

# CURRENT TENSOR WITH HEAVY PHOTON FOR DOUBLE HARD PHOTON EMISSION BY LONGITUDINALLY POLARIZED ELECTRON

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The electron current tensor for the scattering of a heavy photon on a longitudinally polarized electron with the emission of two hard real photons is considered. The contributions of collinear and semicollinear kinematics are computed. The result allows one to calculate the corresponding contribution to the second-order radiative correction to the deep inelastic scattering or electron-positron annihilation cross sections with next-to-leading-order accuracy.

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1. The recent polarized experiments on deep inelastic scattering (DIS) [1,2] cover the kinematical region  $y \simeq 0.9$ , where the electromagnetic corrections to the cross section are extremely large. The first-order QED correction has been computed in [3,4], and it is of the order of the Born cross section in this region. For this reason the calculation of the second-order QED correction has become very important for interpretation of these experiments. The first step in such calculation was made in [5], where the one-loop-corrected Compton tensor with a heavy photon was considered. That is one of the contributions to the polarized electron current tensor which appear in the second order of perturbation theory. Other contributions arise due to double hard photon emission and pair production.

Here we calculate the contribution to the polarized electron current tensor from the emission of two hard photons. We investigate the double collinear and semicollinear kinematics. This allows us to compute the corresponding second-order radiative correction to various observables with next-to-leading-order accuracy, in the same manner as has been done, for example, for small-angle Bhabha scattering [6], tagged photon cross sections in DIS [7], and electron-positron annihilation [8] in the unpolarized case.

In the Born approximation the electron current tensor for a longitudinally polarized electron has the form

$$L_{\mu\nu}^B = Q_{\mu\nu} + i\lambda E_{\mu\nu}, \quad Q_{\mu\nu} = -4(p_1 p_2) g_{\mu\nu} + 4p_{1\mu} p_{2\nu} + 4p_{1\nu} p_{2\mu}, \quad E_{\mu\nu} = 4\epsilon_{\mu\nu\rho\sigma} p_{1\rho} p_{2\sigma}, \quad (1)$$

where  $p_1$  ( $p_2$ ) is the 4-momentum of the initial (final) electron, and  $\lambda = 1(-1)$  if the initial electron is polarized along (against) its 3-momentum direction.

In the case of single collinear photon emission the corresponding contribution to the electron current tensor conserves the Born structure for radiation along the scattered electron momentum direction,

$$L_{\mu\nu}^{(1)f} = \frac{\alpha}{2\pi} \left[ \frac{1 + (1+y)^2}{y} \tilde{L}_0 - \frac{2(1+y)}{y} \right] dy L_{\mu\nu}^B, \quad y = \frac{\omega}{\varepsilon_2}, \quad \tilde{L}_0 = \ln \frac{\varepsilon_2^2 \theta_0^2}{m^2}, \quad (2)$$

and acquires an additional part (which is proportional to  $i\lambda E_{\mu\nu}$ ) for radiation along the initial electron momentum direction [9];

$$L_{\mu\nu}^{(1)i} = \frac{\alpha}{2\pi} \left\{ \left[ \frac{1 + (1-x)^2}{x} L_0 - \frac{2(1-x)}{x} \right] L_{\mu\nu}^B - 2xi\lambda E_{\mu\nu} \right\} dx, \quad x = \frac{\omega}{\varepsilon_1},$$

$$L_0 = \ln \frac{\varepsilon_1^2 \theta_0^2}{m^2}. \quad (3)$$

In Eqs. (2) and (3)  $\omega$  is the photon energy,  $\varepsilon_1(\varepsilon_2)$  is the energy of the initial (final) electron,  $m$  is the electron mass, and the parameter  $\theta_0$  defines the angular phase space of the hard collinear photon. The index  $i(f)$  labels the initial (final) electron state.

Looking at Eq. (3), we see that the additional part does not contribute in the leading logarithmic approximation and does not have infrared divergence. In other words, the Born structure of the electron current tensor in the case of a longitudinally polarized electron is disturbed only in the next-to-leading approximation due to radiation by the initial polarized electron itself.

In general, the contribution to the current tensor  $L_{\mu\nu}$  due to the emission of  $n$  collinear photons can be written as follows:

$$L_{\mu\nu}^{(n)} = \left( \frac{\alpha}{2\pi^2} \right)^n [I^{(n)} L_{\mu\nu}^B + K^{(n)} i\lambda E_{\mu\nu}] \prod_{i=1}^n \frac{d^3 k_i}{\omega_i}, \quad (4)$$

where the quantity  $K^{(n)}$  equals zero if (and only if) all  $n$  collinear photons are emitted along the scattered (unpolarized) electron momentum direction. The first term in the right-hand side of Eq. (4) was obtained in [10] with next-to-leading-order accuracy. Our goal is to find the second term with the same accuracy.

2. We use the covariant method of calculation and start from the general expression for the polarized current tensor which arises due the emission of two hard photons,

$$L_{\mu\nu}^{(2)} = \left( \frac{\alpha}{4\pi^2} \right)^2 \frac{d^3 k_1 d^3 k_2}{\omega_1 \omega_2} Sp(\hat{p}_2 + m) Q_{\mu}^{\lambda\rho} (\hat{p}_1 + m) (1 - \gamma_5 \hat{P}) (Q_{\nu}^{\lambda\rho})^+, \quad (5)$$

where  $P$  is the polarization 4-vector of initial electron. The quantity  $Q_{\mu}^{\lambda\rho}$  reads

$$Q_{\mu}^{\lambda\rho} = \gamma_{\mu} \frac{\hat{\Delta} + m}{\Delta^2 - m^2} \gamma_{\rho} \frac{\hat{p}_1 - \hat{k}_1 + m}{-2p_1 k_1} \gamma_{\lambda} + \gamma_{\rho} \frac{\hat{p}_2 + \hat{k}_2 + m}{2p_2 k_2} \gamma_{\mu} \frac{\hat{p}_1 - \hat{k}_1 + m}{-2p_1 k_1} \gamma_{\lambda} +$$

$$+ \gamma_{\rho} \frac{\hat{p}_2 + \hat{k}_2 + m}{2p_2 k_2} \gamma_{\lambda} \frac{\hat{\Sigma} + m}{\Sigma^2 - m^2} \gamma_{\mu} + (1 \leftrightarrow 2), \quad \Delta = p_1 - k_1 - k_2, \quad \Sigma = p_2 + k_1 + k_2. \quad (6)$$

For the important case of a longitudinally polarized electron the polarization vector can be written, in the framework of the adopted accuracy, as

$$P = \frac{\lambda}{m} \left( p_1 - \frac{m^2 k}{p_1 k} \right), \quad (7)$$

where  $\lambda$  is the doubled electron helicity, and the 4-vector  $k$  has components  $(\varepsilon_1, -\mathbf{p}_1)$ ,  $k^2 = m^2$ . It is easy to see that

$$P^2 = -1 + O(m^4/\varepsilon^4), \quad P p_1 = 0.$$

Note that for calculations in the leading approximation we can neglect the second term in the right-hand side of Eq. (7), as was done in [5].

There are four collinear regions in the case of double photon emission:  $(\mathbf{k}_1, \mathbf{k}_2 \parallel \mathbf{p}_1)$ ;  $(\mathbf{k}_1, \mathbf{k}_2 \parallel \mathbf{p}_2)$ ;  $(\mathbf{k}_1 \parallel \mathbf{p}_1, \mathbf{k}_2 \parallel \mathbf{p}_2)$  and  $(\mathbf{k}_1 \parallel \mathbf{p}_2, \mathbf{k}_2 \parallel \mathbf{p}_1)$ . A straightforward calculation in the region  $(\mathbf{k}_1, \mathbf{k}_2 \parallel \mathbf{p}_1)$ , when both hard collinear photons are emitted by the initial-state polarized electron, gives

$$\begin{aligned} \frac{m^4}{4} I_{ii}^{(2)} &= \frac{1+y^2}{2x_1x_2\eta_1\eta_2} + \frac{1}{d\eta_1} \left[ -(1-x_2) + 2y \left(1 - \frac{x_1}{x_2}\right) + \frac{1-x_1}{x_1x_2} ((1-y)(x_1-x_2) - 2y) \right] - \\ &- \frac{y\eta_2}{d^2\eta_1} + \frac{2}{d\eta_1^2} \left( x_2 + \frac{2y(1-x_1)}{x_2} \right) - \frac{4y}{d^2\eta_1} + \frac{(1-y)(2y+x_1x_2)}{x_1x_2d\eta_1\eta_2} + \frac{4y}{d^2\eta_1} \left( \frac{1}{\eta_1} + \frac{1}{\eta_2} \right) + (1 \leftrightarrow 2), \end{aligned} \quad (8)$$

$$\begin{aligned} \frac{m^4}{4} K_{ii}^{(2)} &= \frac{2}{d\eta_1^2} \left( 1 - x_2 - x_1x_2 + \frac{2yx_1^2}{x_2} \right) + \frac{1}{d\eta_1\eta_2} \left( 3 - 3y + 2y^2 + \frac{4x_2^2}{x_1} \right) + \\ &+ \frac{2}{d^2\eta_1} (3y - x_1^2 - x_2^2) + \frac{2y\eta_2}{d^2\eta_1^2} + \frac{4}{d^2\eta_1} \left( \frac{1}{\eta_1} + \frac{1}{\eta_2} \right) [(1-y)^2 - x_1x_2] + (1 \leftrightarrow 2), \quad y = 1 - x_1 - x_2. \end{aligned} \quad (9)$$

In writing Eqs. (8) and (9) we used the following notation:

$$2p_1k_{1,2} = m^2\eta_{1,2}, \quad \Delta^2 - m^2 = m^2d, \quad x_{1,2} = \omega_{1,2}/\varepsilon_1.$$

In the region  $(\mathbf{k}_1, \mathbf{k}_2 \parallel \mathbf{p}_2)$ , when both hard collinear photons are emitted by the final-state unpolarized electron, we have

$$K_{ff}^{(2)} = 0, \quad I_{ff}^{(2)} = I_{ii}^{(2)}(x_{1,2} \rightarrow -y_{1,2}, \quad \eta_{1,2} \rightarrow -\sigma_{1,2}, \quad d \rightarrow \sigma, \quad \rightarrow \eta = 1 + y_1 + y_2), \quad (10)$$

where

$$y_{1,2} = \omega_{1,2}/\varepsilon_2, \quad 2p_2k_{1,2} = m^2\sigma_{1,2}, \quad \Sigma^2 - m^2 = m^2\sigma.$$

In accordance with the quasireal electron method [9] we can express the electron current tensor in the region  $(\mathbf{k}_1 \parallel \mathbf{p}_1, \mathbf{k}_2 \parallel \mathbf{p}_2)$  as a product of the probability of radiation of a collinear photon with energy  $\omega_2$  by the scattered electron (which is the coefficient at  $L_{\mu\nu}^B$  in the right-hand side of Eq. (2) with  $y = y_2$ ) and the electron current tensor due to single photon emission by the initial electron, as given by Eq. (3) with  $x = x_1$ . Therefore the contribution of the regions  $(\mathbf{k}_1 \parallel \mathbf{p}_1, \mathbf{k}_2 \parallel \mathbf{p}_2)$  and  $(\mathbf{k}_2 \parallel \mathbf{p}_1, \mathbf{k}_1 \parallel \mathbf{p}_2)$  reads

$$\begin{aligned} L_{\mu\nu}^{(2)if} &= \left( \frac{\alpha}{2\pi} \right)^2 \left[ \frac{1 + (1+y_2)^2}{y_2} \tilde{L}_0 - \frac{2(1+y_2)}{y_2} \right] \times \\ &\times \left\{ \left[ \frac{1 + (1-x_1)^2}{x_1} L_0 - \frac{2(1-x_1)}{x_1} \right] L_{\mu\nu}^B - 2x_1 i \lambda E_{\mu\nu} \right\} dy_2 dx_1 + (1 \leftrightarrow 2). \end{aligned} \quad (11)$$

In order to derive the corresponding contributions in the regions  $(\mathbf{k}_1, \mathbf{k}_2 \parallel \mathbf{p}_1)$  and  $(\mathbf{k}_1, \mathbf{k}_2 \parallel \mathbf{p}_2)$  we have to perform the angular integration in Eq. (4) using Eqs. (8) and (9). Moreover, we can also integrate over the energy fraction  $x_1$  ( $y_1$ ) in the region  $(\mathbf{k}_1, \mathbf{k}_2 \parallel \mathbf{p}_1)$  ( $(\mathbf{k}_1, \mathbf{k}_2 \parallel \mathbf{p}_2)$ ) at a fixed value of the quantity  $x_1 + x_2 = 1 - y$  ( $y_1 + y_2 = \eta - 1$ ), because the 4-momentum of the heavy photon, which interacts with the hadronic part of the amplitude, depends on  $1 - y(\eta - 1)$  in this case.

The expressions (8) and (9) for  $I_{ii}^{(2)}$  and  $K_{ii}^{(2)}$  are suitable for calculations with a power-law accuracy (up to terms of order  $m^2/\varepsilon_1^2$ ). But here we restrict ourselves to logarithmic accuracy and can therefore omit terms proportional to  $1/d\eta_1\eta_2$ ,  $1/d^2\eta_1$ ,  $1/d^2\eta_1^2$ , and  $1/d^2\eta_1\eta_2$  in the right-hand sides of Eqs. (8) and (9). In this approximation the integration of  $I_{ii}^{(2)}$  leads to (see [10])

$$\int \frac{d^3 k_1 d^3 k_2}{\omega_1 \omega_2} \frac{I_{ii}^{(2)}}{m^4} = \pi^2 \left[ \frac{1}{2} L_0^2 A(y, \delta) + L_0 B(y, \delta) \right] dy, \quad (12)$$

$$A = 4 \frac{1+y^2}{1-y} \ln \frac{1-y-\delta}{\delta} + (1+y) \ln y - 2(1-y), \quad (13)$$

$$B = 3(1-y) + \frac{3+y^2}{2(1-y)} \ln^2 y - \frac{2(1+y)^2}{1-y} \ln \frac{1-y-\delta}{\delta}, \quad (14)$$

where  $\delta \ll 1$  is the infrared cutoff for the energy fraction of each photon. Analogously, the integration of  $K_{ii}^{(2)}$  reads

$$\int \frac{d^3 k_1 d^3 k_2}{\omega_1 \omega_2} \frac{K_{ii}^{(2)}}{m^4} = \pi^2 L_0 C(y, \delta) dy, \quad C = 2(1-y) \left[ 2 - \ln y - 2 \ln \frac{1-y}{\delta} \right] dy. \quad (15)$$

Using Eqs. (12) and (15) together with Eq. (4), we obtain

$$L_{\mu\nu}^{(2)ii} = \left( \frac{\alpha}{2\pi} \right)^2 \left[ \left( \frac{1}{2} L_0^2 A(y, \delta) + L_0 B(y, \delta) \right) L_{\mu\nu}^B + C(y, \delta) L_0 i \lambda E_{\mu\nu} \right] dy \quad (16)$$

for the contribution of the region ( $\mathbf{k}_1, \mathbf{k}_2 \parallel \bar{\mathbf{p}}_1$ ) to the current tensor of the longitudinally polarized electron. In some applications the quantity  $y$  remains fixed (for example, for calculation of the tagged photon cross sections). In this case we can write  $\ln((1-y)/\delta)$  instead of  $\ln((1-y-\delta)/\delta)$  in the expressions for  $A$  and  $B$ .

The corresponding contribution of the region ( $\mathbf{k}_1, \mathbf{k}_2 \parallel \mathbf{p}_2$ ) can be written as follows:

$$L_{\mu\nu}^{(2)ff} = \left( \frac{\alpha}{2\pi} \right)^2 \left[ \frac{1}{2} \tilde{L}_0^2 \tilde{A}(\eta, \delta') + \tilde{L}_0 \tilde{B}(\eta, \delta') \right] L_{\mu\nu}^B d\eta, \quad \delta' = \frac{\delta \varepsilon_1}{\varepsilon_2}, \quad (17)$$

where

$$\tilde{A} = 4 \frac{1+\eta^2}{\eta-1} \ln \frac{\eta-1-\delta'}{\delta'} - (1+\eta) \ln \eta - 2(\eta-1), \quad (18)$$

$$\tilde{B} = 3(\eta-1) + \frac{3+\eta^2}{2(\eta-1)} \ln^2 \eta - 2 \frac{(1+\eta)^2}{\eta-1} \ln \frac{\eta-1-\delta'}{\delta'}. \quad (19)$$

Note that the quantities  $\tilde{A}$  and  $\tilde{B}$  can be reconstructed from the quantities  $A$  and  $B$  by the rule

$$\tilde{A}(\eta, \delta') = -A(\eta, -\delta'), \quad \tilde{B}(\eta, \delta') = -B(\eta, -\delta').$$

As we saw above (Eq. (3)), the additional part to the Born structure of the polarized electron current tensor due to single collinear photon emission has neither collinear (does not contain a large logarithm) nor infrared (is finite in the limit  $x \rightarrow 0$ ) singularities. These singularities do appear, however, in the corresponding contribution due to double collinear photon emission (Eqs. (11) and (16)). Nevertheless, the additional part never contributes in the leading approximation.

The infrared parameter  $\delta$  must cancel out in any physical application if the photons are unobserved. Such cancellation takes place because of contributions due to double virtual and soft photon emission as well as virtual and soft corrections to the single hard photon emission. The last contributions have been considered recently [5] within the approximation  $m^2 = 0$ , which describes large-angle photon radiation. If we put  $m^2 = 0$  in our calculations, then we will be left with only the Born-like structure in Eqs. (3), (11), and (16). Moreover, the quantities  $B$  and  $\bar{B}$  in Eqs. (16) and (17) will be changed in this approximation. Consequently, we see that the electron mass must be kept finite in order to be correct in the next-to-leading approximation in any physical application with unobserved photons (for example, in classical DIS). We conclude, therefore, that the results of Ref. [5] need to be improved for applications of this kind.

On the other hand, the case of the loop-corrected large-angle single photon radiation is described in [5] with a very high accuracy (only terms of order  $m^2/\varepsilon_1^2$  are neglected). In order to reach adequate accuracy for double hard photon emission one needs to take into account the contributions of semicollinear (see paragraph 3 below) and noncollinear kinematics. The last case, when both hard photons are radiated at large angles with respect to the directions of the initial and scattered electron momenta, will be considered by us in another publication.

3. Let us consider double hard photon emission in the semicollinear regions where  $\mathbf{k}_1 \parallel \mathbf{p}_1$  or  $\mathbf{p}_2$ , and  $\mathbf{k}_2$  is arbitrary. In this situation we can use the quasireal electron method for a longitudinally polarized initial electron [9]. In accordance with this method the contribution of the region  $\mathbf{k}_1 \parallel \mathbf{p}_2$  to the electron current tensor is defined by its Born-like structure  $L_{\mu\nu}^\gamma$  as follows:

$$L_{\mu\nu}(\mathbf{k}_1 \parallel \mathbf{p}_2) = \frac{\alpha^2}{8\pi^3} \frac{d^3 k_2}{\omega_2} \frac{dy_1}{1+y_1} \left[ \frac{1+(1+y_1)^2}{y_1} \tilde{L}_0 - \frac{2(1+y_1)}{y_1} \right] L_{\mu\nu}^\gamma(p_1, p_2(1+y_1), k_2), \quad (20)$$

where for the large-angle emission tensor  $L_{\mu\nu}^\gamma$  we can use the approximation  $m^2 = 0$  [3, 5, 11]

$$\begin{aligned} L_{\mu\nu}^\gamma(p_1, p_2, k_2) &= 4(B_{\mu\nu} + i\lambda E_{\mu\nu}^\gamma) \\ B_{\mu\nu} &= \frac{1}{st} [(s+u)^2 + (t+u)^2] \tilde{g}_{\mu\nu} + \frac{4q^2}{st} (\tilde{p}_{1\mu} \tilde{p}_{2\nu} + \tilde{p}_{1\nu} p_{2\mu}) \\ E_{\mu\nu}^\gamma &= \frac{2\varepsilon_{\mu\nu\rho\sigma}}{st} [(u+t)p_{1\rho} q_\sigma + (u+s)p_{2\rho} q_\sigma], \quad \tilde{g}_{\mu\nu} = g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}, \quad (21) \\ \tilde{p}_\mu &= p - \frac{(pq)q_\mu}{q^2}, \quad u = -2p_1 p_2, \quad s = 2p_2 k_2, \quad t = -2p_1 k_2, \quad q = p_2 + k_2 - p_1. \end{aligned}$$

As above, the emission of a collinear photon by the initial electron disturbs the Born structure of the electron current tensor in just the same manner as in Eq. (11):

$$\begin{aligned} L_{\mu\nu}(\mathbf{k}_1 \parallel \mathbf{p}_1) &= \frac{\alpha^2}{8\pi^3} \frac{d^3 k_2}{\omega_2} \frac{dx_1}{1-x_1} \left\{ \left[ \frac{1+(1-x_1)^2}{x_1} L_0 - \frac{2(1-x_1)}{x_1} \right] \times \right. \\ &\quad \left. \times L_{\mu\nu}^\gamma(p_1(1-x_1), p_2, k_2) - 2x_1 i\lambda E_{\mu\nu}^\gamma(p_1(1-x_1), p_2, k_2) \right\} \quad (22) \end{aligned}$$

Formulas (20) and (22) have been derived by us independently in the quasireal electron method, starting from the general expression for the current tensor as given by Eqs. (5), (6), and (7).

When calculating the radiative corrections to the polarized DIS cross section we have to integrate over all of the phase space of photons. In this case the angular cutoff parameter  $\theta_0$  is unphysical and must vanish in the sum of contributions of the double collinear and semicollinear regions. At the adopted accuracy this implies the cancellation of terms of the type  $L_0 \ln \theta_0^2$ , and that can be verified by separation of  $\ln \theta_0^2$  in the integration of  $L_{\mu\nu}^\gamma(p_1(1-x_1), p_2, k_2)$  in the limit  $\mathbf{k}_2 \parallel \mathbf{p}_1$ :

$$\int \frac{d^3 k_2}{\omega_2} L_{\mu\nu}^\gamma(p_1(1-x_1), p_2, k_2 \approx x_2 p_1) = -2\pi \ln \theta_0^2 dx_2 \frac{y^2 + (1-x_1)^2}{(1-x_1)x_2} L_{\mu\nu}^B. \quad (23)$$

Taking into account that at fixed  $x_1 + x_2 = 1 - y$

$$\int dx_1 dx_2 \frac{[1 + (1-x_1)^2][y^2 + (1-x_1)^2]}{x_1 x_2 (1-x_1)^2} = A(y, \delta) dy, \quad (24)$$

we are convinced that the terms of the type  $L_0 \ln \theta_0^2$  indeed vanish in the sum of contributions due to the collinear and semicollinear kinematics. Of course, an analogous cancellation takes place for the radiation in the final state.

We note in conclusion that the electron current tensor has an universal character. It can be used for calculation of cross sections in different processes, including the most interesting DIS and  $e^+e^-$  annihilation into hadrons. To obtain the corresponding cross sections we have to multiply the electron current tensor by the hadron one. The hadron tensor itself carries important information about the hadronic structure and fragmentation functions [12], and the study of the radiative corrections to the electron current tensor is necessary for interpretation of experimental data in terms of these hadronic functions.

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