

П И С Ь М А
В ЖУРНАЛ ЭКСПЕРИМЕНТАЛЬНОЙ
И ТЕОРЕТИЧЕСКОЙ ФИЗИКИ

ОСНОВАН В 1965 ГОДУ
ВЫХОДИТ 24 РАЗА В ГОД

ТОМ 69, ВЫПУСК 12
25 ИЮНЯ, 1999

Pis'ma v ZhETF, vol.69, iss.12, pp.839 - 844

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CHERN-SIMONS CORRELATIONS ON (2+1)D LATTICE

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Submitted 1 April 1999

Resubmitted 11 May 1999

We have computed the contribution of zero modes to the value of the number of particles in the model of discrete $(2 + 1)$ -dimensional nonlinear Schrödinger equation. It is shown for the first time that in the region of small values of the CS coefficient k there exists a universal attraction between field configurations. For $k = 2$ this phenomenon may be a dynamic origin of the semion pairing in high temperature superconducting state of planar systems. The preliminary results of this paper were presented in the proceedings of the XI Intern. Conf. "Problems of Quantum Field Theory" (Dubna, 13-17 July 1998).

PACS: 11.15.Kc, 47.20.Ky, 47.27.Ak, 52.35.Ra

The cooperative behavior based on universal topological features of planar systems is the subject of a number of papers [1, 2]. The topological properties of $(2+1)$ -dimensional (D) systems are described by a Chern-Simons (CS) term in the Lagrangian of the model. This term displays the phenomenon of the chiral invariance violation in such systems. The magnetic CS field leads usually [3–8] to the effective repulsion between field configurations. This property was described in detail in Refs. [9, 10] where the structure of zero modes in continuous [9] and spacial discrete [10] models of $(2 + 1)D$ nonlinear Schrödinger equation was studied. The competition of the basic nonlinearity corresponding to attraction, diffraction and additional nonlinearity, describing the repulsion caused by CS interaction, resulted in the increase of the critical value of the number of particles N in the region of small values of coefficient k . The increase of N represents the existence of additional repulsion.

We argue in this paper that on the $(2 + 1)D$ lattice there is universal attraction due to CS correlations between field configurations and show the conditions when this phenomenon takes place. We shall show that consideration of complete contribution of the

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statistical CS fields in the form of the holonomies causes the additional attraction (to the bare one) at small numbers of links k . The necessary condition for that is naturally the existence of bare attraction between field configurations. This result is the first indication that there exists the universal attraction due to CS correlation. The comparison with the results of our previous papers [9, 10] shows that the condition for obtaining the attraction is the inclusion in the consideration the Wilson and Polyakov exponents on the equal foot. In other words, we consider the model which takes into account the conditions of the compactness for the temporary component of the gauge potential as well as for its spatial component. The gauge invariance requirements lead immediately that discrete evolution should be considered (for detail see [10]). We want to emphasize that the gauged discrete (2+1)D nonlinear Schrödinger equation gives us a convenient tool to display this (2+1)D system phenomenon of additional attraction which has a general and universal character.

The equation of motion in the model of the gauged discrete (2+1)D nonlinear Schrödinger equation has the form

$$(\hat{t}_x + \hat{t}_y + \text{h.c.} - 4)\rho_{m,n} = -2C\rho_{m,n}^3 - \rho_{m,n} \sin(w_{m,n} - 1), \quad (1)$$

$$\hat{t}_x \rho_{m,n} \equiv e^{iA_{\hat{m},n}} \rho_{m+1,n}.$$

Here \hat{t}_x is the operator of so-called magnetic translations. The parameter $C = g|k|$ in Eq.(1) contains the coupling constant g of the classical nonlinear Schrödinger equation and CS coefficient k . Besides the consideration of the model on 2D lattice [10] we included the discrete time $t \in \mathbb{Z}$ in the description. This leads for the stationary states $\Psi(m, n, t) = \rho_{m,n} e^{it}$ to the existence of nonlinearity expressed by the sine function in Eq.(1). The multiconnection of the 2D manifold has been taken into account by the gauge field $A^\mu(m, n) = (w_{m,n}, A_{\hat{m},n}, A_{m,\hat{n}})$ where

$$w_{m,n} = \sum_{m',n'} [(\Delta_2 G(m-m', n-n'))(\rho_{m',n'}^2 + \rho_{m',n'+1}^2)A_{m',\hat{n}'} - (\Delta_1 G(m-m', n-n'))(\rho_{m',n'}^2 + \rho_{m'+1,n'}^2)A_{\hat{m}',n'}] \quad (2)$$

is the temporal component of CS potential and

$$A_{\hat{m},n} = \sum_{m',n'} \Delta_2 G(m-m', n-n') \rho_{m',n'}^2 \quad (3)$$

is the x -component of the vector potential. The notation $A_{\hat{m},n}$ denotes that the components $A_x(m, n)$ are determined on the links connecting the sites $(m, n), (m+1, n)$. In Eqs.(2),(3) $\Delta_{1,2} f(\mathbf{r}) \equiv f(\mathbf{r} + \mathbf{e}_{1,2}) - f(\mathbf{r})$ is the gradient on the lattice with coordinates $\mathbf{r} = (m, n) \in \mathbb{Z}^2$; \mathbf{e}_i is the unit vector. Green function on the lattice in Eqs.(2),(3) has the form:

$$G(m-m', n-n') = \int_{-\pi}^{\pi} \frac{d^2 k}{(2\pi)^2} \frac{e^{i\{k_x(m-m') + k_y(n-n')\}} - 1}{4 - 2 \cos k_x - 2 \cos k_y}. \quad (4)$$

The main purpose of this paper is to study the dependence of the critical number of particles $N = \sum_{m',n'} \rho_{m',n'}^2$ on the parameter $C = g|k|$ considering the arbitrary large contribution of the temporal component of the gauge potential as well as of its spatial

component. To solve the problem we compute this dependence using the zero modes found for various values of CS coefficient k .

We performed the simulation of the problem (1)–(4) on the lattice with linear size $L \leq 20$ using the method of the stabilizing Petviashvili multiplier [11]. The description of the block diagram of the method and the details of calculations may be found in Refs. [9, 10]. In accordance with the rules of the gauge field theory on the lattice we assumed that while the gauge field is defined on the lattice links, the curl of the field $A_\mu(u)$ and the density ρ^2 are defined on the sites of the dual lattice.

We used the function $\rho_{m,n}$ for the calculation of the dependence $N(C)$ which is shown in Fig. 1. The form of the functions $\rho_{m,n}, A_{m,n}, w_{m,n}$ found numerically is displayed in Ref.[10]. The form of these functions in the present paper is qualitatively the same. Note that the limit $k \rightarrow \infty$ is equivalent to the zero contribution of the gauge fields $A_{m,n}, w_{m,n}$, when in the continuous limit $N_{cr} = 11.703$. This number of particles separates the 2D collapse regime at $N > N_{cr}$ and its absence at $N < N_{cr}$. On the lattice the values N_{cr}^{lat} are always less [12] than the critical number of particles in the continuous limit even if CS fields are neglected. Therefore the main problem of interest is whether $N_{cr}^{lat}(A_\mu \neq 0)$ is smaller or greater than $N_{cr}^{lat}(A_\mu = 0)$ if the CS gauge fields are taken into account.

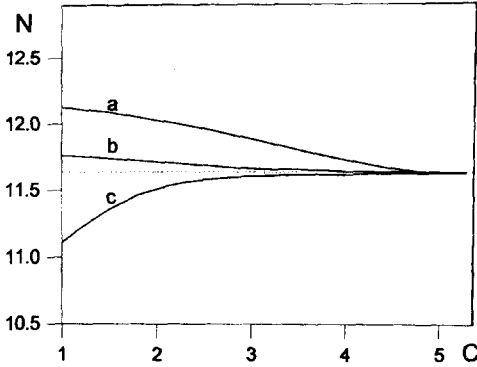


Fig.1. The dependence of the critical number of the particles on the parameter C for the three types of the nonlinearity in Eq.(1) (see for details the text). The dashed line shows the value $N_{cr}^{lat}(A_\mu = 0) = 11.605$

The curve a in Fig.1 shows the result of the paper [10] when we considered only the part $4 - 2 \cos A_x - 2 \cos A_y$ of the contribution to Eq.(1) of the spatial gauge field components and did not consider the discrete time. The case b in Fig.1 corresponds to complete consideration of spatial gauge field contribution to the l.h.s of Eq.(1) with the same properties of the time as it is above. The dependence $N(C)$ in the case c in Fig.1 presents the result of computations on the $(2 + 1)D$ lattice with consideration of the complete contribution of all (arbitrary large) gauge field components A_μ taking into account the discrete time.

From dependence $N(C)$ in the case c in Fig.1 one can infer that the decrease of the critical number of particles N with decreasing of the parameter C is equivalent to the increase of the attraction in comparison with the case when we do not take into account the contribution of the CS fields.

It is known [1, 2] that in CS systems there arises the induced angular momentum proportional to $1/k$. The calculated lines of the equal value of the field $\rho_{m,n}$ for $C = 1$ are shown in Fig.2. Here we would like to pay attention to the d -symmetry of the field $\rho_{m,n}$ for a small value of the parameter $C = 1$ at great distance from the origin. We found weak display of this phenomenon (see Fig.2). The accuracy of the performed calculations

according to our estimates is several percents. The comparison of the results on Fig.2 for the different value of the parameter C shows that the display of d -wave symmetry increases with decrease of parameter C .

The origin of the considered phenomena is as follows. We consider the nonlinear nonlocal dependence of the components $A_\mu(m, n)$ of CS gauge field via the field $\rho_{m, n}$ in its complete form presented in Eq.(1) for the Laplacian. In particular, if we extract [10] a part of this contribution to nonlinearity, specifically taking into account the compact version $\rho(4 - 2 \cos A_x - 2 \cos A_y)$ of the nonlinearity $\rho(A_x^2 + A_y^2)$ [9], the rest part in the discrete Laplacian in the continuous limit has the form

$$\cos A_x \frac{\partial^2}{\partial x^2} + \cos A_y \frac{\partial^2}{\partial y^2}.$$

The decrease of the coefficients in this expression in comparison with the unity for the value of $A_{m, n}$ in the region of small coefficients k leads to the decrease of diffraction. It is seen that the origin of additional attraction due to CS fields is the decrease of the diffraction. Neglecting this effect we observe [10] only the repulsion due to CS fields. The anisotropy of this operator is the reason of d -symmetry of the ground state.

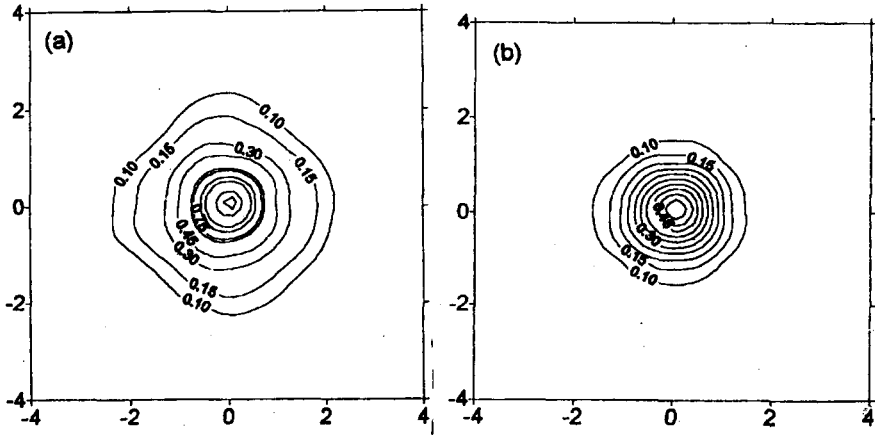


Fig.2. The lines of equal value of the field $\rho_{m, n}$ for $C = 1$ (a) and $C = 5$ (b)

As it was pointed out above the considered phenomenon exists on the lattice under the condition of the complete taking into account the gauge field by the holonomies e^{iA_μ} . The physical interpretation is clear: because the zero component of gauge potential plays the role of chemical potential the arbitrary large value of the one corresponds to arbitrary large value of the energy which we add to the system adding a particle.

Note that discrete models are characterized by the features which are absent in the continuous limit. In particular, the localized states can exist in $(1+1)$ D discrete nonlinear Schrödinger equation [13]. In our case the dimensionality of the problem as well as the discrete character of the space and the time are important. Using the arguments in inverse order we have to include in the model discrete space and time on the equal foot in order to consider the large magnitude of the gauge field in the form of Wilson exponent as well as Polyakov exponent having in mind the gauge invariance. Our simulation shows for example that without the condition of the discretization of the time we can not obtain the attraction due to CS fields.

Finally, we should like to make a general remark. The attraction between particles in the systems with CS interaction was a subject of extensive studies during the last ten years. The attention was focused [14–16] on analyzing the symmetry of the state (s -, p - or d -wave states) with non-zero value of the superconducting gap in the framework of the perturbation theory when the parameter $\alpha = 4\pi/|k|$ was small, i.e. in the limit $|k| \gg 1$. The discovery [17] of the time-reversal symmetry-breaking p -wave superconductivity in Sr_2RuO_4 (see also Refs.[18–20]) stimulated the recent paper [21] where the search for the induced CS term in P - and T -violating superconductors was performed.

The picture of the Chern-Simons correlations, described in the present paper, is in some sense beyond the above-mentioned approaches. We found the attraction due to the CS gauge field in the essentially non-perturbative region of *the small* value of the coefficient k irrespectively of the symmetry of the ground state. In other words, this phenomenon takes place for d -wave as well as for p -wave field configurations. Another distinction from the above-mentioned papers is that the choice of the coefficient k itself determines the symmetry of the considered state. In this respect, Fig.2 presents only one of the possible symmetry of the ground state characterized by the specific value of the parameter C .

Let us suppose in the Eq.(1) that $C = 0$. We obtain in this case the model describing the nonlocal interaction of CS vortices. This case corresponds in continuous limit to universal nonlinearity of ρ^5 kind and differs from Gross – Pitaevskii model [22] when the nonlinearity in the equation of motion is a local one and proportional to ρ^3 . We plan to study this interesting limit in a separate paper.

In conclusion, we studied the dependence of the critical particle number on the link numbers of the field configurations. Using the model of the discrete $(2 + 1)\text{D}$ nonlinear Schrödinger equation we found for the first time the existence of the attraction due to CS fields. It was shown that the origin of this phenomenon is the suppression of the free propagation by CS fields at small link numbers. Note that for $g = 1$ the semion value $k = 2$, which is of topical interest, lies inside this region. Therefore the found attraction may be the dynamical reason of the semion pairing and phase transition to a superconducting state.

We are grateful to S.N.Vlasov, E.A.Kuznetsov, A.G.Litvak, A.M.Satanin, V.I.Talanov and V.E.Zakharov for very interesting discussions and useful advices. The work was supported in part by the RFBR under Grant #98-02-16237.

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