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THREE NONDISSIPATIVE FORCES ON A MOVING VORTEX LINE IN SUPERFLUIDS AND SUPERCONDUCTORS

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In the hydrodynamic limit the nondissipative force $\mathbf{F}_{nd}$, which acts on the vortex in superconductors and Fermi superfluids, contains three different contributions of the topological origin. These are (i) the Magnus force, (ii) the Iordanskii force, which origin is analogous to the Aharonov-Bohm effect for the spinning cosmic strings, and (iii) the force resulting from the spectral flow of fermion zero modes in the vortex core, which leads to the production of the momentum of quasiparticles when the vortex moves with respect to the normal component. The latter force leads to the anomaly in the thermodynamics of the moving vortices, and the possible relation of this anomaly to the Unruh effect is discussed.

1. General expression for the nondissipative forces

Recent developments in high-T_c superconductivity and in the physics of superfluid ^{3}He have renewed interest in the dynamics of vortices in these pair-correlated Fermi systems [1–5]. Theoretical investigation of the vortex dynamics was started 3 decades ago (see classic papers [6–9]). In the linear regime this dynamics was finally calculated by Kopnin and coauthors [10, 1, 11]. However the correct interpretation of the nondissipative forces acting on the moving vortex line of the nondissipative forces was missing.

Here we consider the hydrodynamical limit case $\omega_0\tau \ll 1$, where ω_0 is the distance between the levels of the fermions localized within the vortex core [12] and τ the lifetime of the fermions on these levels. In this case the dissipative force can be neglected and we also neglect the pinning forces. Our statement is that in this limit the nondissipative force $\mathbf{F}_{nd}$, which acts on the vortex, contains three different contributions. $\mathbf{F}_{nd}$ is expressed in terms of 3 different combinations of the relevant velocities, the velocity of the vortex line $\mathbf{v}_L$, the velocity $\mathbf{v}_s$ of the superfluid vacuum (or superfluid velocity) and the velocity $\mathbf{v}_n$ of the normal component of the liquid (the velocity of the heat bath):

$$\mathbf{F}_{nd} = \mathbf{F}_{\rm Magnus} + \mathbf{F}_{\rm Iordanskii} + \mathbf{F}_{\rm spectral\ flow} \ , \quad \mathbf{F}_{\rm Magnus} = \vec{\kappa} \times \rho(\mathbf{v}_L - \mathbf{v}_s) \ ,$$

$$\mathbf{F}_{\rm Iordanskii} = \vec{\kappa} \times \breve{\rho}_n(T)(\mathbf{v}_s - \mathbf{v}_n) \ , \quad \mathbf{F}_{\rm spectral\ flow} = \vec{\kappa} \times C_0(\mathbf{v}_n - \mathbf{v}_L) \ . \qquad (1)$$

Each of the three forces is of the topological origin.

(i) According to the Landau picture of the superfluid liquid its motion consists of the motion of the superfluid vacuum (with the total mass density ρ and the superfluid velocity $\mathbf{v}_s$) and the dynamics of the elementary excitations. The Magnus force in Eq.(1) acts on the vortex if it moves with respect to the superfluid vacuum. Here $\vec{\kappa}$ is the circulation vector: for the Fermi (Bose) superfluids $\kappa = \pi N\hbar/m$ ($\kappa = 2\pi N\hbar/m$), where N is the vortex winding number, m is the bare mass of the fermion (boson). The Magnus force comes from the flux of the linear momentum from the vortex to infinity. The topological origin of this force was discussed in Refs.[3, 4].

(ii) The Iordanskii force [11, 9] results from the elementary excitations outside the vortex core: the vortex line produces for them the Aharonov-Bohm potential. This force can be obtained as a sum of the forces acting on the individual particles according to the equation $\partial_t \mathbf{p} = (\vec{\nabla} \times \mathbf{v}_s) \times \mathbf{p}$, where $\mathbf{p}$ is the quasiparticle momentum and the vorticity $\vec{\nabla} \times \mathbf{v}_s = \vec{\kappa}\delta_2(\mathbf{r})$ is concentrated in a thin tube (vortex core). The Iordanskii force is thus

$$-\sum_{\mathbf{p}} \partial_t \mathbf{p} = -\vec{\kappa} \times \int d^3p/(4\pi^3) \ f[E_{\mathbf{p}} + \mathbf{p}\cdot(\mathbf{v}_s - \mathbf{v}_n)] = \vec{\kappa} \times \breve{\rho}_n(T)(\mathbf{v}_n - \mathbf{v}_s) \ .$$

Here f is Fermi or Bose function depending on the type of the elementary excitations, which is Doppler shifted due to the counterflow $\mathbf{v}_n - \mathbf{v}_s$; and $\breve{\rho}_n(T)$ is the density of the normal component, which can be an anisotropic tensor. The Iordanskii force is the only nondissipative force in Eq.(1), which depends on temperature T. In Section 2 we discuss the analogy of the Iordanskii effect with the Aharonov-Bohm effect for spinning cosmic strings [13].

(iii) The third term exists only in the fermionic systems. It is the result of the momentum exchange between the fermions localized in the vortex core and the fermions in the heat bath. It is described by the spectral flow of the fermion zero modes within the vortex core and is related to the Callan-Harvey mechanism of the anomaly cancellation applied to the fermion zero modes on vortices[2]. In Ref.[2] the parameter C_0 was obtained in the zero temperature limit, where for the spherical Fermi surface it is expressed in terms of the Fermi momentum p_F: $C_0 = mp_F^3/3\pi^2$. It was found recently that the spectral flow is unaffected by T, since the temperature does not change the topology of the spectrum of fermion zero modes, and thus the parameter C_0 equals its zero temperature value [14]. This finding allowed to accomplish the construction of the general expression for the nondissipative force $\mathbf{F}_{nd}$ in Eq.(1) in the limit $\omega_0\tau \ll 1$, in which the spectral flow of the momentum is not suppressed.

In Section 3 we consider the anomaly in the thermodynamics of the mixed state (state with vortices) caused by the spectral flow contribution to the nondissipative force, which possibly leads to the effective temperature of the moving vortex and to the analog of the Hawking-Unruh radiation [15, 16] (Section 4).

If one adds the dissipative force to the Eq.(1) one obtains the conventional expression for the balance of forces acting on the vortex

$$\rho_s(\mathbf{v}_s - \mathbf{v}_L) \times \vec{\kappa} + D\kappa(\mathbf{v}_n - \mathbf{v}_L) - D'(\mathbf{v}_n - \mathbf{v}_L) \times \vec{\kappa} = 0 \quad . \tag{2}$$

Here $\rho_s = \rho - \rho_n$; the parameters D and D' were calculated by Kopnin, the factor D in the dissipative force is small in the limit $\omega_0\tau \ll 1$ [1]. From the comparison with the Eq.(1) it follows that the parameter $D' = C_0 - \rho_n$. This is slightly different from Kopnin's result $D' = \rho_s$, which follows if one neglects the difference between the anomaly parameter C_0 and the mass density ρ. In conventional superconductors and in superfluid ^{3}He the difference between C_0 and ρ is very small, of order$(\Delta/E_F)^2$, where Δ is the gap in the quasiparticle spectrum and E_F the Fermi energy. However one can imagine the systems where this difference is sizable.

2. Aharonov-Bohm effect and analog of the spinning string

To clarify the analogy between the Iordanskii force and Aharonov-Bohm effect, let us consider the simplest cases of phonons propagating in the velocity field of the quantized vortex in the Bose superfluid ^{4}He and fermions propagating in the velocity field of the quantized vortex in the Fermi superfluid ^{3}He-A. According to Unruh [17] the dynamics of the phonons in the presence of the velocity field is the same as the dynamics of photons in the gravity field. For the velocity field of quantized vortex the phonons obey the equation of motion of the scalar wave in the metric

$$ds^2 = (c^2 - v^2(r))(dt + \frac{\kappa}{2\pi(c^2 - v^2(r))}d\phi)^2 - dr^2 - dz^2 - \frac{c^2}{c^2 - v^2(r)}r^2 d\phi^2 \quad . \tag{3}$$

Here c is the sound velocity, $\mathbf{v} = \hat{\phi}\kappa/2\pi r$ is the superfluid velocity around the vortex, and we use the cylindrical system of coordinates with the axis z along the vortex line. The same metric takes place for the gapless Bogoliubov fermions propagating in the field of the axisymmetric phase vortex in the A-phase of superfluid ^{3}He (see Eq.(4.5) in Ref.[18]). In this case the "velocity of light" is anisotropic and its transverse component is $c = \Delta/p_F$ where Δ is the amplitude of the gap.

Far from the vortex, where $v(r)$ is small and can be neglected, this metric corresponds to that of the so called rotating cosmic string. The spinning cosmic string (see the latest references [19, 20]) is such a string which has the rotational angular momentum. The metric in Eq.(3) corresponds to the string with the angular momentum $J = \kappa/8\pi G$ per unit lehgth and with zero mass.

The effect peculiar for the spinning string is the gravitational Aharonov-Bohm topological effect [13]. Though the metric outside the string is flat, there is the time difference for the particles propagating around the spinning string in the opposite directions. For the vortex (at large distances from the core) this time delay approaches

$$2\tau = 2\kappa/c^2 \quad . \tag{4}$$

This asymmetry between the particles moving on different sides of the vortex is just the origin of the Iordanskii force acting on the vortex in the presence of the net momentum of the quasiparticles: $2\int d^3p/(2\pi)^3\ \mathbf{p} f(\mathbf{p})$.

3. Anomaly in the low temperature hydrodynamics

The parameter C_0 in Eq.(1) results in the anomaly in the hydrodynamics of rotating liquid, which becomes pronounced at low temperature, $T \rightarrow 0$ (ie $T \ll T_c$). Actually we consider the intermediate asymptote at which the system is still in the hydrodynamical regime and the condition $\omega_0\tau \ll 1$ is satisfied. This intermediate limit of $T \ll T_c$ can be obtained for example in ^{3}He-A [1], where ω_0 is extremely small for continuous vortices, and also in superconductors where the lifetime τ is defined by impurities.

Let us consider the state in which the normal and superfluid components rotate with different angular velocities:

$$\mathbf{v}_s = \vec{\Omega}_s \times \mathbf{r} \ , \quad \mathbf{v}_n = \vec{\Omega}_n \times \mathbf{r} \ . \tag{5}$$

For the superfluid component the uniform (in average) rotation is simulated by the system of identical rectilinear vortices with density $n = (2/\kappa)\Omega_s$. These vortices are rotated with the velocity $\mathbf{v}_L$, found from the force balance Eq.(2). The equilibrium situation with $\Omega_n \neq \Omega_s$ is possible because we neglect the dissipation ($D = 0$) and thus the friction between the normal and the superfluid rotations is absent.

The immediate result of the nonzero parameter C_0 is that in addition to the radial gradient of pressure:

$$P(r) - P(0) = \frac{1}{2}\rho_s v_s^2(r) + \frac{1}{2}\rho_n v_n^2(r) \ , \tag{6}$$

from the hydrodynamic equations for rotating superfluids (see Rev. [21]) it also follows the gradient of the temperature

$$-S\partial_r T = \partial_r\{v_s(\rho v_L - (\rho_n v_n + \rho_s v_s))\} \quad , \tag{7}$$

where S is the entropy density of the rotating liquid. This is the so called thermorotation effect [21], and it takes place when the vortex velocity deviates from the center-of-mass velocity $(\rho_n v_n + \rho_s v_s)/\rho$ (see also discussion in [22] where the variant of this effect was observed in the rotating ^{3}He-B and was used for the experimental estimation of the parameters D and D'). As follows from Eqs.(2) and (7), in the absence of D the thermorotation effect comes solely from the spectral flow

$$-S\partial_r T = C_0\partial_r(v_s(v_L - v_n)) = 2C_0\Omega_s(v_L - v_n)) = \hbar n\frac{p_F^3}{3\pi}(v_L - v_n) \quad . \tag{8}$$

The rhs of Eq.(8) is temperature independent and thus is valid in a proper $T \rightarrow 0$ limit. This could mean that even in this limit the moving (rotating) vortices have a nonzero entropy and a nonzero intrinsic temperature. The effect is governed by the spectral flow of fermions from the vortex into the bulk liquid. Thus T and S are related to the radiation of the fermions from the vortex, if the vortex moves relatively to the heat bath, ie $\mathbf{v}_L \neq \mathbf{v}_n$. This reminds the Unruh effect in quantum field theory[16]: the object moving with constant acceleration a radiates the particles as a black body with temperature $T_{\rm Unruh} = \hbar a/2\pi c$. This results from the apparent change of the vacuum state, as measured by an accelerating particle detector. The main difference is that in superfluids the vacuum state can change even if the object moves with constant velocity: this leads to the radiation of particles if, say, the critical velocity is exceeded.

If one considers this analogy seriously, the moving vortex should have an effective temperature, which defines the radiation of the fermions by the moving vortex. This temperature corresponds to the average energy of the fermion zero mode in the vortex as measured in the frame of the heat bath:

$$T_{\rm eff} = \overline{|(\mathbf{v}_L - \mathbf{v}_n) \cdot \mathbf{p}|} = \frac{2}{\pi}|\mathbf{v}_L - \mathbf{v}_n|p_F \int_0^{\pi/2} d\phi \cos\phi = \frac{2}{\pi}|\mathbf{v}_L - \mathbf{v}_n|p_F \ . \qquad (9)$$

4. Tunneling of particles and effective temperature

Let us consider the regime when $\omega_0\tau$ is large, and the discrete nature of the generalized angular momentum Q, which describes the fermions in the vortex core, becomes important. In this case the spectral flow is suppressed and occurs only due to the tunneling of the particles between the discrete levels, which is caused by the vortex motion. The exponential suppression of the forces acting on the vortex was recently discussed in [5], which corresponds to the effective temperature in Eq.(9), however without the factor $2/\pi$. We show now that just the factor $2/\pi$ was missing in [5] and the tunneling rate between the neighbouring levels is defined by $\exp -(\omega_0/T_{\rm eff})$.

The Hamiltonian, which describes the problem at low T is related only to the low-energy anomalous branch of spectrum, ie to the fermion zero mode:

$$\mathcal{H} = \mathrm{Q}\omega_0 + \omega_0 t(\mathbf{v}_L \times \mathbf{p}) \cdot \hat{z} \ . \qquad (10)$$

Here Q is the operator of the generalized angular momentum of rotations about the vortex axis z [12] and p is the operator of the linear momentum of the particle. For simplicity we consider the 2-dimensional case, ie ω_0 does not depend on p_z. The second term comes from the change of the angular momentum if it is observed in the heat bath frame, $\mathrm{Q} \rightarrow \mathrm{Q} + \mathbf{r}(t) \times \mathbf{p}$, and $\mathbf{r}(t) = (\mathbf{v}_L - \mathbf{v}_n)t$ (further we choose $\mathbf{v}_n = 0$). The operators Q, and transverse linear momentum $\mathbf{p} = \mathbf{p}_\perp$ do not commute:

$$[\mathrm{Q}, \mathbf{p}] = i\hat{z} \times \mathbf{p} \ . \qquad (11)$$

In terms of the matrix elements between the states with different Q the Hamiltonian is:

$$\mathcal{H}_{QQ'} = Q\delta_{QQ'}\omega_0 + \omega_0 t(\hat{z} \times \mathbf{v}_L) < Q|\mathbf{p}|Q' > \ ,$$

$$< Q|\mathbf{p}|Q' >= \frac{1}{2}p_F((\hat{y} + i\hat{x})\delta_{Q,Q'+1} + (\hat{y} - i\hat{x})\delta_{Q,Q'-1}) \ , \qquad (12)$$

where we used the condition $p^2 = p_F^2$.

If the vortex velocity v_L is small compared to ω_0/p_F, the semiclassical approach becomes valid. In this case the level flow is determined by the exponentially small transition probability between two neighbouring levels. Let us find this exponent. The Hamiltonian for the two-level system, Q and $Q+1$, is

$$\mathcal{H} = (Q + \frac{1}{2})\omega_0 + \frac{1}{2}\omega_0 \begin{pmatrix} 1 & v_L t p_F \\ v_L t p_F & -1 \end{pmatrix} \ . \qquad (13)$$

The square of the energy counted from the position in the middle between these two states is

$$(E-(Q+\frac{1}{2})\omega_0)^2=\frac{1}{4}\omega_0^2(1+(v_L t p_F^2))\ . \qquad (14)$$

The trajectory $t = i\tau$ on the imaginary time axis, which connects two states, gives the following transition probability between the states in the exponential approximation:

$$w \propto \exp -2\mathrm{Im}S\ ,\quad \mathrm{Im}S = 2\int_0^{\tau_0} d\tau \frac{1}{2}\omega_0\sqrt{1-\frac{\tau^2}{\tau_0^2}}\ ,\quad \tau_0 = \frac{1}{v_L p_F}\ . \qquad (15)$$

This gives

$$w \propto \exp -\frac{\pi}{2}\frac{\omega_0}{v_L p_F}\ . \qquad (16)$$

Thus the tunneling rate is equivalent to the thermal distribution of quasiparticles on the levels of the anomalous branch of the spectrum with the effective temperature in Eq.(9).

5. Discussion

We found three topologically different contributions to the nondissipative force acting on the moving vortex. The third contribution, which comes from the spectral flow, is anomalous: it reflects the Callan-Harvey effect of anomaly cancellation and leads to the low temperature anomaly in the vortex thermodynamics. This anomaly can be described by some effective temperature of the vortex cores in Eq.(9), which defines the radiation of the fernions from the moving vortex. If the Eq.(9) is taken seriously, then from Eq.(8) it follows that there is an entropy density of the system of vortices related to the spectral flow

$$S(r) = -m_3\frac{p_F^2}{3\pi}\Omega_s r = -\frac{1}{6}p_F^2 n r\ . \qquad (17)$$

This corresponds to the entropy per one vortex $\frac{1}{6}p_F^2 r L$, where L is the length of the vortex, or $\frac{1}{6}p_F^2 A$, where A is the area of the surface between the vortex line and the axis of rotation.

The effect is possibly somewhat similar to the temperature and entropy of the black holes. For the black hole[15] the entropy is related to the process of the crossing the surface of the hole's horizon and equlas the area A of the horizon multiplied by some fundamental constant (for the black hole this is the square of Planck momentum) $S=(1/4)Ap_{Planck}^2$ [15]. In our case the Planck momentum is substituted by the Fermi momentum, and also the factor 1/6 is different from the black hole value 1/4, which can reflect the difference in the case of geometry.

Possible interpretaion of the area law for the vortex entropy can be related to the multivaluedness of the condensate phase around the vortex which according to Eq.(3) produces the Aharonov-Bohm jump in the phase of the quasiparticle wave function on some surface bounded by the vortex loop.

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GENERATION OF RING DOMAINS AND FORMATION OF DYNAMIC GRATING OF THEM IN FERRITE-GARNET FILM

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A formation of a dynamic grating of ring domains as a result of a functioning of the local domain source in an alternating magnetic field has been observed in the ferrite-garnet film. Dynamic characteristics of the domain source and quantitative parameters of the ring domains grating have been determined.

It is known [1] that in the ferrite-garnet (FG) films with perpendicular anisotropy a self-organization of a system of magnetic domains takes place under the influence of an external magnetic field. Stable regular dynamic domain structures (DDS) of different configuration are formed in a region of an anger state (AS). At the same time a self-generation of the periodical processes of an appearance/disappearance of the DDS with a certain geometry takes place.