FLAVOURED MULTISKYRMIONS

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Static properties of multiskyrmions with baryon numbers up to 8 are calculated, based on the recently given rational map ansaetze. The spectra of baryonic systems with strangeness, charm and bottom are considered within a "rigid oscillator" version of the bound state soliton model. It is suggested that the recently observed negatively charged nuclear fragment can be considered as a quantized strange multiskyrmion with $B = 6$ or 7. In agreement with previous observation, baryonic systems with charm or bottom have more chance to be bound by the strong interactions than strange baryonic systems.

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1. The topological soliton models, and the Skyrme model among them [1], are attractive because of their simplicity and the possibility that they may describe well various properties of low energy baryons. The models of this kind provide also a very good framework within which to investigate the possibility of the existence of nuclear matter fragments with unusual properties, such as flavour being different from u and d quarks. In addition to being important by itself, this issue can have consequences in astrophysics and cosmology. It is well known that the relativistic many-body problems cannot be solved directly using the existing methods, and the chiral soliton approach may allow to overcome some of these difficulties.

The description of skyrmions with large baryon numbers is complicated because the explicit form of the fields was not known. A recent remarkable observation [2] that the fields of the $SU(2)$ skyrmions can be approximated accurately by rational map ansaetze giving the values of masses close to their precise values, has simplified considerably their studies. Similar ansaetze have also been recently presented for $SU(N)$ skyrmions (which are not embeddings of $SU(2)$ fields) [3].

Here we use the $SU(2)$ rational map ansaetze as the starting points for the calculation of static properties of bound states of skyrmions necessary for their quantization in the $SU(3)$ collective coordinates space. The energy and baryon number densities of the $B = 3$ configuration have tetrahedral symmetry, for $B = 4$ – the octahedral (cubic) one [4], for $B = 5$ – D_{2d}-symmetry, for $B = 6$ – D_{4d}, for $B = 7$ – dodecahedral symmetry, and for $B = 8 - D_{6d}$ – symmetry [5,2], etc. The minimization, with the help of a 3-dimensional variational $SU(3)$ program [6], lowers the energies of these configurations by few hundreds of MeV and shows that they are local minima in the $SU(3)$ configuration space. The knowledge of the "flavour" moment of inertia and the Σ-term allows us then to estimate the flavour excitation energies. The mass splittings of the lowest states with different values of strangeness, charm or bottom are calculated within the rigid oscillator version of the bound state approach. The binding energies of baryonic systems (BS) with different flavours are also estimated.

2. Let us consider simple $SU(3)$ extentions of the Skyrme model [1]: we start with $SU(2)$ skyrmions (with flavour corresponding to (u, d) quarks) and extend them to various

$SU(3)$ groups, (u, d, s), (u, d, c), or (u, d, b). We take the Lagrangian density of the Skyrme model, which in its well known form depends on parameters F_π, F_D and e and can be written in the following way [7]:

$$\mathcal{L} = \frac{F_\pi^2}{16} \mathrm{Tr} l_\mu l^\mu + \frac{1}{32e^2} \mathrm{Tr}[l_\mu, l_\nu]^2 + \frac{F_\pi^2 m_\pi^2}{16} \mathrm{Tr}(U + U^\dagger - 2) +$$

$$+ \frac{F_D^2 m_D^2 - F_\pi^2 m_\pi^2}{24} \mathrm{Tr}(1 - \sqrt{3}\lambda_8)(U + U^\dagger - 2) + \frac{F_D^2 - F_\pi^2}{48} \mathrm{Tr}(1 - \sqrt{3}\lambda_8)(U l_\mu l^\mu + l_\mu l^\mu U^\dagger). \quad (1)$$

$U \in SU(3)$ is a unitary matrix incorporating chiral (meson) fields, and $l_\mu = U^\dagger \partial_\mu U$. In this model F_π is fixed at the physical value: $F_\pi = 186\,\mathrm{MeV}$. M_D is the mass of K, D or B meson.

The flavour symmetry breaking (FSB) in the Lagrangian is of the usual form, and was sufficient to describe the mass splittings of the octet and decuplets of baryons [7]. The Wess-Zumino term, not shown here, plays an important role in the quantization procedure, but it does not contribute to the static masses of classical configurations [8].

We begin our calculations with $U \in SU(2)$, as was mentioned above. The classical mass of $SU(2)$ solitons, in most general case, depends on 3 profile functions: f, α and β. The general parametrization of U_0 for an $SU(2)$ soliton we use here is given by $U_0 = c_f + s_f \tau \mathbf{n}$ with $n_z = c_\alpha$, $n_x = s_\alpha c_\beta$, $n_y = s_\alpha s_\beta$, $s_f = \sin f$, $c_f = \cos f$, etc.

The masses of solitons, moments of inertia, Γ and $\tilde{\Gamma}$ are presented in the Table below.

B	M_{cl}	$\Theta_F^{(0)}$	Θ_T	Γ	$\tilde{\Gamma}$	ω_s	ω_c	ω_b	$\Delta\epsilon_s$	$\Delta\epsilon_c$	$\Delta\epsilon_b$
1	1.702	2.04	5.55	4.83	15.6	0.309	1.542	4.82	—	—	—
3	4.80	6.34	14.4	14.0	26.7	0.289	1.504	4.75	−0.041	−0.01	0.03
4	6.20	8.27	16.8	18.0	31.4	0.283	1.493	4.74	−0.020	0.019	0.06
5	7.78	10.8	23.5	23.8	35.3	0.287	1.505	4.75	−0.027	0.006	0.05
6	9.24	13.1	25.4	29.0	38.2	0.287	1.504	4.75	−0.019	0.017	0.05
7	10.6	14.7	28.7	32.3	44.4	0.282	1.497	4.75	−0.017	0.021	0.06
8	12.2	17.4	33.4	38.9	46.9	0.288	1.510	4.77	−0.018	0.014	0.02

Characteristics of the bound states of skyrmions with baryon numbers up to $B = 8$. The classical mass of solitons M_{cl} is in GeV, moments of inertia, Γ and $\tilde{\Gamma}$ – in GeV^{-1}, the excitation frequences for flavour F, ω_F in GeV. The parameters of the model $F_\pi = 186\,\mathrm{MeV}$, $e = 4.12$. The accuracy of calculations is better than 1% for the masses and few % for other quantities. The $B = 1$ quantities are shown for comparison. $\Delta\epsilon_{s,c,b}$, in GeV, are the changes of binding energies of lowest BS with flavour s, c or b, $|F| = 1$, in comparison with usual (u, d) nuclei (see Eq.(14)).

The flavour moment of inertia enters directly in the procedure of quantization [9 − 17], and for arbitrary $SU(2)$ skyrmions is given by [15, 17]:

$$\Theta_F = \frac{1}{8} \int (1 - c_f) \left[F_D^2 + \frac{1}{e^2} ((\partial f)^2 + s_f^2 (\partial\alpha)^2 + s_f^2 s_\alpha^2 (\partial\beta)^2) \right] d^3\mathbf{r}. \quad (2)$$

It is simply connected with $\Theta_F^{(0)}$ for the flavour symmetric case: $\Theta_F = \Theta_F^{(0)} + (F_D^2/F_\pi^2 - 1)\Gamma/4$, Γ is defined in (3) below. The isotopic moments of inertia are the components of the corresponding tensor of inertia [9, 10], in our case this tensor of inertia is close to unit matrix multiplied by Θ_T. The quantities Γ (or Σ-term), which defines the contribution of the mass term to the classical mass of solitons, and $\tilde{\Gamma}$ also are used in the quantization procedure:

$$\Gamma = \frac{F_\pi^2}{2} \int (1 - c_f) d^3\mathbf{r}, \quad (3)$$

$$\tilde{\Gamma} = \frac{1}{4} \int c_f \left[(\boldsymbol{\partial} f)^2 + s_f^2 (\boldsymbol{\partial}\alpha)^2 + s_f^2 s_\alpha^2 (\boldsymbol{\partial}\beta)^2 \right] d^3 \mathbf{r}. \tag{4}$$

3. To quantize the solitons in $SU(3)$ configuration space, in the spirit of the bound state approach to the description of strangeness proposed in [11, 12] and used in [13, 14], we consider the collective coordinates motion of the meson fields incorporated into the matrix U:

$$U(r,t) = R(t) U_0(O(t)\mathbf{r}) R^\dagger(t), \qquad R(t) = A(t) S(t), \tag{5}$$

where U_0 is the $SU(2)$ soliton embedded into $SU(3)$ in the usual way (into the left upper corner), $A(t) \in SU(2)$ describes $SU(2)$ rotations, $S(t) \in SU(3)$ describes rotations in the "strange", "charm" or "bottom" directions, and $O(t)$ describes rigid rotations in real space;

$$S(t) = \exp(i\mathcal{D}(t)), \qquad \mathcal{D}(t) = \sum_{a=4,\dots 7} D_a(t)\lambda_a, \tag{6}$$

λ_a are Gell-Mann matrices of the (u, d, s), (u, d, c) or (u, d, b) $SU(3)$ groups. The (u, d, c) and (u, d, b) $SU(3)$ groups are quite analogous to the (u, d, s) one. For the (u, d, c) group a simple redefiniton of hypercharge should be made. For the (u, d, s) group, $D_4 = (K^+ + K^-)/\sqrt{2}$, $D_5 = i(K^+ - K^-)/\sqrt{2}$, etc. And for the (u, d, c) group $D_4 = (D^0 + \bar{D}^0)/\sqrt{2}$, etc.

The angular velocities of the isospin rotations are defined in the standard way: $A^\dagger \dot{A} = -i\boldsymbol{\omega}\boldsymbol{\tau}/2$. We shall not consider here the usual space rotations explicitly because the corresponding moments of inertia for BS are much greater than isospin moments of inertia, and for lowest possible values of angular momentum J the corresponding quantum correction is either exactly zero (for even B), or small.

The field D is small in magnitude, at least, of order $1/\sqrt{N_c}$, where N_c is the number of colours in QCD. Therefore, an expansion of the matrix S in D can be made safely. To the lowest order in field D the Lagrangian of the model (1) can be written as

$$L = -M_{cl,B} + 4\Theta_{F,B}\dot{D}^\dagger\dot{D} -$$

$$- \left[\Gamma_B \left(\frac{F_D^2}{F_\pi^2} m_D^2 - m_\pi^2 \right) + \tilde{\Gamma}_B (F_D^2 - F_\pi^2) \right] D^\dagger D - i\frac{N_c B}{2} (D^\dagger \dot{D} - \dot{D}^\dagger D). \tag{7}$$

Here and below D is the doublet K^+, K^0 (D^0, D^-, or B^+, B^0). We have kept the standard notation for the moment of inertia of the rotation into the "flavour" direction $\Theta_F = \Theta_s$, Θ_c or Θ_b [10, 15] (the index c denotes the charm quantum number, except in N_c). The contribution proportional to $\tilde{\Gamma}_B$ is suppressed in comparison with the term $\sim \Gamma$ by the small factor $\sim F_D^2/m_D^2$, and is more important for strangeness. The term proportional to $N_c B$ in (7) arises from the Wess-Zumino term in the action and is responsible for the difference of the excitation energies of strangeness and antistrangeness (flavour and antiflavour in general case) [13, 14].

Following the canonical quantization procedure the Hamiltonian of the system, including the terms of the order of N_c^0, takes the form [11, 12]:

$$H_B = M_{cl,B} + \frac{1}{4\Theta_{F,B}}\Pi^\dagger\Pi + \left(\Gamma_B \bar{m}_D^2 + \tilde{\Gamma}_B (F_D^2 - F_\pi^2) + \frac{N_c^2 B^2}{16\Theta_{F,B}} \right) D^\dagger D + i\frac{N_c B}{8\Theta_{F,B}}(D^\dagger\Pi - \Pi^\dagger D); \tag{8}$$

$\bar{m}_D^2 = (F_D^2/F_\pi^2)m_D^2 - m_\pi^2$. The momentum Π is canonically conjugate to variable D. Eq. (8) describes an oscillator-type motion of the field D in the background formed by

the (u, d) $SU(2)$ soliton. After the diagonalization which can be done explicitly following [13, 14], the normal-ordered Hamiltonian can be written as

$$H_B = M_{cl,B} + \omega_{F,B} a^\dagger a + \bar{\omega}_{F,B} b^\dagger b + O(1/N_c), \tag{9}$$

with $a^\dagger$, $b^\dagger$ being the operators of creation of strangeness, i.e., antikaons, and antistrangeness (flavour and antiflavour) quantum number, $\omega_{F,B}$ and $\bar{\omega}_{F,B}$ being the frequences of flavour (antiflavour) excitations. D and Π are connected with a and b in the following way [13, 14]:

$$D^i = (b^i + a^{\dagger i})/\sqrt{N_c B \mu_{F,B}}, \qquad \Pi^i = \sqrt{N_c B \mu_{F,B}}(b^i - a^{\dagger i})/2i \tag{10}$$

with $\mu_{F,B} = [1 + 16(\bar{m}_D^2 \Gamma_B + (F_D^2 - F_\pi^2)\tilde{\Gamma}_B)\Theta_{F,B}/(N_c B)^2]^{1/2}$. For the lowest states the values of D are small: $D \sim [16\Gamma_B \Theta_{F,B} \bar{m}_D^2 + N_c^2 B^2]^{-1/4}$, and increase, with increasing flavour number $|F|$ like $(2|F| + 1)^{1/2}$. As was noted in [14], deviations of the field D from the vacuum decrease with increasing mass m_D, as well as with increasing number of colours N_c, and the method works for any m_D (and also for charm and bottom quantum numbers).

The excitation frequences ω and $\bar{\omega}$ are:

$$\omega_{F,B} = N_c B(\mu_{F,B} - 1)/8\Theta_{F,B}, \qquad \bar{\omega}_{F,B} = N_c B(\mu_{F,B} + 1)/8\Theta_{F,B}. \tag{11}$$

As was observed in [15], the difference $\bar{\omega}_{F,B} - \omega_{F,B} = N_c B/4\Theta_{F,B}$ coincides, to the leading order in N_c with the expression obtained in the collective coordinates approach [16].

The FSB in the flavour decay constants, i.e. the fact that $F_K/F_\pi \simeq 1.22$ and $F_D/F_\pi = = 1.7 \pm 0.2$ (we take $F_D/F_\pi = 1.5$ and $F_B/F_\pi = 2$) leads to the increase of the flavour excitation frequences, in better agreement with data for charm and bottom [18]. It also leads to some increase of the binding energies of BS [15].

The behaviour of static characteristics of multiskyrmions and flavour excitation frequences shown in the Table is similar to that obtained in [19] for toroidal configurations with $B = 2, 3, 4$. The flavour inertia increases with B almost proportionally to B. The frequences ω_F are smaller for $B \geq 3$ than for $B = 1$.

4. The terms of the order of N_c^{-1} in the Hamiltonian, which depend on the angular velocities of rotations in the isospin and the usual space and which describe the zero-mode contributions are not crucial but important for the numerical estimates of spectra of baryonic systems.

In the rigid oscillator model the states predicted do not correspond to the definite $SU(3)$ or $SU(4)$ representations. How this can be remedied was shown in [14]. For example, the state with $B = 1$, $|F| = 1$, $I = 0$ should belong to the octet of (u, d, s), or (u, d, c), $SU(3)$ group, if $N_c = 3$.

Here we consider quantized states of BS which belong to the lowest possible $SU(3)$ irreps (p, q), $p + 2q = 3B$: $p = 0$, $q = 3B/2$ for even B, and $p = 1$, $q = (3B - 1)/2$ for odd B. For $B = 3, 5$ and 7 they are 35, 80 and 143-plets, for $B = 4, 6$ and $8 - 28$, 55 and 91-plets. Since we are interested in the lowest energy states, we discuss here the baryonic systems with the lowest allowed angular momentum, i.e. $J = 0$, for $B = 4, 6$ and 8. For odd B the quantization of BS meets some difficulties, but the correction to the energy of quantized states due to nonzero angular momentum is small and decreases

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with increasing B since the corresponding moment of inertia increases proportionally to $\sim B^2$. Moreover, the J-dependent correction to the energy cancels in the differences of energies of flavoured and flavourless states which we discuss.

For the energy difference between the state with flavour F belonging to the (p,q) irrep, and the ground state with $F = 0$ and the same angular momentum and (p,q) we obtain:

$$\Delta E_{B,F} = |F|\omega_{F,B} + \frac{\mu_{F,B} - 1}{4\mu_{F,B}\Theta_{F,B}}[I(I+1) - T_r(T_r+1)] + \frac{(\mu_{F,B} - 1)(\mu_{F,B} - 2)}{4\mu_{F,B}^2\Theta_{F,B}}I_F(I_F+1),$$
(12)

$T_r = p/2$ is the quantity analogous to the "right" isospin T_r, in the collective coordinates approach [9, 10], and $\mathbf{T}_r = \mathbf{I}_{bf} - \mathbf{I}_F$. Clearly, the binding energy of multiskyrmions is cancelled in Eq. (12). For the states with maximal isospin $I = T_r + |F|/2$ the energy difference can be simplified to:

$$\Delta E_{B,F} = |F|\left[\omega_{F,B} + T_r\frac{\mu_{F,B} - 1}{4\mu_{F,B}\Theta_{F,B}} + \frac{(|F| + 2)}{8\Theta_{F,B}}\frac{(\mu_{F,B} - 1)^2}{\mu_{F,B}^2}\right].$$
(13)

This difference depends on the flavour moment of inertia but not on Θ_T. In the case of antiflavour excitations we have the same formulas, with the substitution $\mu \to -\mu$. For even B, $T_r = 0$, for odd B, $T_r = 1/2$ for the lowest $SU(3)$ irreps. It follows from (12) and (13) that when some nucleons are replaced by flavoured hyperons in BS the binding energy of the system changes by

$$\Delta\epsilon_{B,F} = |F|\left[\omega_{F,1} - \omega_{F,B} - \frac{3(\mu_{F,1} - 1)}{8\mu_{F,1}^2\Theta_{F,1}} - T_r\frac{\mu_{F,B} - 1}{4\mu_{F,B}\Theta_{F,B}} - \frac{(|F| + 2)}{8\Theta_{F,B}}\frac{(\mu_{F,B} - 1)^2}{\mu_{F,B}^2}\right].$$
(14)

For strangeness Eq. (14) is negative indicating that stranglets should have binding energies smaller than those of nuclei, or can be unbound. Since $\Theta_{F,B}$ increases with increasing B and m_D this leads to the increase of binding with increasing B and mass of the "flavour", in agreement with [15]. For charm and bottom Eq. (14) is positive for $B \geq 3$, see the Table for the case $|F| = 1$.

The nuclear fragments with sufficiently large values of strangeness (or bottom) can be found in experiments as fragments with negative charge Q, according to the well known relation $Q = T_3 + (B+S)/2$ (similarly for the bottom number). One event of a long lived nuclear fragment with mass about $7.4\,\text{GeV}$ was reported in [20]. Using the above formulas it is not difficult to establish that this fragment can be the state with $B = -S = 6$, or $B = 7$ and strangeness $S = -3$. Greater values of strangeness are not excluded.

As in the $B = 1$ case [21] the absolute values of masses of multiskyrmions are controlled by the poorly known loop corrections to the classic masses, or the Casimir energy. And as was done for the $B = 2$ states, [16], the renormalization procedure is necessary to obtain physically reasonable values of the masses. This generates an uncertainty of about several tens of MeV, as the binding energy of the deuteron is $30\,\text{MeV}$ instead of the measured value $2.23\,\text{MeV}$, so $\sim 30\,\text{MeV}$ characterises the uncertainty of our approach [16, 17]. But this uncertainty is cancelled in the differences of binding energies $\Delta\epsilon$ shown in the Table.

5. Using rational map ansaetze as starting configurations we have calculated the static characteristics of bound skyrmions with baryon numbers up to 8. The excitation frequences for different flavours-strangeness, charm and bottom-have been calculated using a rigid oscillator version of the bound state approach of the chiral soliton models. This variant of the model overestimates the mass splitting of strange hyperons when

FSB in decay constant F_K is included, but works better for c and b flavours [18]. Our previous conclusion that BS with charm and bottom have more chances to be bound respectively to strong decay than strange BS [15] is reinforced by the present investigation. This conclusion takes place also in FS case, $F_D = F_\pi$.

Consideration of the BS with "mixed" flavours is possible in principle, but would be technically more involved. Our results agree qualitatively with the results of [22] where the strangeness excitation frequences had been calculated within the bound state approach. The difference is, however, in the behaviour of excitation frequences: we have found that they decrease when the baryon number increases from $B = 1$ thus increasing the binding energy of corresponding BS.

The charmed baryonic systems with $B = 3$, 4 were considered in [23] within a potential approach. The $B = 3$ systems were found to be very near the threshold and the $B = 4$ system was found to be stable with respect to the strong decay, with a binding energy of $\sim 10\,\text{MeV}$. Further experimental searches for the baryonic systems with flavour different from u and d could shed more light on the dynamics of heavy flavours in baryonic systems.

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