ON THE "SUPERCOLLIMATION" OF X-RAY BEAMS IN ROUGH INTERFACIAL CHANNELS

T.A.Bobrova, L.I.Ognev[1)]

Nuclear Fusion Institute, Russian Research Center "Kurchatov Institute"

123182 Moscow, Russia

Submitted 15 March 1999

Resubmitted 15 April 1999

The transmission of x-rays through rough submicron narrow channels is investigated by numerical simulation with diffraction and decay of coherence taken into account. It is found that transmission is strongly increased for directions within the diffraction limit λ/d (d is the channel width). For larger angles strong roughness scattering results in rapid decay of coherence and absorption of the x-ray beams. When the coherent part is a significant portion of the transmitted beam, its divergence is also within the diffraction limit, which can be an order of magnitude smaller than the Fresnel angle of total external reflection. The effects are explained with the statistical theory of x-ray scattering in a rough transitional layer. Such "supercollimation" can be used for fine angular discrimination of x radiation and for the production of very narrow diffraction-quality x-ray beams.

PACS: 29.30.Kv

Monitoring of x-ray beams by capture into a narrow dielectric channel is used in waveguide x-ray laser physics [1], in the production of thin x-ray probe beams [2], and other applications based on the total external reflection effect. The minimum angle of divergence of the beam in this case is limited by the Fresnel angle ϑ_F, and roughness scattering can only degrade this parameter. In this letter we consider the role of diffraction, which can be important for narrow beams, especially when the roughness is high. Scattering from surfaces with high roughness requires a special approach because small-perturbation methods fail [3]. X-ray scattering at rough surfaces is usually investigated within the well-known Andronov – Leontovich approach [4], but for very small angles of incidence the "parabolic equation" model for slowly varying scalar amplitudes $A(x,z)$ of the electric field vector should be used. Within this model, scattering and absorption do not disappear in the small grazing angle limit that results from the Andronov – Leontovich approach [4]. In this case large-angle scattering is neglected, so

$$\partial^2 A(x,z)/\partial z^2 \ll k \cdot \partial A(x,z)/\partial z,$$

and because the beam is narrow

$$\partial^2 A(x,z)/\partial z^2 \ll \partial^2 A(x,z)/\partial x^2,$$

where z and x are the coordinates along and across the channel. Here consideration will be restricted to two-dimensional channels (gaps), although the same approach can be applied to capillaries. The assumption results in the "parabolic equation" of quasioptics:

$$2ik\frac{\partial A}{\partial z} = \Delta_\perp A + k^2 \frac{\varepsilon - \varepsilon_0}{\varepsilon_0} A \quad (1)$$

1) e-mail: ognev@nfi.kiae.su

$$A(x, z = 0) = A_0(x),$$

where $k = \sqrt{\varepsilon_0}(\omega/c)$. (Here ε_0 is the dielectric permittivity of air, and ε_1 is the dielectric permittivity of glass.) The evolution of the channeled x-ray beam was calculated by direct integration of the "parabolic" equation [5]. The dielectric permittivity on the rough boundary, with the random shape $x = \xi(z)$, was represented as $\varepsilon(x, z) = \varepsilon_1 + (\varepsilon_0 - \varepsilon_1)H(x - \xi(z))$, where $H(x)$ is a step function. The distribution of roughness heights is assumed to be normal. It is known from the results of Ref.[4] that at grazing incidence the effect of scattering is very small. Therefore special surfaces are needed to observe scattering effects in the gap interface at a reasonable distance. In the calculations we used roughness amplitudes up to 400 Å. The results of direct simulation of scattering with the model rough surface by integration of Eq.(1), calculated for an x-ray energy $E = 10$ keV, channel width $d = 0.5$ μm, $\sigma = 400$ Å, and roughness correlation length $z_{\text{corr}} = 5$ μm and averaged over 40 realizations, are shown in Fig.1 as the incoherent part r_{inc} normalized to the initial value of the total intensity of the beam r_{tot}, where

$$r_i = \int_{-\infty}^{\infty} I_i(x)dx \Big/ \int_{-d/2}^{d/2} I_0(x)dx.$$

Initial angles of incidence were $\vartheta = 0$, $3 \cdot 10^{-4}$, and $6 \cdot 10^{-4}$ rad.

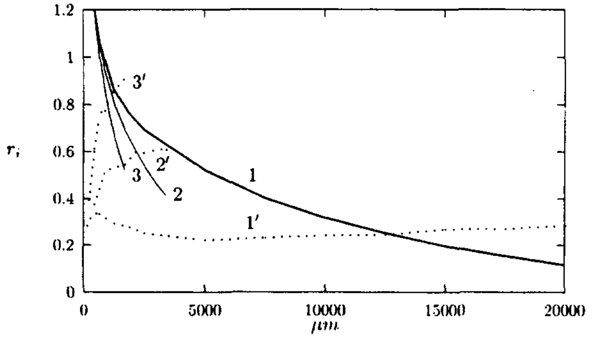

Fig.1. Evolution of the total integral normalized intensity of the beam r_{tot} and normalized incoherent part $r_{part} = r_{inc}/r_{tot}$ for different angles of incidence ϑ. $\vartheta = 0$, r_{tot} (curve *1*), r_{part} (curve *1′*); $\vartheta_F/10$ (curves *2* and *2′*); $\vartheta_F/5$ (curve *3* and *3′*)

Angular spectra of the coherent and incoherent parts of the wave amplitude averaged over 40 realizations are shown in Fig.2.

Statistical averaging of Eq.(1) gives for the last term the expression (the angle brackets correspond to averaging)

$$k^2\chi(x, z)\langle A(x, z)\rangle + k^2\langle \delta\varepsilon' \cdot A(x, z)\rangle,$$

where

$$\chi(x, z) = (\langle \varepsilon(x)\rangle - \varepsilon_0)/\varepsilon_0, \qquad \delta\varepsilon'(x, z) = (\varepsilon(x, z) - \langle \varepsilon(x)\rangle)/\varepsilon_0.$$

The first term in the sum corresponds to absorption of the x-ray beam and the last term to decay of coherence due to incoherent scattering. Assuming that the variation of $A(x, z)$ over the roughness correlation length z_{corr} is small and that $\langle \delta\varepsilon(x, z)\rangle = 0$, we can use the statistical method of Tatarsky (see [6]), which is valid for δ-correlated fluctuating media. Thus

$$\langle \delta\varepsilon'(x, z) \cdot A(x, z)\rangle = \langle A(x, z)\rangle(-ik/4) \int_{-\infty}^{\infty} \langle \delta\varepsilon'(x, z)\delta\varepsilon'(x, z')\rangle dz'.$$

The same generalization of the method to include stratified media has been used in the case of electron channeling in single crystals [7]. Thus the coefficient multiplying $\langle A(x,z)\rangle$ has the meaning of a "scattering potential" that causes decay of coherence and can be written as

$$W(x) = (-ik/4)\int_{-\infty}^{\infty} \langle \delta\varepsilon'(x,z)\delta\varepsilon'(x,z')\rangle dz' =$$

$$= -\frac{k}{4}\frac{(\varepsilon_0-\varepsilon_1)^2}{\pi(\varepsilon_0)^2}\int_{-\infty}^{+\infty} dz' \int_{-\infty}^{x/\sigma} \exp(-\xi^2)d\xi \int_{x/\sigma}^{\frac{x/\sigma - R(z')\xi}{(1-R^2(z'))^{1/2}}} \exp(-\eta^2)d\eta, \qquad (2)$$

where $R(z)$ is the autocorrelation coefficient, and σ is the variance of the $\xi(z)$ distribution. Thus the coherent part of the amplitude $A(x,z)$ can be calculated from the statistically averaged equation

$$\begin{aligned} &2ik\partial\langle A(x,z)\rangle/\partial z - \Delta_\perp\langle A(x,z)\rangle - \\ &- k^2\chi(x)\langle A(x,z)\rangle - ik^2W(x)\langle A(x,z)\rangle = 0, \end{aligned} \qquad (3)$$

and

$$\langle A(x, z=0)\rangle = A_0(x).$$

It can be shown that the value of $W(x)$ in the middle of the transitional layer ($x = 0$) does not depend on σ and is nearly proportional to the roughness correlation length z_{corr}. The same is true for $x = d$. This results in a weak dependence of the = phase shifts of the incoherently scattered wave on the value of σ, unlike the case for the higher angles of incidence of the beam which have usually been investigated in experiments.

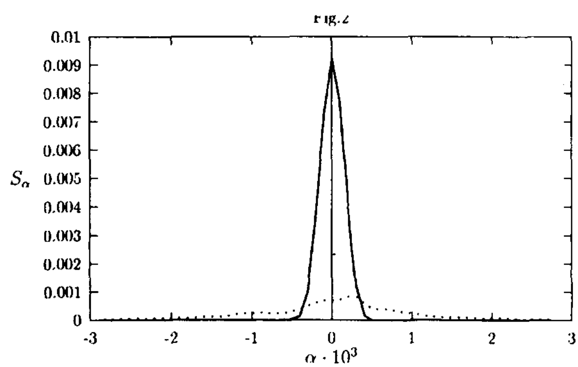

Fig.2. Angular spectra of coherent (solid curve) and incoherent (dots) components of the x-ray beam after transmission through a 2 cm rough gap

The inner double integrals in Eq.(2) can be simplified for small R, and an approximation for $W(x)$ can be written as

$$W(x) \approx -\frac{k}{4}\frac{(\varepsilon_0-\varepsilon_1)^2}{\pi(\varepsilon_0)^2}\int_{-\infty}^{\infty} dz' \int_{-\infty}^{0} \exp(-\xi^2)d\xi \int_{0}^{\frac{-R(z')\xi}{(1-R^2(z'))^{1/2}}} \exp(-\eta^2)d\eta$$

$$\cdot \exp(-\frac{x^2}{\sigma^2}) \qquad (4)$$

with a clear dependence on the vertical coordinate x.

The results for the coherent part of the beam obtained by averaging the solution of equation (1) and integrating Eq.(3) with approximation (4) are shown of Fig.3.

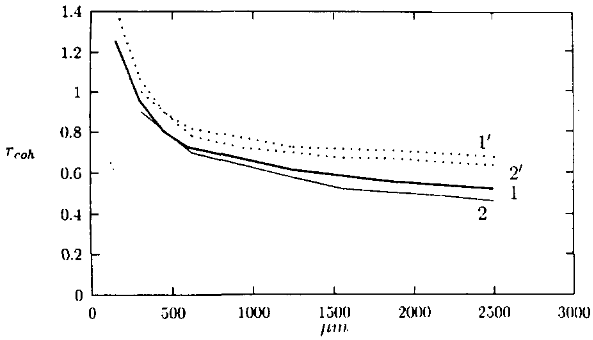

Fig.3. Decay of the coherent part of the radiation, r_{coh}, calculated with statistical averaging (solid curves) and in the transitional layer model (2) with the assumption (4) (dashed curves), $\vartheta = 0$. $z_{corr} = 5$ μm (curves *1* and *1'*); $z_{corr} = 2$ μm (curves *2* and *2'*); $\sigma = 400$ Å

The decay of coherence can be described with attenuation coefficients β_l. The attenuation coefficients can be found as overlap integrals,

$$\beta_l = -\frac{k}{2}\int \varphi_l{}^*(x)[\mathrm{Im}(\chi(x)) + W(x)]\varphi_l(x)dx,$$

where the eigenfunctions $\varphi_j(x)$ are solutions of the equations

$$\Delta_\perp \varphi_j(x) = k[2k_{jz} - k\mathrm{Re}(\chi(x))]\varphi_j(x).$$

It can be shown for lower channeled modes that the incoherent scattering attenuation coefficient is proportional to σ (see the discussion above about the dependence of $W(x)$ on σ):

$$\beta_{\text{scatter}} \sim k^2(\varepsilon_0 - \varepsilon_1)^2\sigma \int_{-\infty}^{\infty} dz' \int_{-\infty}^{0} \exp(-\xi^2/2)d\xi \int_0^{\frac{-R(z')\xi}{(1-R^2(z'))^{1/2}}} \exp(-\eta/2^2)d\eta.$$

The results for the attenuation coefficients [$\mu\mathrm{m}^{-1}$] for various wave modes are shown in Fig.4 separately for absorption and incoherent scattering within a 0.5 μm quartz glass channel. The angles of incidence ϑ_N corresponding to the center of the N mode ($N \geq 1$) are $\vartheta_N = \lambda/2d \cdot (N + 1/2)$.

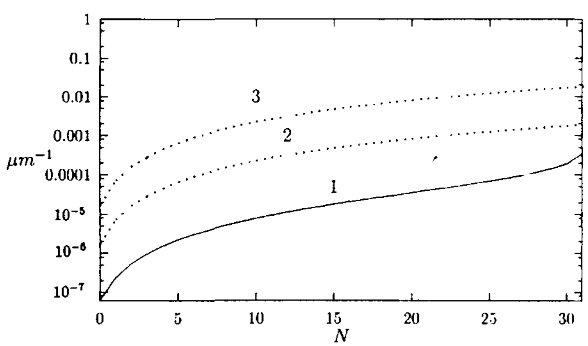

Fig.4. Dependence of the attenuation coefficients for incoherent scattering $\beta_{scatter}$ (dashed lines *2*, *3*) and absorption β_{absorp} (solid line *1*) on the mode number N; $\sigma = 100$ Å, $z_{corr} = 2$ μm (curve *2*); $\sigma = 400$ Å, $z_{corr} = 5$ μm (curve *3*). The center of mode N corresponds to $\vartheta_N \approx 1.24 \cdot 10^{-4} \cdot N$

It is seen from Fig.4 that the effect of incoherent scattering for given correlation length is an order of magnitude higher than the effect of real absorption. Coherence effects for modes number "1" and higher will decay after several millimeters of channel length.

And decreasing of correlation length will result in a nearly proportional decrease of the incoherent scattering. The rate of coherence decay of the "0" mode agrees well with the results shown in Fig.3.

The "supercollimation effect" can be measured as the strong sharpening of the angular dependence of the transmission of x-rays through an interface, to below the Fresnel angle of total external reflection. Such "supercollimation" can be used for fine angular discrimination of x radiation and for the production of very narrow diffraction-quality x-ray beams for use as probes and also of high-quality soft x-ray beams for x-ray laser amplifiers.

---

1. S.V.Kukhlevsky, G.Lubkovics, K.Negrea, and L.Kozma, Pure Appl. Opt. **6**, 97 (1999).
2. S.A.Hoffman, D.J.Thiel, and D.H.Bilderback, Nucl. Instrum. Methods Phys. Res. **A347**, 384 (1994).
3. S.S.Fanchenko and A.A.Nefedov, phys. status solidi **B212/1**, R3 (1999).
4. A.V.Vinogradov, N.N.Zorev, I.V.Kozhevnikov, and I.G.Yakushkin, ZhETF **89**, 2124 (1985).
5. T.A.Bobrova and L.I.Ognev, phys. status solidi **B203/2**, R11 (1997).
6. V.Holý and K.T.Gabrielyan, phys. status solidi **B140**, 39 (1987).
7. L.I.Ognev, Radiation Effects and Defects in Solids **25**, 81 (1993).

Edited by Steve Torstveit