Pis'ma v ZhETF, vol.69, iss.10, pp.681 - 685

THE EFFECT OF γ-RADIATION ON THE DETERMINATION OF THE ANTINEUTRINO ANGULAR DISTRIBUTION FROM EXPERIMENTS ON THE β-DECAY OF POLARIZED NEUTRONS

G.G.Bunatian[1)]

Joint Institute for Nuclear Research

141980, Dubna, Moscov reg., Russia

Submitted 8 April 1999

In experiments on the β-decay of polarized neutrons where only the electron and proton momentum distributions are observed and the γ-radiation is not registered, the asymmetry factor B of the antineutrino angular distribution cannot be obtained rigorously – the value of B is only estimated on the average by taking into consideration the expectation (mean) value $\langle B\rangle$ and the rms deviation ΔB. The resulting unavoidable ambiguities in the determination of B amount to several per cent, which is significant for the present-day experimental attempts to obtain B to very high precision $\sim$(0.1–1)%.

PACS: 12.15.Lk, 13.10.+q, 13.40.Ks

Recently there has been a great deal of interest in high-precision measurement of the neutron β-decay characteristics, first, the lifetime τ [1], and, if the neutron is polarized, the asymmetry factors A and B, respectively, of the electron [2] and antineutrino [3] angular distributions with respect to the neutron polarization vector $\boldsymbol{\xi}$. The rigorous determination of the β-decay characteristics τ, A, B, ... is well understood nowadays to be of fundamental importance for the general elementary particle theory (see, e.g., Refs.[4 – 7]).

The electron and antineutrino momentum distribution[4, 5]

$$d\mathbf{W}(\varepsilon,\mathbf{p},\mathbf{n}_\nu,\boldsymbol{\xi}) = d\mathbf{w}\frac{d\mathbf{n}_\nu}{4\pi}(g_V^2+3g_A^2)\Big\{1+(\mathbf{v}\boldsymbol{\xi})A(g_V,g_A,\varepsilon)+ \\ +B(g_V,g_A,\varepsilon)(\mathbf{n}_\nu\boldsymbol{\xi})+a(g_V,g_A,\varepsilon)(\mathbf{n}_\nu\mathbf{v})\Big\} \qquad (1)$$

is usually what haunts us whenever we consider the β-decay process. In Eq. (1) we have

$$d\mathbf{w} = \frac{\tilde{G}^2}{2\pi^3}\varepsilon p\omega_\nu^2 d\varepsilon(d\mathbf{n}_e/4\pi),\quad \mathbf{n}_e=\mathbf{p}/p,\quad \mathbf{v}=\mathbf{p}/\varepsilon,\quad \mathbf{n}_\nu=\mathbf{p}_\nu/\omega_\nu,$$

where $\tilde{G}$ stands for the effective β-decay amplitude [4, 5] and $\varepsilon,\omega_\nu,\mathbf{p},\mathbf{p}_\nu$ are the electron and antineutrino energies and momenta, respectively; a system of units with $h=c=1$ is adopted. But so far as antineutrino registration is unfeasible, Eq. (1), immediately as it stands, is useless for obtaining the value of B from experiment. Since an experiment for obtaining the antineutrino angular distribution without registering the antineutrino itself is expounded thoroughly in Ref. [3], here we only recall that in its ideal scheme, which is sufficient for our purposes, the registered electron momentum $\mathbf{p}$ is directed strictly along the $\mathbf{x}$ axis (see Fig. 1), the at-rest neutron polarization vector $\boldsymbol{\xi}$ is also directed exactly along or opposite the $\mathbf{x}$ axis direction, and the proton momentum projection on

1) e-mail: bunat@cv.jinr.dubna.su

the $\mathbf{x}$ axis, P_x, is registered in coincidence with the electron momentum $\mathbf{p}$, while the components of the proton momentum $\mathbf{P}$ perpendicular to $\mathbf{x}$ are not observed at all, nor is the γ-radiation. If for a moment we leave aside the γ-radiation and neglect the kinetic energy of the proton on account of its very large mass, the antineutrino energy $\omega_{\nu 0}$ and the cosine of the angle between the $\mathbf{x}$ axis and the direction of the antineutrino emission are clearly given by

$$\omega_{\nu 0} = \Delta - \varepsilon, \quad y_0 \equiv \cos\Theta_{\nu x} = (-P_x - |\mathbf{p}|)/\omega_{\nu 0}, \tag{2}$$

with the corresponding momentum distribution taking the form

$$d\mathbf{W}^z(P_x, \mathbf{p}) = dP_x \frac{d\mathbf{w}}{2\omega_{\nu 0}} w^z(P_x, \mathbf{p}),$$
$$w^z(P_x, \mathbf{p}) = (g_V^2 + 3g_A^2)[1 + Azv + B_0 y_0 z + a y_0 v]. \tag{3}$$

In (3) and hereafter, the value $z = +$ stands for neutron polarization along the $\mathbf{x}$ axis and $z = -$ for the opposite direction. We have appended a subscript 0 on B to stress that it is the value that would be obtained if the γ-radiation were turned off. In the experiment of Ref. [3], the distribution

$$d\mathbf{W}^z_{\text{exp}}(P_x, \mathbf{p}) = W^z_{\text{exp}}(P_x, \mathbf{p}) \cdot d\mathbf{p}\, dP_x \tag{4}$$

was obtained. Using Eqs. (1)–(4), for which the γ-radiation has been left aside, one would infer the equation

$$W^z_{\text{exp}}(P_x, \mathbf{p}) = f_0(\omega_{\nu 0})(1 + zAv) + f_0(\omega_{\nu 0}) y_0 (zB_0 + av), \tag{5}$$

and, consequently, one would arrive at the following expression, in terms of W^z_{exp} (4), for the coefficient multiplying $(\boldsymbol{\xi}\mathbf{n}_\nu) = zy_0$ in Eqs. (1) and (5):

$$B_0 = \frac{1}{zy_0 f_0}[W^z_{\text{exp}} - f_0(1 + zAv) - f_0 a v y_0], \quad f_0 = \frac{\tilde{G}^2 \omega_{\nu 0}}{16\pi^4}(g_V^2 + 3g_A^2). \tag{6}$$

Accordingly [3], $B_0 = 0.9821 \pm 0.004$.

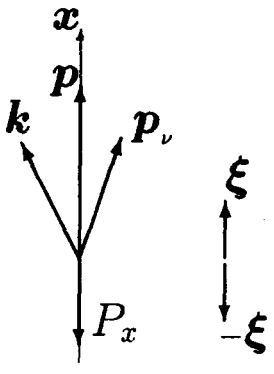

Рис.1

However, the experiment of Ref. [3] deals with the β-decay probability for given $P_x, \mathbf{p}$ values, involving γ-radiation with all the allowed momenta $\mathbf{k}$. In describing each single event, the expressions for $y_0, \omega_{\nu 0}$ in (2) will be replaced (see Fig.1) by the following ones:

$$y_0 \longrightarrow y(\omega) = \cos\Theta_{\nu x} = \frac{-P_x - |\mathbf{p}| - x\omega}{\omega_\nu}, \quad x = \cos\Theta_{\gamma x}, \tag{7}$$

$$f_0 \longrightarrow f(\omega) = \frac{\bar{G}^2\omega_\nu}{16\pi^4}(g_V^2 + 3g_A^2)\,, \quad \omega_{\nu 0} \longrightarrow \omega_\nu(\omega) = \Delta - \varepsilon - \omega,$$

where $\omega = |\mathbf{k}|$ is the γ-ray energy, and $\Theta_{\gamma x}$ stands for the angle of the γ-radiation direction relative to the $\mathbf{x}$ axis. It is natural to estimate the quantity B in (1) via the expectation value $\langle B\rangle$ expressed in terms of the expectation values $\langle yf\rangle, \langle f\rangle$, which are to be calculated by averaging $f(\omega)$, $f(\omega)y(\omega, x)$ over the momentum distribution $W_\gamma^z(P_x, \mathbf{p}, \mathbf{k})$ of the γ-radiation accompanying the decay event with given $P_x, \mathbf{p}, z$. Each single decay event with a given $\mathbf{k}$ value enters into the experimental $W_{\exp}^z(P_x, \mathbf{p})$ value with its own weight, its own probability $W_\gamma^z(P_x, \mathbf{p}, \mathbf{k})d\mathbf{k}$, which is the probability of γ-radiation with a given momentum $\mathbf{k}$ accompanying β-decay with the given $P_x, \mathbf{p}$ values. Consequently, Eq. (5) is replaced by a new relation in which the experimentally observed quantity $W_{\exp}^z(P_x, \mathbf{p})$ is equated to the β-decay probability averaged with the weight $W_\gamma^z(P_x, \mathbf{p}, \mathbf{k})$, namely:

$$\begin{aligned} W_{\exp}^z(P_x, \mathbf{p}) &= \frac{\int d\mathbf{k} W_\gamma^z(P_x, \mathbf{p}, \mathbf{k}) f(\omega)[1 + zAv + z\langle B\rangle^z y(\omega, x) + avy(\omega, x)]}{\int d\mathbf{k} W_\gamma^z(P_x, \mathbf{p}, \mathbf{k})} = \\ &= \langle f\rangle^z(1 + zAv) + \langle yf\rangle^z(z\langle B\rangle^z + av), \end{aligned} \tag{8}$$

where the familiar notation of averaging is introduced:

$$\langle F\rangle^z(P_x, \mathbf{p}) = \frac{\int_0^{\Delta-\varepsilon} d\omega^2 \int_{x_1}^{x_2} dx F(P_x, \mathbf{p}, \omega, x) \int_0^{2\pi} d\phi W_\gamma^z(P_x, \mathbf{p}, \omega, x, \phi)}{\int_0^{\Delta-\varepsilon} d\omega^2 \int_{x_1}^{x_2} dx \int_0^{2\pi} d\phi W_\gamma^z(P_x, \mathbf{p}, \omega, x, \phi)}. \tag{9}$$

Here the limits x_1, x_2 emerge merely from kinematics of the process under consideration, the quantities to be averaged, $f(\omega), f(\omega)y(\omega, x)$, being independent of the azimuth ϕ of the γ-radiation (see Fig. 1).

Thus we have derived Eq. (8) to replace the former equation (5). In the absence of an immediate one-to-one correspondence between the distribution (8) involving $\langle B\rangle^z$ and the antineutrino angular distribution (1) involving B, the quantity $\langle B\rangle^z$ is seen, nevertheless, to be relevant for our goal, which is to estimate, on the average, the value of B in (1):

$$\langle B\rangle^z = z[(1 + zAv)(f_0 - \langle f\rangle^z) + y_0 f_0(av + zB_0)]/\langle yf\rangle^z - zav. \tag{10}$$

To judge with full confidence the accuracy and even the very validity of the aforementioned estimation of B in terms of $\langle B\rangle^z$, let us visualize the distributions of the quantities $f(\omega), f(\omega)y(\omega, x)$ around their mean or expectation values $\langle f\rangle, \langle fy\rangle$, that is, let us evaluate the rms deviations of $f(\omega), f(\omega)y(\omega, x)$. In short, in addition to the quantities $\langle f\rangle, \langle fy\rangle$ themselves, we must calculate the mean square deviations of $f(\omega), f(\omega)y(\omega, x)$ from their expectation values $\langle f\rangle, \langle fy\rangle$ (i.e., the variances of these quantitites):

$$\begin{aligned} \langle(\Delta f)^2\rangle^z = \langle f^2\rangle^z - (\langle f\rangle^z)^2, \quad \langle(\Delta(yf))^2\rangle^z &= \langle(yf)^2\rangle^z - (\langle yf\rangle^z)^2, \\ \langle\Delta(f{\cdot}yf)\rangle^z &= \langle f{\cdot}yf\rangle^z - \langle f\rangle^z{\cdot}\langle yf\rangle^z. \end{aligned} \tag{11}$$

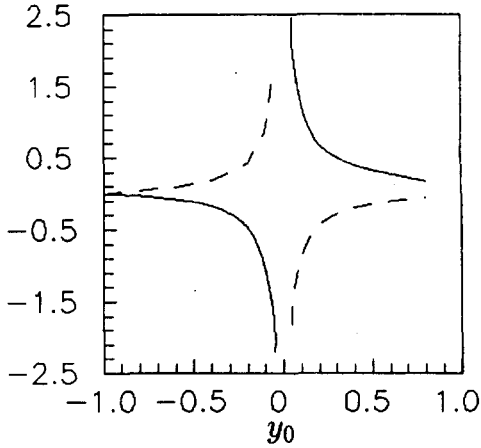

Fig.2. The y_0-dependence of the quantity $(\langle B\rangle^z - B_0)/B_0$, in %, at the value $\varepsilon = 1\,\mathrm{MeV}$. The solid line stands for $z = +$, the dashed line for $z = -$

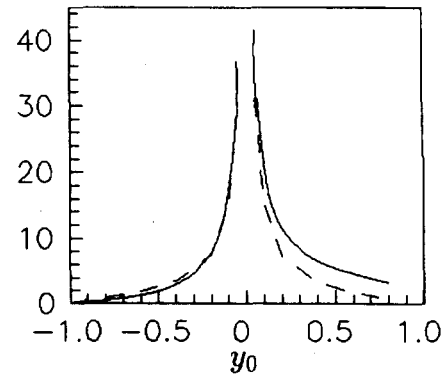

Fig.3. The same as in Fig.2, but for the quantity $\Delta B^z/\langle B\rangle^z$

Accordingly, the attainable accuracy

$$\Delta B^z \equiv \sqrt{\langle(\Delta B^z)^2\rangle} = \sqrt{\langle B^2\rangle^z - (\langle B\rangle^z)^2}$$

of the B value estimation (10) is expressed in the usual way (see, for instance, [8]) in terms of the quantities (11) and the derivatives

$$\partial\langle B\rangle^z/\partial\langle f\rangle^z\,, \quad \partial\langle B\rangle^z/\partial\langle yf\rangle^z\,.$$

Thus the ambiguities in estimating the true value of B from the expectation values $\langle B\rangle^\pm$ stem from the difference between the quantitites $\langle B\rangle^+$ and $\langle B\rangle^-$ themselves and from the emergence of an rms deviation $\Delta B^\pm$.

Upon integrating over $d\phi$ in (9), the γ-radiation distribution takes the form[9]

$$\begin{aligned}
&\omega^2 d\omega dx d\mathbf{p} dP_x \int_0^{2\pi} d\phi W_\gamma^z(P_x, \mathbf{p}, \omega, x, \phi) = \\
&= (\frac{e\tilde{G}}{2\sqrt{2}})^2 \frac{8}{(2\pi)^7} \frac{1}{4\varepsilon^2} \frac{\varepsilon_\nu}{[1-xv]^2} \frac{1}{m} \left(\frac{m}{\omega}\right)^{(1-o)} dx d\omega dP_x d\mathbf{p} \times \\
&\times\{(1-x^2)\varepsilon v[v(\varepsilon+\omega)(g_V^2+3g_A^2) + y(\omega+v^2\varepsilon)(g_V^2-g_A^2)] + \\
&+\omega^2[(g_V^2+3g_A^2) + yx(g_V^2-g_A^2)](1-vx) + \\
&+2zg_A[(1-x^2)\varepsilon v[(g_V-g_A)(v^2\varepsilon+\omega) + (g_V+g_A)vy(\varepsilon+\omega)] + \\
&+\omega^2(1-vx)[(g_V-g_A)x + (g_V+g_A)y]]\}, \quad o = \frac{2\alpha}{\pi}[\frac{1}{v} ln(\frac{\varepsilon+|\mathbf{p}|}{m}) - 1].
\end{aligned} \tag{12}$$

It should be noted that it is the presence of the quantity o in Eq. (12) that governs the true infrared ($\omega \to 0$) behaviour of $W_\gamma^z(P_x, \mathbf{p}, \omega, x, \phi)$ (see Refs. [4, 9, 10]).

It is pertinent to present the calculated quantities $(B^\pm - B_0)/B_0$, $\Delta B^\pm/\langle B\rangle^\pm$ as functions of the electron energy ε and of the quantity y_0 (2), as was done in Ref. [3].

Here the dependence on ε proves to be rather smooth, whereas the dependence on y_0, in contrast, becomes strong, as is seen in Figs. 2 and 3, which typify the results of the calculations.

The expectation value $\langle B\rangle^z$ is relevant for ascertaining the true value of B in (1) when the distributions of the values of $f(\omega), f(\omega)y(\omega,x)$ are sharp enough, that is, when, at given $P_x, \mathbf{p}$, the ratios $\Delta f/\langle f\rangle, \Delta(fy)/\langle fy\rangle$ and, hence, $\Delta B/\langle B\rangle$ turn out to be substantially smaller than (i.e., negligible in comparison with) the desired accuracy of determination of B [3]. The magnitude of the ratio $\Delta B/\langle B\rangle$ sets the bound on the precision of obtaining the value of B (1) from the processing [3] of the experimental data (4). Yet when, at certain $P_x, \mathbf{p}$, the distributions of $f(\omega), f(\omega)y(\omega,x)$ around $\langle f\rangle, \langle fy\rangle$ turn out to be so smoothed that $\Delta f/\langle f\rangle \sim 1, \Delta(fy)/\langle fy\rangle \sim 1$, and, consequently, $\Delta B/\langle B\rangle \sim 1$, there will apparently be no reason et all to estimate the quantity B (1) in terms of $\langle B\rangle^z$. In that case, the antineutrino kinematics, the antineutrino angular distribution (1), can't be reconstructed from the experimentally observed [3] distribution (4) even on the average. Of course, it is no wonder that the values in Figs. 2 and 3 increase sharply as y_0 tends to zero, $y_0 \to 0$, the physical reason for such behaviour of $\langle B\rangle, \Delta B$ being quite visible. Indeed, when $y_0 \approx 0$, that is $|\mathbf{p}| + P_x \approx 0$, the inclusion of the term $x\omega$ in $y(\omega,x)$ (7) gives rise to appreciable values of the ratios $(y-y_0)/y_0, \Delta y/\langle y\rangle$ at any ω, even a very tiny one. In this case, any γ-radiation absolutely destroys the antineutrino kinematics which would hold in the absence of electromagnetic interactions. In turn, the values of $(\langle B\rangle^{\pm} - B_0)/B_0, \Delta B^{\pm}/\langle B\rangle^{\pm}$ increase significantly and can even get arbitrary large at $|y_0| \to 0$. Of course, under such circumstances one can say nothing about the expectation (mean) values themselves. By processing all the experimental data beyond these small $|y_0|$ values, we can claim to acquire a semiquantitative estimate of B to an accuracy of a few per cent. At best, with allowance for the events with $|y_0| \approx 0.8$–1.0 only, an accuracy better than 1% is thought to be attainable in recovering the antineutrino asymmetry coefficient B.

Thus there is, alas, no justification for glossing over the effect of γ-radiation on the determination of B and touting the achievement of very high accuracy $\approx 0.4\%$ in the measurement of B, as proclaimed in [3].

---

1. J.Byrne et al., Europhys. Lett. **38**, 187 (1996).
2. P.Liaud et al., Nucl. Phys. **A612**, 53 (1997).
3. I.A.Kuznetsov et al., JETP Lett. **60**, 311 (1994); I.A.Kuznetsov et al., Phys. Rev. Lett. **75**, 794 (1995); A.P.Serebrov et al., JETP **113**, 1963 (1998).
4. E.M.Lifshitz and L.P.Pitajevsky, *Relativistic Quantum Field Theory*, part II, M.: Nauka, 1971.
5. E.D.Commins and P.H.Bucksbaum, *Weak Interactions of Leptons and Quarks*, Cambridge Univ. Press, Cambridge, 1983.
6. M.Kobajashi and T.Maskawa, Prog. Theor. Phys. **49**, 625 (1973).
7. A.P.Serebrov and N.V.Romanenko, JETP Lett. **55**, 503 (1992); B. R.Hostein and S.B.Treiman, Phys. Rev. **D16**, 2369 (1977).
8. L.D.Landau and E.M.Lifshitz, *Statistical Pysics*, M.: Gostexizdat, 1951.
9. G.G.Bunatian, Yad. Fiz. **62**, 697 (1999); /*aps1999mar*11_005 .
10. D.R.Yennie, S.C.Frautschi, and H.Suura, Ann. Phys. (N.Y.) **13**, 379 (1961).

Edited by Steve Torstveit