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ANDREEV REFLECTIONS AND MAGNETORESISTANCE IN FERROMAGNET-SUPERCONDUCTOR MESOSCOPIC STRUCTURES

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We analyze the change in the resistance of a nanostructure consisting of a diffusive ferromagnetic (F) wire and normal metal electrodes, due to the onset of superconductivity (S) in the normal electrode and Andreev scattering processes. The superconducting transition results in an additional contact resistance arising from the necessity to match the spin-polarized current in the F-wire to the spinless current in the S-reservoir, which is comparable to the resistance of a piece of a F-wire with the length equal to the spin-relaxation length. It is also shown that in the absence of spin relaxation the resistance of a two-domain structure is the same for a ferro- or antiferromagnetic configuration if one electrode is in the superconducting state.

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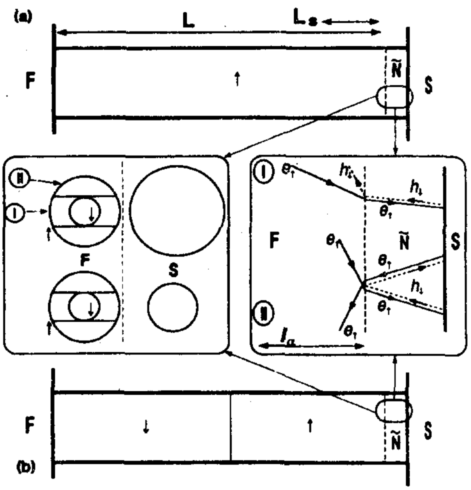
In recent years, studies of transport in mesoscopic conductors with strongly correlated electrons have revealed a number of novel phenomena, including the occurrence of a giant magnetoresistance (GMR) in multilayer FN structures [1], where F(N) are ferromagnetic (normal) metals. At the same time a variety of new transport properties arising from superconductivity (S) in mesoscopic NS structures have been identified [2, 3]. More recently the effect of superconductivity on transport properties of spin-polarized electrons in magnetic materials was studied [4 – 8] and it was observed that the onset of superconductivity may lead to both an increase and decrease of the conductance of an F film [4 – 6]. This change may be as much as 10% of the normal state conductance and is too large to be attributed to the superconducting proximity effect (in magnetic materials, such as Ni and Co, the exchange energy ϵ_{ex} is two orders of magnitude larger than the superconducting gap Δ, which suppresses the proximity effect).

It has been pointed out by de Jong and Beenakker [9] that, when the conductivities $\sigma_\uparrow$ and $\sigma_\downarrow$ for spin-up and spin-down electrons in a ferromagnetic material are different, then the resistance of a ferromagnetic wire increases due to contact with a superconductor. This is because the electrical current in the s-wave superconductor is spin-less, and matching the spin-polarized current in the ferromagnet to the spin-less current in the S-reservoir involves the Andreev scattering process, which increases the resistance of a system. When the ferromagnetic wire is long and when spin-relaxation processes in it are efficient, the resistance variation of a diffusive FS structure caused by this mechanism has the form of an additional contact resistance [10] of the FS interface, which can be also extended onto multi-terminal geometry [11]. The necessity to match the spin-less and spin-polarized currents at the FS interface also results in a different non-equilibrium population of spin-up and -down states within the spin-relaxation length, L_s near the superconducting contact.

In the present paper, we consider a nanostructure consisting of a ferromagnetic wire with one ($F_{\uparrow\uparrow}$) or two anti-collinear ($F_{\uparrow\downarrow}$) domains embedded between two normal reservoirs, one of which becomes superconducting at $T = T_c$. The sequence of domains in a ferromagnetic wire represents our simplified view of a multilayer GMR structure. We calculate the resistances of these structures, $R_{\uparrow\uparrow N}$, $R_{\uparrow\downarrow N}$, $R_{\uparrow\uparrow S}$ and $R_{\uparrow\downarrow S}$ in[1)] the limit of a long and short spin-relaxation length ($L_s >> L$, $L_s << L$) and in the case when the FS interface itself causes the spin-relaxation (for example, due to spin-orbit coupling). We find that, in the absence of spin-relaxation, $R_{\uparrow\uparrow N} < R_{\uparrow\downarrow N} = R_{\uparrow\uparrow S} = R_{\uparrow\downarrow S}$, so that an applied magnetic field (which polarizes domains) yields a non-zero resistance variation above T_c, as in typical giant magnetoresistance systems, and gives no resistance variation at $T \ll T_c$. Spin-relaxation processes of any kind (either due to spin-orbit coupling in the bulk of a ferromagnetic metal and its surface, or caused by a non-collinearity of ferromagnetic domains in the wire) change the resistance $R_{\uparrow\uparrow S}$, leading to a non-zero magnetoresistance at $T < T_c$.



Pictorial representation of the two-domain FS structure with ferromagnetic (a) and antiferromagnetic (b) alignment of domains, a double Andreev reflection process in it, and of possible relations between Fermi surfaces of spin-up and -down electrons in the F-wire (left) and in the N(S) metal (right)

First of all, we consider a structure $F_{\uparrow\uparrow}N(S)$ shown in Figure a, which consists of a single ferromagnetic domain. The resistance of a disordered F-wire can be found by solving diffusion equations for the isotropic part of the electron distribution function, $n_\alpha(z,\varepsilon) = \int d\Omega_{\mathbf{p}} n_\alpha(z,\mathbf{p})$. Using the electron-hole symmetry, we restrict our analysis to the calculation of a symmetrized function $N_\alpha(\varepsilon,z) = 1/2\,[n_\alpha(z,\varepsilon) + n_\alpha(z,-\varepsilon)]$, where ε is determined with respect to the chemical potential in the S(N) electrode. In terms of $N_\alpha(\varepsilon,z)$, the electric and spin current densities are given by

$$j_{Q,M} = j_\alpha \pm j_{\overline{\alpha}}, \qquad j_\alpha = \sigma_\alpha \int_{-\infty}^{\infty} \frac{d\varepsilon}{e} \partial_z N_\alpha(\varepsilon,z), \tag{1}$$

where $\overline{\alpha} = (\downarrow,\uparrow)$ for $\alpha = (\uparrow,\downarrow)$, $\sigma_\alpha = e^2\nu_\alpha D_\alpha$, ν_α and D_α are the density of states and diffusion coefficient for electrons in the spin-state α. Functions $N_\alpha(\epsilon,z)$ obey the diffusion

1) Indices $\uparrow(\downarrow)$ stand for the alignment of magnetization of the domains, and S(N) – for the normal and superconducting state of the right-hand reservoir, respectively.

equation

$$D_\alpha \partial_z^2 N_\alpha(z,\varepsilon) = w_{\uparrow\downarrow}\nu_{\overline{\alpha}}\left[N_\alpha(z,\varepsilon) - N_{\overline{\alpha}}(z,\varepsilon)\right], \tag{2}$$

which is more convenient to use in the equivalent form

$$\partial_z^2 \sum_{\alpha=\uparrow\downarrow} D_\alpha \nu_\alpha N_\alpha = 0, \; \left[\partial_z^2 - L_s^{-2}\right](N_\uparrow - N_\downarrow) = 0. \tag{3}$$

The term on the right hand side of Eq. (2) accounts for spin relaxation, which may result from both spin-orbit or spin-flip scattering at defects. It can be used to define the effective spin-relaxation length, L_s as $L_s^{-2} = w_{\uparrow\downarrow}\left[\nu_\uparrow/D_\downarrow + \nu_\downarrow/D_\uparrow\right]$. This pair of equations, which ignore any energy relaxation, should be complemented by four boundary conditions, two on each side of the ferromagnetic wire.

The boundary conditions for Eqs. (2), (3) can be obtained in various ways. We employ the model shown in Figure, where the FS junction is replaced by a sandwich of three layers: (i) a ferromagnetic (F) wire of the length L connected to the bulk F reservoir, (ii) a normal metal layer ($\widetilde{N}$) which never undergoes a superconducting transition by itself and has a negligible resistance, and (iii) a bulk electrode S(N) which undergoes the superconducting transition. The insertion of a normal metal layer $\widetilde{N}$ between the F and S(N) parts allows us to formulate the boundary conditions at the FS interface using known boundary conditions at the $\widetilde{N}$S interface [3]. For the sake of simplicity, we consider $\widetilde{N}$ to be ballistic and the F$\widetilde{N}$ junction to be semiclassically transparent, so that electrons either pass from one side to the other, or are fully reflected, depending on whether this process is allowed by energy-momentum conservation near the Fermi surface. The latter approximation avoids resonances through the 'surface states' [12] due to multiple passage through the normal layer inserted between S and F. As illustrated in Figure, we approximate the spectrum of electrons by parabolic bands - two for spin-down and spin-up electrons in F, and one in the N-part, which we take into account by introducing the parameters $\delta_{\alpha\mathrm{N}}^2 = p_{\mathrm{FN}}^2/p_{\mathrm{F}\alpha}^2$ and $\delta^2 = \left(p_{\mathrm{F}\downarrow}/p_{\mathrm{F}\uparrow}\right)^2 < 1$. The $\widetilde{N}$N interface is assumed to be ideal, and the Fermi surfaces in $\widetilde{N}$ and N layers to be the same, so that $\widetilde{N}$S Andreev reflection has unit probability. In such a model, the momentum of an electron in the plane of the junction is conserved.

The boundary conditions on the left end are given by the equilibrium distribution of electrons in the F-electrode,

$$N_\alpha(-L/2,\varepsilon) = \frac{1}{2}\left[n_T(\varepsilon - eV) + n_T(-\varepsilon - eV)\right]. \tag{4}$$

The boundary condition on the other end depends on the state of the electrode, and in the superconducting state takes into account Andreev reflection at the NS interface [13]. Since in our model of an ideal F$\widetilde{N}$ interface, the parallel component of the electron momentum is conserved, the effective reflection/transmission of electrons in parts I and II of the ferromagnet Fermi surface sketched in Figure are different. Although non-equilibrium quasi-particles from F pass inside $\widetilde{N}$ and generate holes by being Andreev reflected at the $\widetilde{N}$S interface, only those holes which are created by quasi-electrons from part I of the Fermi surface in F may escape into the F-wire. The spin-down holes which were generated by spin-up electrons from part II of the Fermi surface cannot find states in F, so that they are fully internally reflected into $\widetilde{N}$. Then, they undergo a second Andreev reflection, convert into spin-up electrons, and return back into the ferromagnetic wire. This results in *complete internal reflection* of spin-up electrons from part II of the Fermi surface inside the F-wire, which nullifies the spin current through its FS edge.

The boundary condition near the $F\widetilde{N}$ junction can be found by matching the iso-energetic electron fluxes determined in the diffusive region found in the ballistic F-region using the reflection/transmission relation between the distributions of incident and Andreev or normal reflected electrons. For quasi-particles with energies $0 < \varepsilon < \Delta$ this can be written in the form

$$\sigma_\uparrow \partial_z N_\uparrow - \sigma_\downarrow \partial_z N_\downarrow = -s(N_\uparrow - N_\downarrow), \tag{5}$$

$$N_\uparrow + N_\downarrow + \frac{2}{3}\varkappa\delta^2 l_\downarrow \partial_z N_\downarrow = 2N_T(\varepsilon), \tag{6}$$

where $\varkappa = (1-\delta^2)^{3/2}/\delta^2$, $\delta^2 = p_{F\downarrow}^2/p_{F\uparrow}^2 < 1$, and $N_T(\varepsilon) = 1/2\,[n_T(\varepsilon) + n_T(-\varepsilon)] = 1/2$ at $T = 0$. The spin-relaxation term on the right hand side of Eq. (5) takes into account the spin-orbit relaxation on the FN interface.

One can obtain boundary conditions in another way, after having considered both F wire and an auxillary N piece of a normal metal in the diffusive limit using the known boundary conditions at the NS interface [3]. Then, Eq. (5) follows from the condition $\partial_z f_\alpha = 0$ at the NS interface [3,14], where $f_\alpha = [n_\alpha + (1 - n_{\overline{\alpha}}(-\epsilon)]/2$ is the sum of the distribution functions of electrons and holes. Eq. (6) emerges from the equilibrium condition for electrons and holes in opposite spin states at the SN interface (if we neglect the third term on the left in Eq. (6) and set the electric potential equal to zero in S). At energies above the superconducting gap Δ, the boundary conditions coincide with Eq. (4).

For a ferromagnetic wire with sufficient intrinsic spin-relaxation, $L_s << L$, we find that the contact resistance of the FS boundary is equal to

$$r_c^S = R_\square \frac{L_s}{L_\perp}\frac{\varsigma^2}{1-\varsigma^2} + \frac{R_\square l_+}{3L_\perp}\frac{\varkappa}{1+\varsigma}, \tag{7}$$

where $\zeta = (\sigma_\uparrow - \sigma_\downarrow)/(\sigma_\uparrow + \sigma_\downarrow)$ is the degree of spin polarization of a current in a mono-domain ferromagnetic wire, $R_\square$ is the resistance per square of a mono-domain ferromagnetic film, and $L_\perp$ is the wire width.

In the normal state of the right hand reservoir, the boundary conditions at the end of an F-wire depend on the relation between the Fermi momenta of electrons in the ferromagnet and normal metal,

$$\left. N_\alpha(z,\varepsilon) + \frac{4\varkappa_{\alpha N} D_\alpha}{v_\alpha}\partial_z N_\alpha(z,\varepsilon)\right|_{z=L/2} = N_T(\varepsilon) \tag{8}$$

where $\varkappa_{\alpha N} = (1-\delta_{\alpha N}^2)^{3/2}/\delta_{\alpha N}^2$, $\delta_{\alpha N} < 1$, and $\varkappa_{\alpha N} = 0$, $\delta_{\alpha N} \geq 1$, $\delta_{\alpha N}^2 = p_{FN}^2/p_{F\alpha}^2$. These result in the contact resistance term

$$r_c^N = R_\square \frac{l_+(1+\varsigma)}{L_\perp}\left\{(1-\varsigma)\,l_+/L_s + \frac{3}{2}\varkappa_{+N}^{-1}\right\}^{-1}, \tag{9}$$

which has sense only when it is larger than the resistance of a short piece of F-wire with length of the order of l_+. Otherwise, it should be neglected.

After comparing the latter result to r_c^S, we find that the resistance of a long ferromagnetic wire attached to a S-electrode exceeds the resistance of the same wire connected to a normal reservoir by the resistance of an F-segment of length of order of L_s. One can extend the result of Eq. (7) to finite temperatures, which yields the resistance variation below the superconducting transition [10]

$$R_{\mathrm{S}}(T) - R_{\mathrm{N}} \approx \frac{\varsigma^2}{1-\varsigma^2}\frac{L_s}{L_\perp}R_\Box \tanh\left(\frac{\Delta(T)}{2T}\right). \tag{10}$$

Note that the increase of the resistance in Eq. (10) originates from *the matching of a spin-polarized current in the highly resistive ferromagnetic wire to a spinless current inside the superconductor*. We expect this robust effect to be present both in the mono-domain and poly-domain wires, with domain size $L_{\mathrm{D}} > L_s$.

The solution of Eqs. (2) – (6) can also be used to describe the contrasting case of a ferromagnetic wire where all spin-relaxation processes take place only at the FS interface. Such a structure may consist of either of one or of two ferromagnetic domains with anti-parallel magnetizations (antiferromagnetic configuration), as shown Figure b. In the latter case, we neglect the local microscopic $F_\uparrow F_\downarrow$ interface resistance, so that the boundary conditions for $N_\alpha(x)$ at the domain wall can be reduced to the continuity equation for the spin-current, $\sigma_\alpha \partial_z N_\alpha$ and for the distribution functions N_α. This yields

$$R_{\uparrow\uparrow\mathrm{N}} = \frac{L}{\sigma_\uparrow + \sigma_\downarrow}, \qquad R_{\uparrow\downarrow\mathrm{N}} = R_{\uparrow\downarrow\mathrm{S}} = \frac{\sigma_\uparrow + \sigma_\downarrow}{4\sigma_\uparrow\sigma_\downarrow}L,$$

and

$$R_{\uparrow\uparrow\mathrm{S}} = \frac{L(\sigma_\uparrow + \sigma_\downarrow + 4s(L/2))}{4(\sigma_\uparrow\sigma_\downarrow + s(L/2)(\sigma_\uparrow + \sigma_\downarrow))}. \tag{11}$$

From this, we deduce that the alignment of magnetizations in two domains results in a significant change of the resistance in the case of normal reservoirs (N) and leaves the conductance unchanged when one of the reservoirs is a superconductor if spin relaxation is completely absent, or the wire is too short: $R_{\uparrow\uparrow S}(sL \to 0) \to R_{\uparrow\downarrow S}$. A similar behavior has been observed in numerical simulations of the transport through the giant magnetoresistance system with S-contacts [15]. In a word, when superconducting leads inject a spin-less electric current into the spin-conserving multi-domain system, the change in the polarization of domains does not affect of resistance of the system. Spin-relaxation at the FS surface restores the sensitivity of the system to the polarization state of domains, and in a long wire ($L \to \infty$) the interplay between Andreev scattering and spin-relaxation results in a contact resistance, similar to that in Eq. (7):

$$R_{\uparrow\uparrow\mathrm{S}}(T) - R_{\uparrow\uparrow\mathrm{N}} \approx \frac{\varsigma^2}{2s}\tanh\left(\frac{\Delta(T)}{2T}\right). \tag{12}$$

Note that the electric current generates a non-equilibrium magnetization, $\delta M = \mu(\nu_\uparrow \int d\epsilon N_\uparrow - \nu_\downarrow \int d\epsilon N_\downarrow)$, which is different for different configurations: $\delta M_{\uparrow\uparrow\mathrm{N}} = 0$,

$$\delta M_{\uparrow\uparrow\mathrm{S}} = (z/L + \frac{1}{2})M_0, \quad \delta M_{\uparrow\downarrow\mathrm{N}} = \delta M_{\uparrow\downarrow\mathrm{S}} = (\frac{1}{2} - |z|/L)M_0, \quad M_0 = eV\frac{4\nu_\uparrow\nu_\downarrow}{\nu_\uparrow + \nu_\downarrow}\varsigma\mu$$

for $T \ll T_c$. Here, μ is the magnetic moment of electrons, $-L/2 < z < L/2$, and $z = 0$ corresponds to the $F_\uparrow F_\downarrow$ domain wall.

In summary, we have shown that in the absence of any spin relaxation the resistances of the structures $F_{\uparrow\downarrow}N$, $F_{\uparrow\downarrow}S$ and $F_{\uparrow\uparrow}S$ coincide, but differ from the resistance of the $F_{\uparrow\downarrow}N$ structure. This can be regarded as a prediction of a suppression of the giant magnetoresistance in multilayer FN structures with superconducting leads and no spin-relaxation. Surface spin-relaxation at the FS interface alters the equivalence between $R_{\uparrow\uparrow S}$ and $R_{\uparrow\downarrow S}$ resistances. When the spin-relaxation is fast in the bulk of the ferromagnetic material, the resistance of the $F_{\uparrow\uparrow}S$ structure changes at the superconducting transition by a contact resistance value which depends on the spin relaxation rate. For example, in a ferromagnetic wire with the size of a ferromagnetic domain larger than the spin-relaxation length L_s, the resistance variation is formed within the L_s-segment of the F wire (where the spin-polarized current from the F-part relaxes to a spin-less current in S), and $R_S(T) - R_N$ increases from zero at T_c to a positive value at $T = 0$.

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