ and, in view of eq. (5), to the decay constant $f_D$. In contrast to that, in the $S$ wave case the spin-flip amplitudes for heavy quarkonia are suppressed by nonrelativistic Fermi motion [8]. The relevant suppression parameter is $\sim 1/(a_S m_V)^2$, where $a_S$ is the radius of the $1S$ state. For this reason, for $D$ wave states the SCHNC effects are much stronger. For instance, for the charmonium $(m_V a_S)^2 \approx 27$, see [12].

To summarize, we found dramatically different spin properties of diffractive leptoproduction of the $S$ and $D$ wave states of vector mesons. We predict very small $R^D = \sigma_L/\sigma_T$ and very strong breaking of $s$-channel helicity conservation in production of $D$ wave states. Higher twist effects in production of longitudinally polarized $D$ wave vector mesons are found to be abnormally large. Consequently, the distinct spin properties of $D$ wave vector mesons in diffractive DIS offer an interesting way to discern $S$ and $D$ wave meson states, which are indistinguishable at $e^+e^-$-colliders.

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