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THE STATIC $Q\bar{Q}$ INTERACTION AT SMALL DISTANCES AND OPE VIOLATING TERMS

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Nonperturbative contribution to the one-gluon exchange produces a universal linear term in the static potential at small distances $\Delta V = 6N_c\alpha_s\sigma r/2\pi$. Its role in the resolution of long-standing discrepancies in the fine splitting of heavy quarkonia and improved agreement with lattice data for static potentials is discussed, as well as implications for operator product expansion (OPE) violating terms in other processes.

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1. Possible nonperturbative contributions from small distances have drawn a lot of attention recently [1, 2]. In terms of interquark potential the appearance of linear terms in the static potential $V(r) = \text{const } r$, where r is the distance between charges, implies violation of OPE, since const $\sim$ (mass)^2 and this dimension is not available in terms of field operators. There is however some analytic [1, 2] and numerical evidence [3] for the possible existence of such terms $O(m^2/Q^2)$ in asymptotic expansion at large Q.

On a more phenomenological side the presence of linear term at small distances, $r < T_g$, where T_g is the gluonic correlation length [4, 5], is required by at least two sets of data.

First, the detailed lattice data [6] do not support much weaker quadratic behaviour of $V(r) \sim \text{const}\cdot r^2$, following from OPE and field correlator method [4, 5], and instead prefer the same linear form $V(r) = \sigma r$ at all distances (in addition to perturbative $-C_2\alpha_s/r$ term). Second, the small-distance linear term is necessary for the description of the fine splitting in heavy quarkonia, since the spin-orbit Thomas term $V_t = -(1/2m^2r)(dV/dr)$ is sensitive to the small r region and additional linear contribution at $r < T_g$ is needed to fit the experimental splitting [7]. Moreover lattice calculations [8] display the $1/r$ behaviour of V_t in all measured region up to $r = 0.1 fm$.

Of crucial importance is the sign of the $O(m^2/Q^2)$ term, since the usual screening correction (real m) leads to negative sign of linear potential, and one needs small-distance nonperturbative (NP) dynamics, which produces negative (tachyonic) sign of m^2 [1, 2]. Phenomenological implications of such contributions have been studied in detail in [1]. In what follows we show that interaction of gluon spin with NP background indeed yields tachyonic gluon mass at small distances.

2. In this letter we report the first application of the systematic background perturbation theory [9,10] to the problem in question. One starts with the decomposition of the full gluon vector potential A_μ into NP background B_μ and perturbative field a_μ,

$$A_\mu = B_\mu + a_\mu, \tag{1}$$

and make use of the 'tHooft identity for the partition function

$$Z = \int DA_\mu e^{-S(A)} = \frac{1}{N}\int DB_\mu \eta(B)\int Da_\mu e^{-S(B+a)} \tag{2}$$

where $\eta(B)$ is the weight for NP fields, defining the vacuum averages, e.g.

$$g^2 < F^B_{\mu\nu}(x)\Phi^B(x,y)F^B_{\lambda\sigma}(y) >_B = \frac{\hat{1}}{N_c}(\delta_{\mu\lambda}\delta_{\nu\sigma} - \delta_{\mu\sigma}\delta_{\nu\lambda})D(x-y) + \Delta_1 \tag{3}$$

where $F^B_{\mu\nu}, \Phi^B$ are field strength and parallel transporter made of B_μ only; Δ_1 is the full derivative term [4] not contributing to the string tension σ, which is

$$\sigma = \frac{1}{2N_c}\int d^2x D(x) + O(< FFFF >). \tag{4}$$

The background perturbation theory is an expansion of the last integral in (2) in powers of ga_μ and averaging over B_μ with the weight $\eta(B_\mu)$, as shown in (3). Referring the reader to [9, 10] for explicit formalism and renormalization, we concentrate below on the static interquark interaction at small r. To this end we consider the Wilson loop of size $r \times T$, where T is large, $T \to \infty$, and define

$$< W >_{B,a} = < P\exp[ig\int_C (B_\mu + a_\mu)dz_\mu] >_{B,a} \equiv \exp\{-V(r)T\}. \tag{5}$$

Expanding (5) in powers of ga_μ, one obtains

$$< W > = W_0 + W_2 + ...; \quad V = V_0(r) + V_2(r) + V_4(r) + ... \tag{6}$$

where $V_n(r)$ corresponds to $(ga_\mu)^n$ and can be expressed through D, Δ_1 and higher correlators [5, 9].

Coming now to $V_2(r)$, describing one exchange of perturbative gluon in the background, one finds from the quadratic in a_μ term in $S(B+a)$ in the background Feynman gauge the gluon Green's function

$$G_{\mu\nu} = -(D^2_\lambda\delta_{\mu\nu} + 2igF^B_{\mu\nu})^{-1}, \quad D^{ca}_\lambda = \partial_\lambda\delta_{ca} + gf^{cba}B^b_\lambda. \tag{7}$$

Expanding in powers of $gF^B_{\mu\nu}, G_{\mu\nu}$ can be written as

$$G = -D^{-2} + D^{-2}2igF^BD^{-2} - D^{-2}2igF^BD^{-2}2igF^BD^{-2} + ..., \tag{8}$$

the first term on the r.h.s. of (8) corresponds to the spinless gluon exchange, propagating in the confining film covering the Wilson loop [9,10]. As it was shown recently [11], the term D^{-2} produces only week corrections $O(r^3)$ to the usual perturbative potential at small distances, while it corresponds to the massive spinless propagator with mass m_0 at large distances.

In what follows we concentrate on the third term in (8), yielding for

$$W_2^{(3)} = T\int \frac{\alpha_s(k^2)}{\pi^2}\frac{d^3ke^{i\mathbf{kr}}\mu^2(k^2)}{(\mathbf{k}^2 + m_0^2)^2} = -\Delta V_2(r)T \tag{9}$$

where we have defined, having in mind (4)

$$\mu^2(k^2) = 6\int \frac{D(z)e^{-ikz}d^4z}{4\pi^2z^2}; \quad \mu^2(0) = \frac{6\sigma N_c}{2\pi}. \tag{10}$$

From (10) one obtains the following positive contribution to the potential $V_2(r)$ at small r (we neglect a constant term $O(1/m_0)$:

$$\Delta V_2(r) = \mu^2(k_{eff})\alpha_s(k_{eff})r + O(r^2), \quad r \lesssim T_g. \tag{11}$$

Analysis of the integral (9) shows that $k_{eff} \sim 1/r$, and therefore $\Delta V_2(r)$ is defined mostly by the short–distance dynamics.

3. The analysis done heretofore concerns static interquark potential and reveals that even at small distances NP background ensures some contributions which is encoded in the negative mass squared term $-\mu^2$.

Applying the same NP background formalism to other processes of interest, one would get similar corrections of the order of μ^2/p^2, as was investigated in [1].

To check the selfconsistency of our results, one can find the contribution of $\mu^2(k)$ to the correlator D,

$$D(q) \sim \alpha_s(q) \int \frac{d^4p\mu^2(p)}{(p^2+m_0^2(p))^2(q-p)^2} \sim \frac{1}{q^2}, \quad q^2 \to \infty \tag{12}$$

which is positive and consistent with recent lattice data [3]. Insertion of (12) into (10) yields constant $\mu^2(p)$ at large p (modulo logarithms), which implies selfconsistent NP dynamics at small distances (large p). It is worthwhile to note also that negative sign of μ^2 contribution is directly connected to the asymptotic freedom, where the same paramagnetic term in the effective action S_{eff} [12] enters with the negative sign, and one can take into account that $-\mu^2(x,y) \sim \delta^2 S_{eff}/\delta a_\mu(x)\delta a_\mu(y)$.

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